

MHD Boundary Layer Flow of Heat and Mass Transfer over a Moving Vertical Plate in a Porous Medium with Suction and Viscous Dissipation

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ABSTRACT

The present paper analyzes the MHD effects on the steady free convective boundary layer flow of a viscous incompressible fluid over a linearly moving porous vertical semi-infinite plate with suction and viscous dissipation. The governing boundary layer equations are reduced to a two-point boundary value problem using similarity transformations. The resultant problem is solved numerically using the Runge - Kutta fourth order method along with shooting technique. The effects of various governing parameters on the fluid velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are shown in figures and tables and discussed in detail.

Keywords- Heat and Mass Transfer, MHD, Moving vertical plate Porous media, Viscous Dissipation

1. INTRODUCTION

Combined heat and mass transfer (or double-diffusion) in fluid-saturated porous media finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering, moisture migration in a fibrous insulation and nuclear waste disposal and others. Double diffusive flow is driven by buoyancy due to temperature and concentration gradients. Bejan and Khair [1] investigated the free convection boundary layer flow in a porous medium owing to combined heat and mass transfer. Lai and Kulacki [2] used the series expansion method to investigate coupled heat and mass transfer in natural convection from a sphere in a porous medium. The suction and blowing effects on free convection coupled heat and mass transfer over a vertical plate in a saturated porous medium were studied by Raptis et al. [3] and Lai and Kulacki [4], respectively.

Magnetohydrodynamic flows have applications in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. In addition from the technological point of view, MHD free convection flows have significant applications in the field of stellar and planetary magnetospheres, aeronautical plasma flows, chemical engineering and electronics.

An excellent summary of applications is given by Huges and Young [5]. Raptis [6] studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Helmy [7] analyzed MHD unsteady free convection flow past a vertical porous plate embedded in a porous medium. Elabashbeshy [8] studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chamkha and Khaled [9] investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption.

Transport of momentum and thermal energy in fluid-saturated porous media with low porosities is commonly described by Darcy's model for conservation of momentum and by the energy equation based on the velocity field found from this model (Kaviany [10]). In contrast to rocks, soil, sand and other media that do fall in this category, certain porous materials, such as foam metals and fibrous media, usually have high porosity. Vajravelu [11] examined the steady flow and heat transfer in a porous medium with high porosity. Hong et al. [12], Chen and Lin [13] and Jaisawal and Soundalgekar [14] studied the natural convection in a porous medium with high porosity. Ibrahim *et al* [15] analytically derived the heat and mass transfer of a chemical convective process assuming an exponentially decreasing suction velocity at the surface of a porous plate and a two terms harmonic function for the rest of the variables. Ali [16] discussed the effect of suction or injection on the free convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with an exponential decaying heat generation.

Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Mahajan and Gebhart [17] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Gebhart and Mollendorf [18] considered the effects of viscous dissipation for

external natural convection flow over a surface. Soundalgekar [19] analyzed viscous dissipative heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Israel Cooney et al. [20] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Ramachandra Prasad and Bhaskar Reddy [21] presented radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate with viscous dissipation.

In this chapter an attempt is made to study the MHD effects on the steady free convective boundary layer flow of a viscous incompressible fluid over a linearly started porous vertical semi-infinite plate with suction and viscous dissipation. The basic equations governing the flow field are partial differential equations and these have been reduced to a set of ordinary differential equations by applying suitable similarity transformations. The resultant equations are coupled and non-linear, and hence are solved numerically using the fourth order Runge - Kutta method along with shooting technique. The effects of various governing parameters on the velocity, temperature, concentration are presented graphically and discussed quantitatively. The local skin-friction coefficient and the heat and mass transfer rates are illustrated for representative values of the major parameters.

To the best of authors' knowledge, MHD effects on a free convective flow over a moving vertical plate with a viscous dissipation have not been reported. This motivated us to carry out the present work.

2. MATHEMATICAL ANALYSIS

A steady two-dimensional laminar free convection flow of a viscous incompressible electrically conducting fluid along a linearly started porous vertical semi-infinite plate embedded in a porous medium, in the presence of suction and viscous dissipation, is considered. The x-axis is taken along the plate in the upward direction and the y-axis is taken normal to it. The velocity of the fluid far away from the plate surface is assumed to be zero for a quiescent state fluid. The variations of surface temperature and concentration are linear. A uniform magnetic field is applied in the direction perpendicular to the plate. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field, Hall effects and Joule heating are negligible. Now, under the usual Boussinesq's approximation, the governing boundary layer equations of the flow field are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + g \beta_T (T - T_\infty) + g \beta_c (C - C_\infty) - \frac{\nu}{K} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} \quad (4)$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u = Bx, v = V, T = T_w = T_\infty + ax, C = C_w = C_\infty + bx \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (5)$$

where B is a constant, "a" and "b" denotes the stratification rate of the gradient of ambient temperature and concentration profiles, u, v, T and C are the fluid x -component of velocity, y -component of velocity, temperature and concentration respectively, ν is the fluid kinematics viscosity, ρ - the density, σ - the electric conductivity of the fluid, β_T and β_c - the coefficients of thermal and concentration expansions respectively, k - the thermal conductivity, C_∞ - the free stream concentration, B_0 - the magnetic induction, D_m - the mass diffusivity and g is the gravitational acceleration, K - the permeability of the porous medium and α - the thermal diffusivity.

To transform equations (2) - (4) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$\eta = y \sqrt{\frac{B}{\nu}}, \quad \psi = x \sqrt{\nu B} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad M = \frac{\sigma B_0^2}{\rho B}, \quad Gt = \frac{g \beta_T (T_w - T_\infty)}{xB^2},$$

$$Gc = \frac{g \beta_c (C_w - C_\infty)}{xB^2}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D_m},$$

$$K = \frac{\nu}{K' x B}, \quad Ec = \frac{B^2 x^2}{c_p (T_w - T_\infty)} \quad (6)$$

where $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ - the dimensionless temperature, $\phi(\eta)$ - the dimensionless concentration, η - the similarity variable, M - the Magnetic parameter, Gt - the local thermal Grashof number, Gc - the local solutal Grashof number, Pr - the Prandtl number, Sc - the Schmidt number, K - the permeability parameter and Ec - the Eckert number.

The mass concentration equation (1) is satisfied by the Cauchy-Riemann equations

$$u = \frac{\partial \psi}{\partial y}, \quad v = - \frac{\partial \psi}{\partial x} \quad (7)$$

where $F_w = \frac{V}{\sqrt{B\nu}}$ is the dimensionless suction velocity.

In view of equations (6) and (7), equations (2) to (4) transform into

$$f''' + ff'' - f'(f' + M) + Gt\theta + Gc\phi - Kf' = 0 \quad (8)$$

$$\theta'' + Pr f\theta' - Pr f'\theta + Ec f''^2 = 0 \quad (9)$$

$$\phi'' + Sc f\phi' - Sc f'\phi = 0 \quad (10)$$

The corresponding boundary conditions are

$$f' = 1, f = -F_w, \theta = 1, \phi = 1 \quad \text{at} \quad \eta = 0$$

$$f' = 0, \theta = \phi = 0 \quad \text{as} \quad \eta \rightarrow \infty \quad (11)$$

where the prime symbol represents the derivative with respect to η .

Other physical quantities of interest for the problem of this type are the local skin-friction coefficient, the local Nusselt number and Sherwood number which are, respectively, proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$.

3. SOLUTION OF THE PROBLEM

The set of coupled non-linear governing boundary layer equations (8) - (10) together with the boundary conditions (11) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (8)-(10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al.* [22]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta = 0.05$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

4. RESULTS AND DISCUSSION

In order to get a physical insight of the problem, a representative set of numerical results are shown graphically in Figs.1-20, to illustrate the influence of physical parameters viz., the suction parameter F_w , magnetic parameter M , permeability parameter K , buoyancy parameters (Gt , Gc), Eckert number Ec and Schmidt number Sc on the velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$. Numerical results for the skin-friction, Nusselt number and Sherwood number are reported in the Tables 1 and 2. The Prandtl number is taken to

be $Pr = 0.72$ which corresponds to air, the value of Schmidt number (Sc) are chosen to be $Sc = 0.24, 0.62, 0.78, 2.62$, representing diffusing chemical species of most common interest in air like H_2 , H_2O , NH_3 and Propyl Benzene respectively.

Fig. 1 shows the dimensionless velocity profiles for different values of magnetic parameter M . It is seen that, as expected, the velocity decreases with an increase in the magnetic parameter M . The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. Fig.2 illustrates the effect of the thermal Grashof number (Gt) on the velocity field. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. The flow is accelerated due to the enhancement in buoyancy force corresponding to an increase in the thermal Grashof number i.e. free convection effects. It is noticed that the thermal Grashof number (Gt) influences the velocity within the boundary layer when compared to far away from the plate. It is seen that as the thermal Grashof number (Gt) increases, the velocity increases.

The effect of mass (solutal) Grashof number (Gc) on the velocity is illustrated in Fig.3. The mass (solutal) Grashof number (Gc) defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with increasing values of the solutal Grashof number. Further as the mass Grashof number (Gc) increases, the velocity field near the boundary layer increases. It is noticed that, for higher values of mass Grashof number (Gc), the profiles are found to be more parabolic. Fig.4 illustrates the effect of the Schmidt number (Sc) on the velocity. The Schmidt number (Sc) embodies the ratio of the momentum diffusivity to the mass (species) diffusivity. It physically relates the relative thickness of the hydrodynamic boundary layer and mass-transfer (concentration) boundary layer. It is observed that as the Schmidt number (Sc) increases, the velocity decreases. Fig.5 illustrates the effect of permeability parameter (K) on the velocity. It is noticed that as the permeability parameter increases, the velocity decreases. As seen in the earlier cases, far away from the plate, the effect is not that much significant.

The effect of the magnetic parameter M on the temperature is illustrated in Fig.7. It is observed that as the magnetic parameter M increases, the temperature increases. From Figs. 8 and 9, it is observed that the thermal boundary layer thickness increases with an increase in thermal or Solutal Grashof number (Gt or Gc). Fig. 10 illustrates the effect of suction parameter (F_w) on the temperature. It is noticed that as the suction parameter increases an

increasing trend in the temperature field is noticed. Fig.11 depicts the effect of the Schmidt number (Sc) on the temperature. It is observed that as the Schmidt number (Sc) increases, the temperature increases. Fig. 12 shows the variation of the thermal boundary-layer with the permeability parameter (K). It is noticed that the thermal boundary layer thickness increases with an increase in the permeability parameter. Fig. 13 shows the variation of the thermal boundary-layer with the Eckert number (Ec). It is observed that the thermal boundary layer thickness increases with an increase in the Eckert number (Ec).

The effect of magnetic parameter M on the concentration field is illustrated Fig.14. As the magnetic parameter M increases, the concentration is found to be increasing. The effect of buoyancy parameters (Gt , Gc) on the concentration field is illustrated Figs. 15 and 16. It is noticed that the concentration boundary layer thickness decreases with an increase in the thermal or Solutal Grashof numbers (Gt or Gc). Fig. 17 illustrates the effect of suction parameter (F_w) on the concentration. As the suction parameter increases, an increasing trend in the concentration field is noticed. The effect of Schmidt number (Sc) on the concentration is illustrated in Fig.18. As expected, as the Schmidt number (Sc) increases, the concentration decreases. The influence of the permeability parameter (K) on the concentration field is shown in Fig.19. It is noticed that the concentration increases monotonically with the increase of the permeability parameter. The influence of the Eckert number Ec on the concentration field is shown in Fig.20. It is noticed that the concentration decreases monotonically with the increase of the Eckert number Ec .

In order to benchmark our numerical results, the present results for the Skin-friction coefficient, Nusselt number, Sherwood number in the absence of both permeability parameter and Eckert number for various values of M , Gt , Gc , F_w and Sc are compared with those of Ibrahim and Makinde [23] and found them in excellent agreement as demonstrated in Table 1. From Table 2, it is found that the local skin friction together with the local heat and mass transfer rates at the moving plate increases with increasing intensity of buoyancy forces (Gt or Gc) or the Schmidt number (Sc), However, an increase in the magnetic field (M) or magnitude of fluid suction (F_w), permeability parameter (K) or Eckert number (Ec) causes a decrease in both the skin friction and surface heat transfer rates and an increase in the surface mass transfer rate.

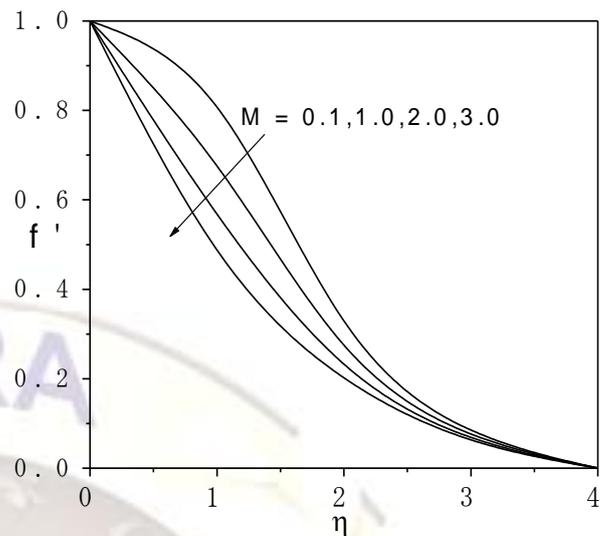


Fig.1: Variation of the velocity f' with M for $Pr=0.72$, $Sc=0.62$, $Gt=Gc=2$, $F_w=0.1$, $K=0.1$, $Ec=0.1$.

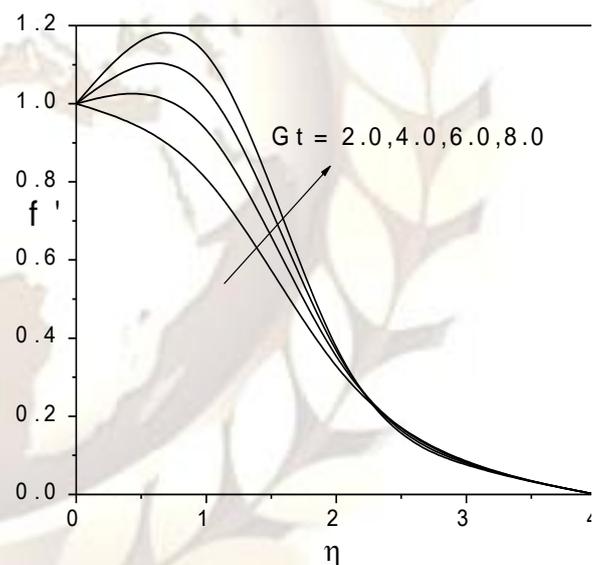


Fig.2: Variation of the velocity f' with Gt for $Pr=0.72$, $Sc=0.62$, $Gc=2$, $Ec=0.1$, $M=K=0.1$.

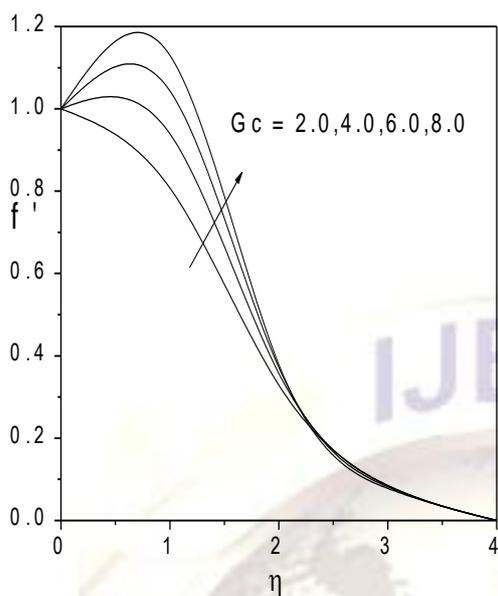


Fig.3: Variation of the velocity f' with Gc for $Pr=0.72, Sc=0.62, Gt=2, Ec=0.1, F_w=M=K=0.1$.

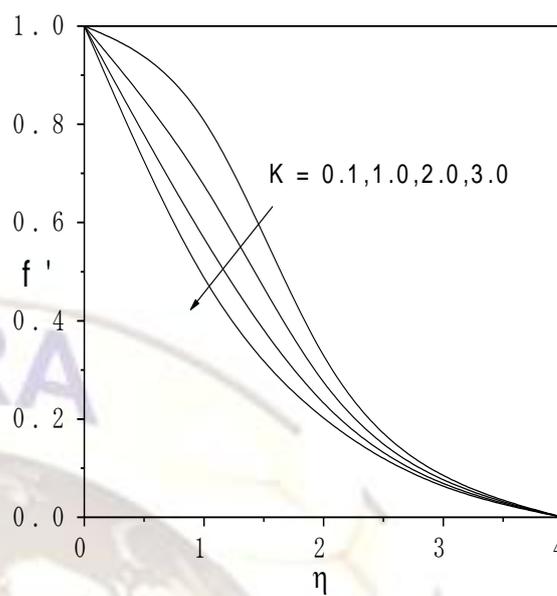


Fig.5: Variation of the velocity f' with K for $Pr=0.72, Sc=0.62, Gt=Gc=2, Ec=0.1, F_w=M=0.1$.

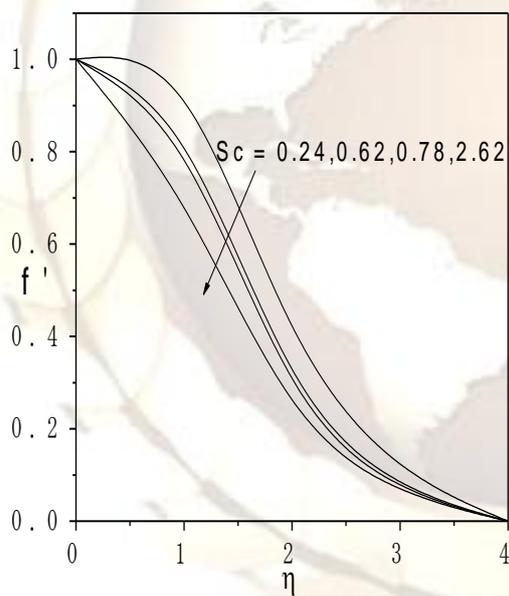


Fig.4: Variation of the velocity f' with Sc for $Pr=0.72, Gt=Gc=2, Ec=0.1, F_w=M=K=0.1$.

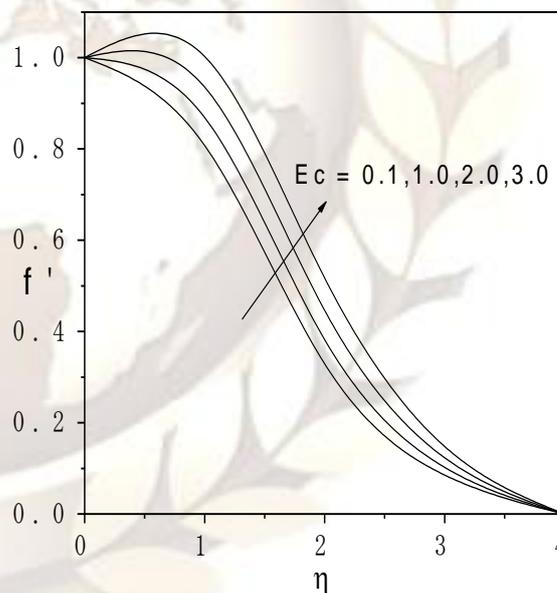


Fig.6: Variation of the velocity f' with Ec for $Pr=0.72, Sc=0.62, Gt=Gc=2, K=F_w=M=0.1$.

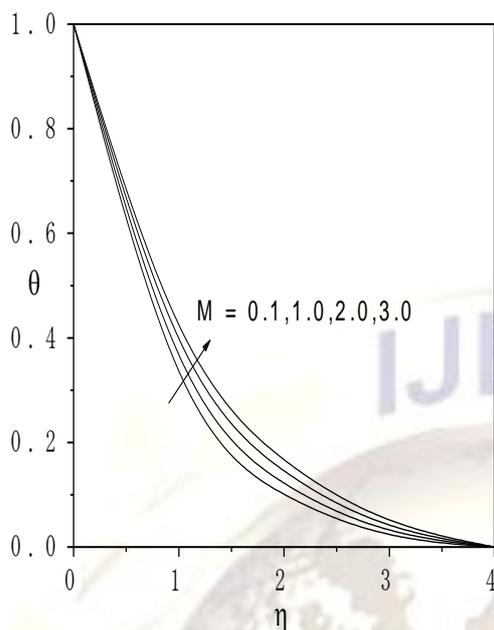


Fig.7: Variation of the temperature θ with M for $Pr=0.72$, $Sc=0.62$, $Gt = Gc = 2$, $Ec=0.1$, $F_w = K=0.1$.

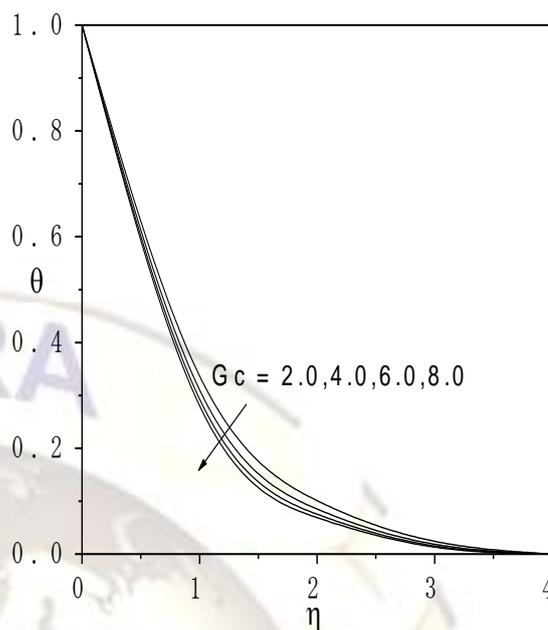


Fig.9: Variation of the temperature θ with Gc for $Pr=0.72$, $Sc=0.62$, $Gt = 2$, $Ec = 0.1$, $F_w = M=K=0.1$.

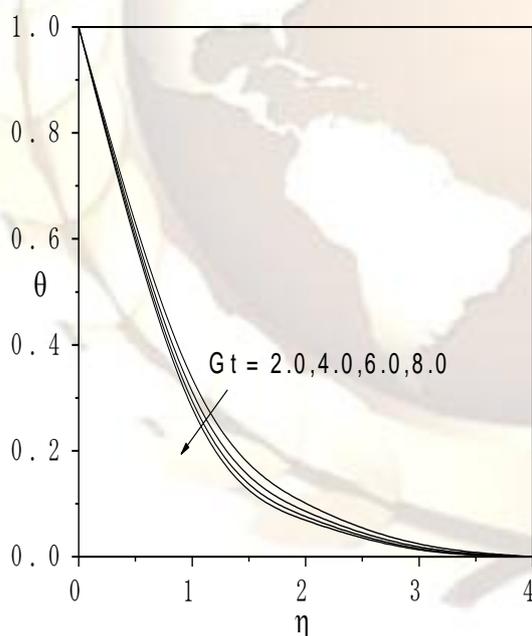


Fig.8: Variation of the temperature θ with Gt for $Pr=0.72$, $Sc=0.62$, $Gc = 2$, $Ec = 0.1$, $F_w = M=K=0.1$.

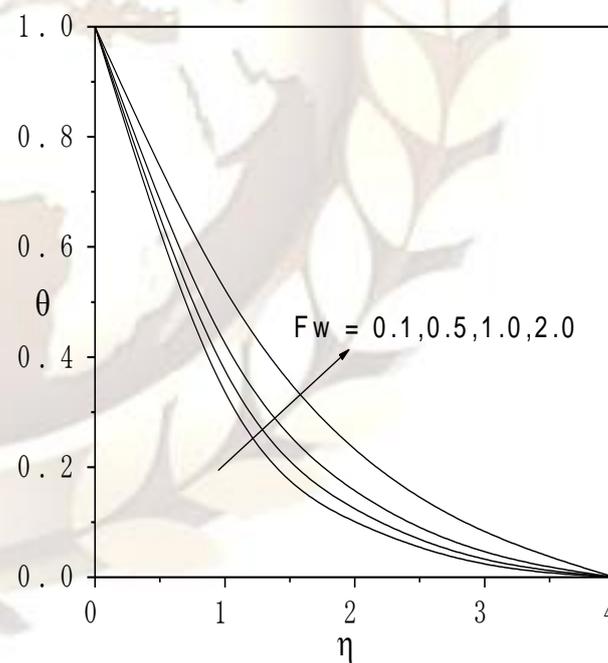


Fig.10: Variation of the temperature θ with F_w for $Pr=0.72$, $Sc=0.62$, $Gc = Gt = 2$, $Ec=0.1$, $M=K=0.1$.

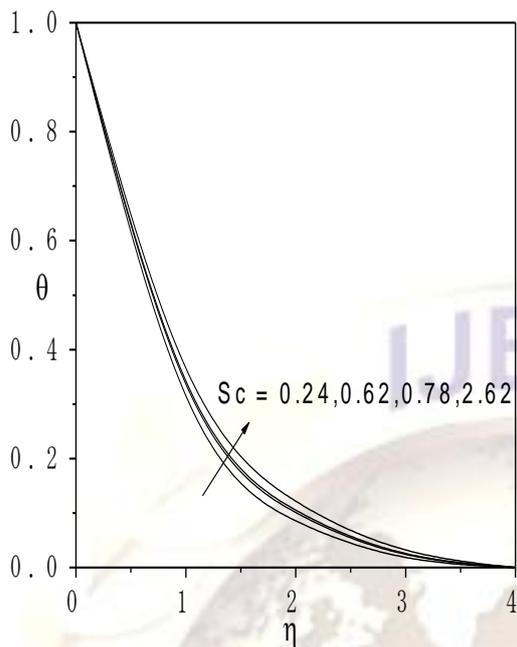


Fig.11: Variation of the temperature θ with Sc For $Pr=0.72, Gt =Gc=2, Ec=0.1, F_w =M=K=0.1$.

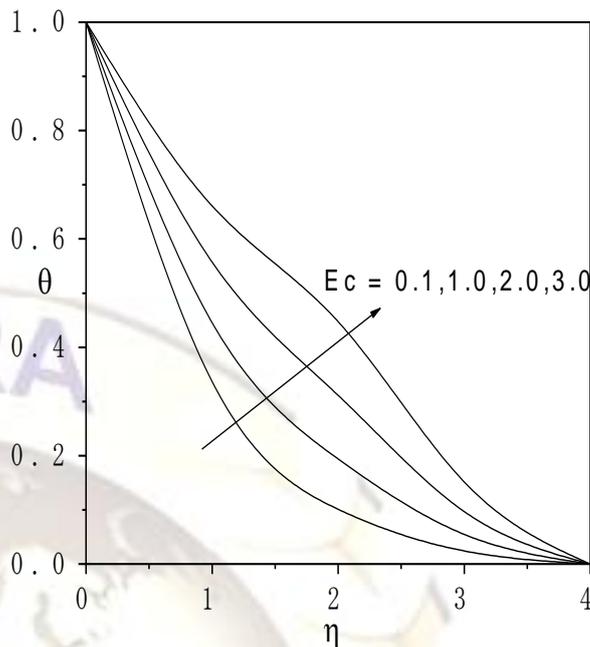


Fig.13: Variation of the temperature θ with Ec For $Pr=0.72, Sc=0.62, Gt =Gc =2, F_w =M=0.1, K=0.1$.

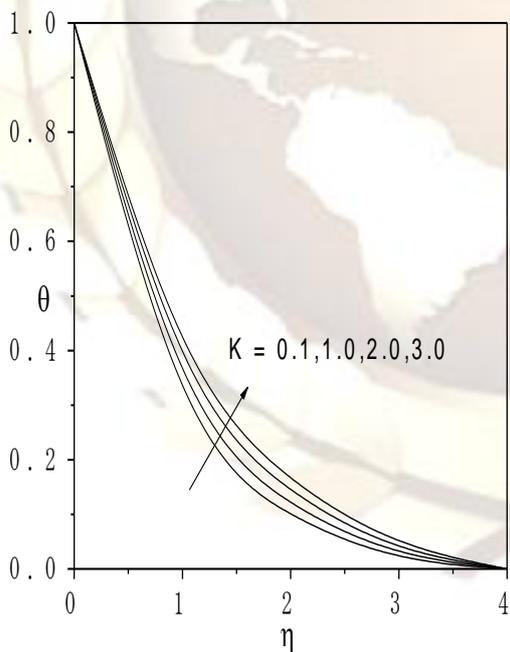


Fig.12: Variation of the temperature θ with K for $Pr=0.72, Sc=0.62, Gt =Gc=2, Ec=0.1, F_w=M=0.1$.

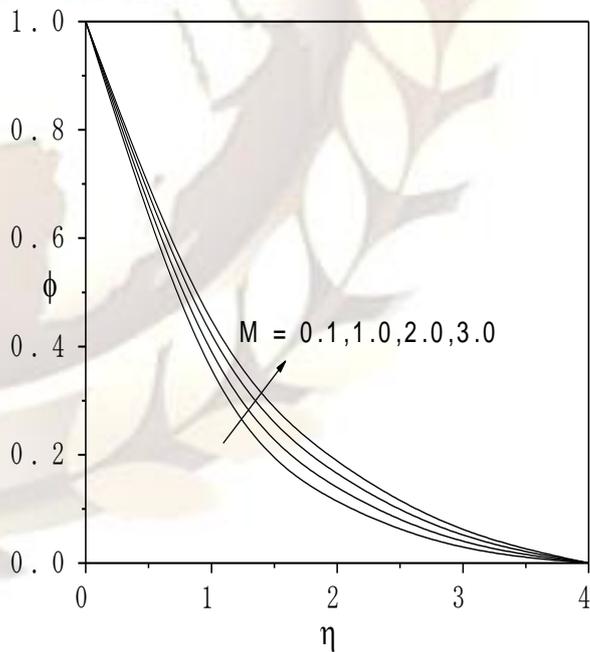


Fig.14: Variation of the concentration ϕ with M For $Pr=0.72, Sc=0.62, Gt =Gc =2, Ec=0.1, F_w =K=0.1$.

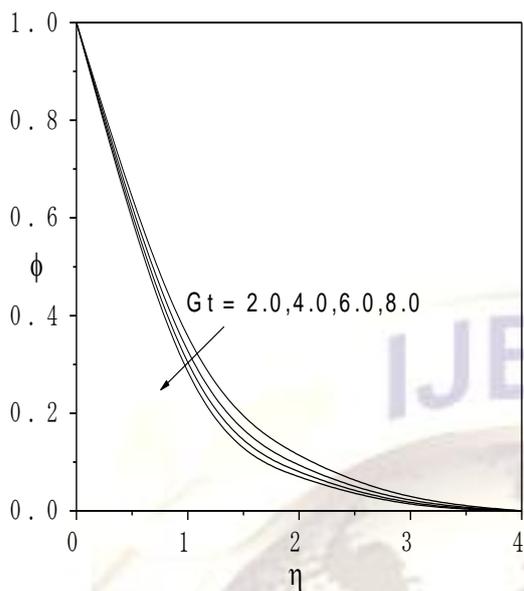


Fig.15: Variation of the concentration ϕ with Gt for $Pr=0.72, Sc=0.62, M=0.1, Gc =2, Ec=0.1, F_w =K=0.1$.

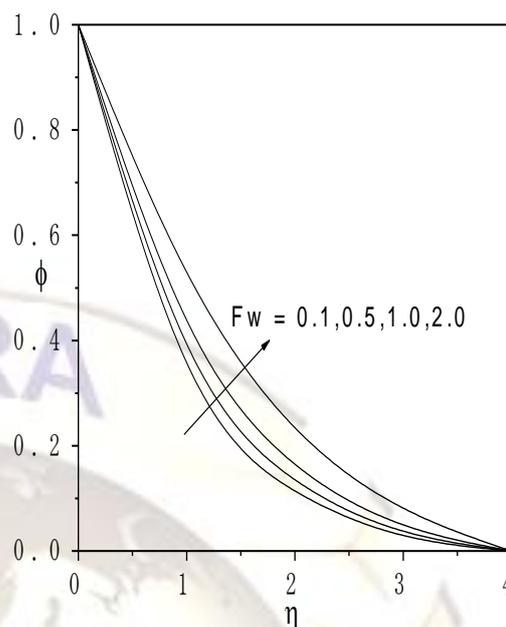


Fig.17: Variation of the concentration ϕ with F_w for $Pr=0.72, Sc=0.62, Gt=Gc=2, Ec=0.1, M=K=0.1$.

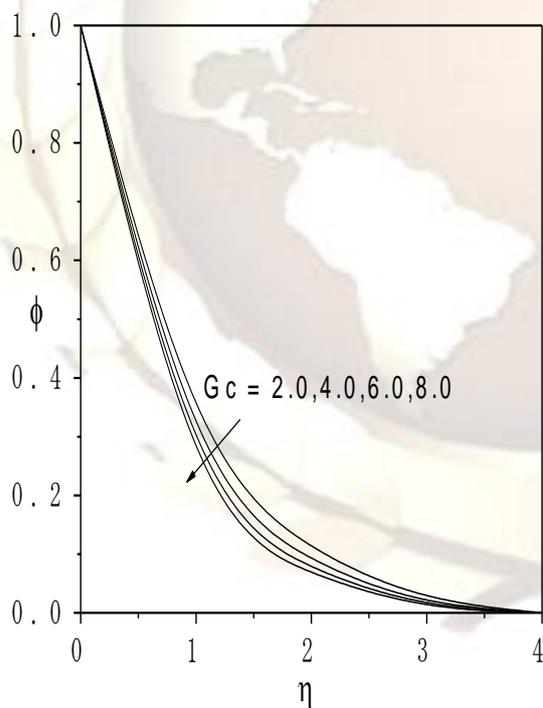


Fig.16: Variation of the concentration ϕ with Gc for $Pr=0.72, Sc=0.62, Gt=2, Ec=0.1$

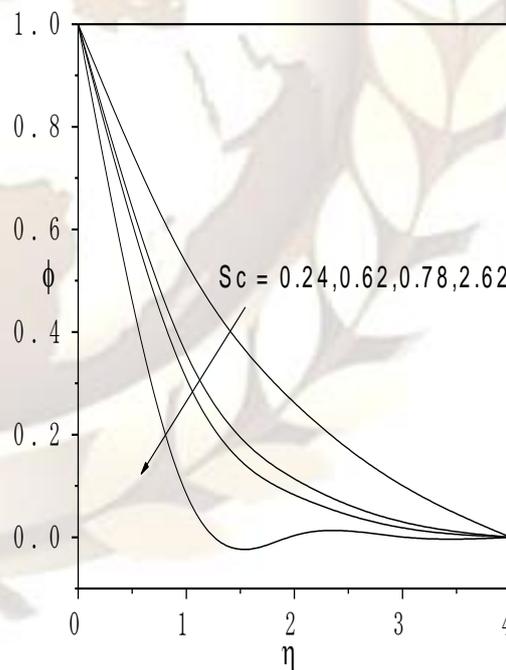


Fig.18: Variation of the concentration ϕ with Sc for $Pr=0.72, Gt=Gc =2, Ec=0.1, F_w =M=K=0.1$.

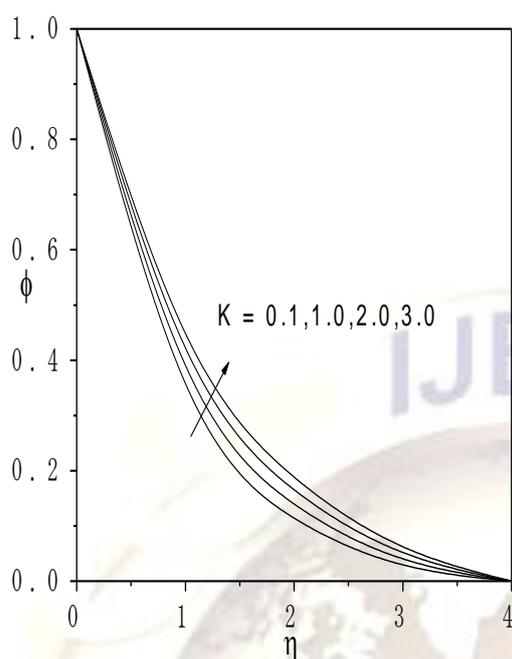


Fig.19: Variation of the concentration ϕ with K for $Pr=0.72, Sc=0.62, Gt =Gc =2, Ec=0.1, F_w=M=0.1$.

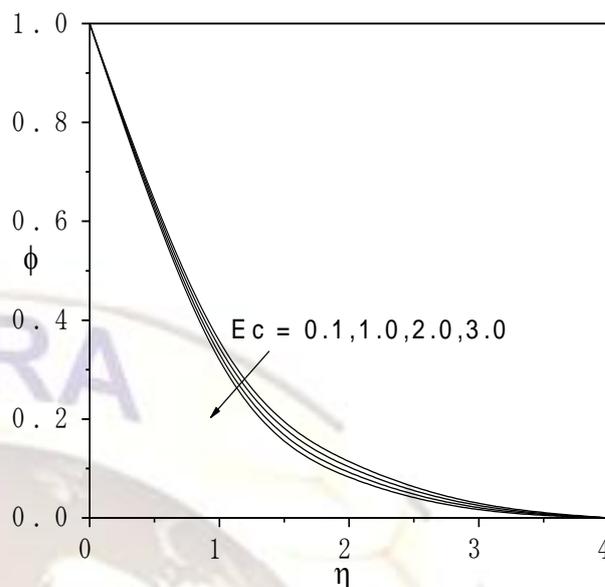


Fig.20: Variation of the concentration ϕ with Ec for $Pr=0.72, Sc=0.62, Gt =Gc =2, K= F_w =M=0.1$.

Table 1 Computations showing comparison with Ibrahim and Makinde [23] results for $f''(0), -\theta'(0), -\phi'(0)$ at the plate with Gr, Gc, M, F_w, Sc for $Pr=0.72, K=0, Ec=0$.

Gr	Gc	M	F_w	Sc	Ibrahim and Makinde [23]			Present work		
					$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.1	0.1	0.62	0.888971	0.7965511	0.7253292	0.897945	0.805082	0.740192
0.5	0.1	0.1	0.1	0.62	0.695974	0.8379008	0.7658018	0.70684	0.841012	0.773803
1.0	0.1	0.1	0.1	0.62	0.475058	0.8752835	0.8020042	0.485886	0.876165	0.806773
0.1	0.5	0.1	0.1	0.62	0.686927	0.8421370	0.7701717	0.699552	0.843609	0.776285
0.1	1.0	0.1	0.1	0.62	0.457723	0.8818619	0.8087332	0.470894	0.880875	0.811303
0.1	0.1	1.0	0.1	0.62	1.264488	0.7089150	0.6400051	1.2696	0.734784	0.675272
0.1	0.1	3.0	0.1	0.62	1.868158	0.5825119	0.5204793	1.87194	0.63906	0.588803
0.1	0.1	0.1	1.0	0.62	0.570663	0.5601256	0.5271504	0.575823	0.566525	0.537923
0.1	0.1	0.1	3.0	0.62	0.275153	0.2955702	0.2902427	0.00301923	0.143197	0.143829
0.1	0.1	0.1	0.1	0.78	0.893454	0.7936791	0.8339779	0.901088	0.803798	0.841183
0.1	0.1	0.1	0.1	2.62	0.912307	0.7847840	1.6504511	0.918401	0.797633	1.64733

Table 2 Variation of $f''(0)$, $-\theta'(0)$, $-\phi'(0)$ at the porous plate with K , Ec for $Gr = Gc = 2$, $M = F_w = 0.1$, $Sc = 0.62$, $Pr = 0.72$.

K	Ec	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.94395	0.761108	0.731902
0.5	0.1	1.11587	0.72043	0.701346
1.0	0.1	1.30489	0.676969	0.669635
1.5	0.1	1.47298	0.639974	0.64341
2.0	0.1	1.62552	0.60796	0.621302
0.1	0.5	0.941408	0.62269	0.732747
0.1	1.0	0.938266	0.451784	0.73379
0.1	1.5	0.935162	0.283174	0.734818
0.1	2.0	0.932094	0.116807	0.735831

5. CONCLUSIONS

From the present study, we arrive at the following significant observations.

By comparing the present results with previous work, it is found that there is a good agreement.

The momentum boundary layer thickness decreases, while both thermal and concentration boundary layer thicknesses increase with an increase in the magnetic field intensity.

An increase in the wall suction enhances the boundary layer thickness and reduces the skin friction together with the heat and mass transfer rates at the moving plate.

As the permeable parameter increases, there is a decrease in the skin friction and Nusselt number and an increase in the surface mass transfer rate.

An increase in the Eckert number causes a decrease in both the skin-friction and Nusselt number and an increase in the Sherwood number.

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