Design Of Discrete Controller Via Novel Model Order Reduction Method

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ABSTRACT

In this paper, a novel mixed method for discrete linear system is used for reducing the higher order system to lower order systems. The denominator polynomials are obtained by the PSO Algorithm and the numerator coefficients are derived by the polynomial method. This method is simple and computer oriented. If the original system is stable then reduced order system is also stable. Finally the lead compensator is designed and connected to original and reduced order systems to improve steady state responses. The proposed method is illustrated with the help of typical numerical examples considered from the literature.

Keywords: PSO optimization, polynomial method, order reduction, transfer function, discrete system, lead compensator, stability.

INTRODUCTION

The order reduction of a system plays an important role in many engineering applications especially in control systems. The use of reduced order model is to implement analysis, simulation and control system designs. The use of original system is tedious and costly. So to avoid the above problems order reduction implementation is necessary.

Many approaches have been proposed to reducing order from higher to lower order system for linear discrete systems. S.Mukherjee, V.Kumar and R.Mitra [1] proposed a reduction method for discrete time systems using Error minimization technique. This method becomes complex when the input polynomial is of high order. Y. Shamash and Feinmesser [2] proposed a method of reduction using Routh array . The disadvantage of this method is, it doesn't guarantees stable reduced models even though original system is stable. Stability Equation Method of R.Prasad[3] requires separation of even and odd parts of denominator polynomial of original high order system which will become computationally tedious when applied to very high-order original systems.

One of the most proposing research fields is "Evolutionary techniques" [9]. Among all the Evolutionary techniques particles swarm optimization appears as a promising algorithm. PSO algorithm shares many similarities with the genetic algorithm. One of the most promising advantages of PSO over the GA is its algorithmic simplicity. In the present paper to overcome above problem a new mixed method is proposed. The denominator polynomials of the reduced order model are obtained by PSO technique and the numerator Coefficients are determined by using polynomial technique. And lead compensator is designed for the original and reduced order systems. The proposed method is compared with the other well known order reduction techniques available in the literature.

REDUCTION PROCEDURAL STEPS FOR THE PROPOSED MIXED METHOD:

STEP:1 Let the transfer function of high order original system of the order 'n' be

$$G_n(z) = \frac{N(z)}{D(z)} = \frac{a_0+a_1z+a_2z^2+\cdots\ldots\ldots a_{m-1}z^{n-1}}{b_0+b_1z+b_2z^2+\cdots\ldots\ldots b_nz^n} \quad (1)$$

STEP:2 Applying bilinear transformation $Z=\frac{1+w}{1-w}$ to $G_n(z)$, to obtain $G_n(w)$

$$G_n(w) = \frac{N(w)}{D(w)} = \frac{A_0+A_1w+a_2w^2+\cdots\ldots\ldots A_nw^n}{B_0+B_1w+B_2w^2+\cdots\ldots\ldots B_nw^n} \quad (2)$$

STEP:3 Let the transfer function of the reduced model of the order 'k' for w-domain is

$$R_k(w) = \frac{N_k(w)}{D_k(w)} = \frac{D_0+D_1w+D_2w^2+\cdots\ldots\ldots D_{k-1}w^{k-1}}{E_0+E_1w+E_2w^2+\cdots\ldots\ldots E_k^k} \quad (3)$$

STEP:4 Determination of denominator by PSO:

The PSO method is a member of wide category of swarm intelligence methods for solving the optimization problems. Particle swarm optimization technique is computationally effective and easier. PSO is started with randomly generated solution as an initial population called particles. Each particle is treated as a point in D dimensional space. Each particle in PSO flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also flying experience of the other particles. Each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it.

The position corresponding to the best fitness is known as pbest and the overall best of all the particles in the population is called gbest .

By using iteration the values of pbest and gbest are calculated. Velocity and particle positions are updated by using below formulae.

$$v_{id}^{k+1} = v_{id}^{k} + c_1 * r_1 * \left(p_{id}^{k} - x_{id}^{k}\right) + c_2 * r_2 * \left(p_{gd}^{k} - x_{id}^{k}\right) \quad (4)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k} \quad (5)$$

Where 'v' is the velocity, 'x 'is the position, p_{id} and g_{id} are the pbest and gbest, 'k' is iteration and c_1,c_2 are the cognition and social parameter. These parameters are variable or constant. Generally these values are '2'and r_1, r_2 are the random numbers in the range (0, 1). The parameters c_1 and c_2 determine the relative pull of pbset and gbset and the parameters r_1 and r_2 help in stochastically varying these pulls..

$$w = w_f + \left(w_f - w_i\right)(\max it - it)/it \quad (6)$$

Where 'it' is the number of iteration

$$v_{id}^{k+1} = w * v_{id}^{k} + c_1 * r_1 * \left(p_{id}^{k} - x_{id}^{k}\right) + c_2 * r_2 * \left(p_{gd}^{k} - x_{id}^{k}\right) \quad (7)$$

STEP:5 Determination of numerator by polynomial method

The numerator polynomial is obtained by equating the original system with its reduced order model.

$$\frac{A + A_1 w + A_2 w^2 + \cdots \ldots\ldots A_n w^n}{B_0 + B_1 w + B_2 w^2 + \cdots \ldots\ldots B_n w^n} = \frac{D_0 + D_1 w + D_2 w^2 + \cdots \ldots\ldots D_{k-1} w^{k-1}}{E_0 + E_1 w + E_2 w^2 + \cdots \ldots\ldots E w^k} \quad (8)$$

Equate the same power's of 's' on both sides, we get

$$A_0 E_0 = B_0 D_o \quad (9)$$

$$A_0 E_1 + A_1 E_0 = B_0 D_1 + B_1 D_0 \quad (10)$$

$$A_0 E_2 + A_1 E_1 + A_2 E_0 = B_0 D_2 + B_1 D +_1 B_2 D_0 \quad (11)$$

$$A_{n-1} E_k = B_n D_{k-1} \quad (12)$$

From the above equations we can get the values of D_0, $D_1 \ldots D_q$.

After finding D_0, $D_1 \ldots D_q$. apply $w = \frac{z-1}{z+1}$ for obtaining reduced order in Z-domain.

$$R_k(z) = \frac{N_k(z)}{D_k(z)} = \frac{d_0 + d_1 z + d_2 z^2 + \cdots \ldots\ldots d_k z^k}{e_0 + e_1 z + e_2 z^2 + \cdots \ldots\ldots e_k z^k} \quad (13)$$

STEPS FOR DESIGNING A LEAD COMPENSATOR:

Step1: Consider the transfer function of the reduced order model.

Step2: Applying the bilinear transformation $Z = \frac{1+w}{1-w}$ to the reduced order model.

Step3: Draw the Bode plot for the reduced order system and find the phase margin α_m and consider the required value of α_m^1.

Step4: If $\alpha_m > \alpha_m^1$ then no compensation is required, Otherwise lead compensator is to be design.

Step5: Determine the phase lead required using

$$\alpha_l = \alpha_m^1 - \alpha_m + € \quad (14)$$

Where $€ = 5^0$ or 6^0

Step6: Determine the β by using $\beta = \frac{1 - \sin \alpha_l}{1 + \sin \alpha_l}$ and determine the ω_m by using $-20\log(1/\sqrt{\beta})$, $T = \frac{1}{\omega_m \sqrt{\beta}}$

Step7: After finding the T value, design the lead compensator, Then

$$G_c(w) = \frac{1}{\beta} \frac{w + \frac{1}{T}}{w + \frac{1}{\beta T}} \quad (15)$$

Step8: Applying inverse bilinear transformation $w = \frac{z-1}{z+1}$, and cascade the compensator with the original and reduced order system. Obtain the closed loop responses of original and reduced order systems.

NUMERICAL EXAMPLE:

Example1: Consider the 8^{th} order discrete system[15]

$$G(z) = \frac{0.165 z^7 + 0.125 z^6 - 0.0025 z^5 + 0.00525 z^4 - 0.02263 z^3 - 0.00088 z^2 + 0.003 z - 0.000413}{z^8 - 0.62075 z^7 - 0.415987 z^6 + 0.076134 z^5 - 0.05915 z^4 + 0.190593 z^3 + 0.097365 z^2 - 0.016349 z + 0.002226}$$

Applying bilinear transformation $Z = \frac{1+w}{1-w}$ to $G_n(z)$, to obtain $G_n(w)$

$$G_n(w) = \frac{-0.0151 w^8 - 0.4607 w^7 - 2.0074 w^6 - 2.3844 w^5 - 1.4988 w^4 + 1.8399 w^3 + 2.7271 w^2 + 1.5338 w + 0.2656}{1.0007 w^8 + 9.8311 w^7 + 36.613 w^6 + 65.846 w^5 + 73.0179 w^4 + 50.0351 w^3 + 17.4061 w^2 + 1.9988 w + 0.2493}$$

For implementing PSO algorithm, to obtain the reduced denominator several parameters are to be considered.

The values of c_1 and c_2 are '2'

The range of random numbers r_1 and r_2 are (0,1).

Swarm size = 20 (Number of reduced order models)

Unknown coefficients = 2

Number of iterations = 200

The denominator polynomial obtained using PSO algorithm is

$$D_2(w) = 0.027957 + 0.107706 w + w^2$$

For finding the numerator values, use the polynomial technique. Equate original transfer function and reduced order transfer function with obtained denominator.

$$\frac{-0.0151w^8-0.4607w^7-2.0074w^6-2.3844w^5-1.4988w^4+1.8399w^3+2.7271w^2+1.5338w+0.2656}{1.0007w^8+9.8311w^7+36.613w^6+65.846w^5+73.0179w^4+50.0351w^3+17.4061w^2+1.9988w+0.2493} = \frac{D_0+D_1w}{0.027957+0.117706w+w^2}$$

On cross multiplying the above equations and comparing the same power of 'w' on the both sides, we get numerator value and multiply the numerator with 'k.'

Therefore, the numerator by polynomial method is

$$N_2(w) = 0.028847-0.004861w$$

The proposed second order reduced model using mixed method is obtained as follows:

$$R_2(w) = \frac{0.028847-0.004861w}{0.027957+0.117706w+w^2}$$

Apply $w = \frac{z-1}{z+1}$ for obtaining reduced order in Z-domain

$$R_2(z) = \frac{0.0238708z^2+0.057609z+0.033739}{1.1445z^2-1.94564z+0.90984}$$

(proposed model)

The comparison is made by computing the error index known as integral square error ISE in between the transient parts of the original and reduced order model is calculated by

$$ISE= \int_0^{t_\infty}[y(t)-y_r(t)]^2 \qquad (16)$$

Where y(t) and $y_r(t)$ are the unit step responses of original and reduced order systems for a second order reduced respectively. The error is calculated for various reduced order models and proposed method is shown below.

Method	**Reduced order models**	**ISE**
Proposed mixed method	$R_2(z) = \frac{0.0238708z^2+0.057609z+0.03373}{1.1445z^2-1.94564z+0.90984}$	**0.026042**
Improved bilinear Routh approximation	$R_2(z) = \frac{0.409429z-0.29467}{1.235z^2-1.9485z+0.8158}$	0.090225
Routh Approximation Method	$R_2(z) = \frac{0.15208z^2+0.049726z-0.102354}{1.203709z^2-1.955526z+0.84076}$	0.069972

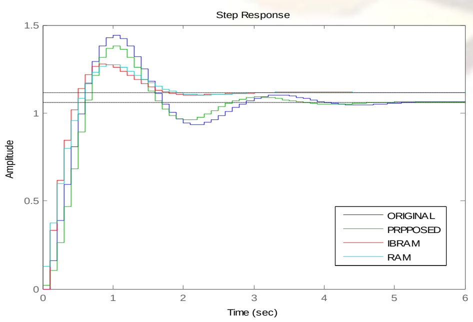

Fig1: Step responses of original and reduced order models

Design procedure for lead compensator:

The transfer function of reduced order system

$$R_2(z) = \frac{0.0238708z^2+0.057609z+0.033739}{1.1445z^2-1.94564z+0.90984} \text{(proposed)}$$

Applying the Bilinear transformation, we get

$$R_2(w) = \frac{0.028847-0.004861w}{0.027957+0.117706w+w^2}$$

From the Bode plot phase margin $\alpha_m = 55.4^0$ and required phase margin $\alpha_m^1 = 70^0$, Then $\alpha_l = 19.6^0$ and $\beta = 0.497$. Then the lead compensator is

$$G_c(w) = \frac{2.012(w+0.169)}{w+0.341}$$

Lead compensator in Z-domain is

$$G_c(z) = \frac{2.012(1.169z-0.831)}{1.341z+0.341}$$

The reduced order system with lead compensator is

$$R(z) = \frac{0.05614zz^3+0.09558z^2-0.01696z-0.05634}{1.591z^3-3.2677z^2+2.4850z-0.5996}$$

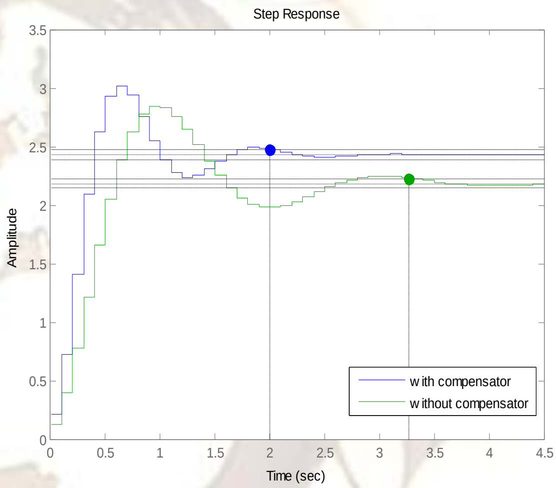

Fig2: Step response of reduced order system with and without compensator

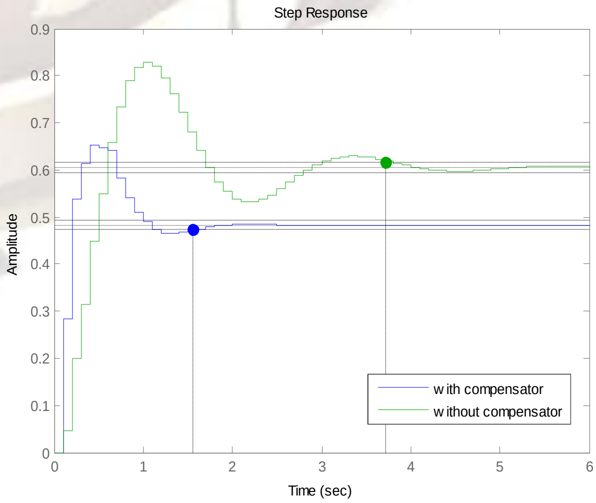

Fig3:Step response of original system with and without compensator

CONCLUSION:

In this paper, design of discrete lead compensator via a novel mixed method is used for discrete linear time system to reduce the high order systems into its lower order systems. In this denominator polynomial is obtained by PSO algorithm technique and numerator coefficients are derived by polynomial method.PSO method is based on the minimization of the integral squared error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input.

This method guarantees stability of the reduced model if the original high order system is stable and which exactly matches the steady state value of the original system. From these comparisons, it is concluded that the proposed method is simple; computer oriented and achieves better approximations than the other existing methods. The design of discrete lead compensator improves the settling time than the proposed mixed model.

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