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Analysis of Efficient square root algorithm for V-blast MIMO Wireless Communications

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Abstract:

Vertical Bell Laboratories Layered Space time (V-BLAST) detection schemes are widely used in time critical application involving high speed packet transfer using MIMO for 3GPP and 3GPP-2 standards. As the number of transmitting and receiving antennas is increased, conventional detection schemes are computationally expensive due to the incorporation of matrix inversion and squaring operations. Reduction in the computational cost is achieved by the efficient square root algorithm which utilizes orthogonal and unitary transformation to avoid matrix inversion and squaring operation .This paper compares the performance between the conventional detection and the efficient square root algorithm when subjected to multi-media application for various numbers of transmitting and receiving antennas with varying SNR. Performance parameters considered include bit error rate (BER), symbol error rate (SER), peak signal-to-noise ratio (PSNR), number of number of floating point operations (FLOPS), Time required for detection. In an attempt to reduce computational complexity the number of FLOPS required for efficient square root detection with MMSE employing 16 transmitting and 16 receiving antennas is 4.8x106 , a reduction of 1x106 FLOPS and 1.4x106 FLOPS is achieved compared to conventional detection scheme employing minimum mean square error (MMSE) and zero-forcing (ZF) respectively, while sustaining the performance of the conventional detection algorithm.

Index Terms— Multiple-input–multiple-output (MIMO) systems, Bell Laboratories Layered Space time (BLAST), vertical BLAST (V-BLAST), Bit error rate (BER), Symbol error rate (SER), peak signal-to-noise ratio (PSNR), Floating point operations (FLOPS).

1. INTRODUCTION

Multiple-Input–Multiple-Output (MIMO) wireless systems, characterized by multiple antenna elements at the transmitter and receiver, have demonstrated the potential for increased capacity in rich multipath environments [1]–[4]. Such systems operate by exploiting the spatial properties of the multipath channel, thereby offering a new dimension which can be used to enable enhanced communication performance. Bell Labs Layered Space-Time architecture (BLAST) [5], including the relative simple vertical BLAST (V-BLAST) [6], is such a system that maximizes the data rate by transmitting independent data streams simultaneously from multiple antennas. V-BLAST often adopts the ordered successive interference cancellation (OSIC) detector [6], which detects the data streams iteratively with the optimal ordering. In each iteration the data stream with the highest signal-to-noise ratio (SNR) among all undetected data streams is detected through zero-forcing (ZF) or minimum mean square error (MMSE) filter. This is referred to as nulling and cancellation. The optimal detection order is from the strongest to the weakest signal, since this minimizes propagation of error from one step of detection to the next step. Further the effect of the detected data stream is subtracted from the received signal vector. This is referred to as interference cancellation.

It turns out that the main computational bottleneck in the conventional detection algorithm is the step where the optimal ordering for the sequential estimation and detection of the transmitted signals, as well as the corresponding so called nulling vector is determined. Current implementations devote 90% of the total computational cost to this step. This high computational cost limits the scope of the application that

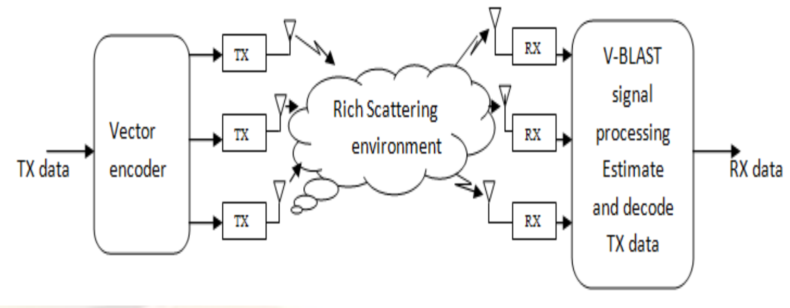


Figure 1: V-BLAST system model

admits inexpensive real time solutions. Moreover, when the numbers of transmitting and receiving antennas are large repeated pseudo-inverse that conventional detection algorithm requires can lead to numerical instability, thus a numerically robust and stable algorithm is required. In an attempt to reduce the computational complexity an efficient square-root [7] algorithm has been proposed. The algorithm is numerically stable since it is division free and uses only Orthogonal transformations such as Householders transformation or sequence of Givens Rotation[8][9]. The numerical stability of the algorithm also makes it attractive for implementation in fixed-point rather than floating-point, architectures.

The remainder of the paper is organized as follows Section II describes the V-BLAST system model Section III introduces different V-BLAST detection schemes which include Conventional Detection Algorithm, An Efficient Square–Root Algorithm, along with their simulation results. Finally we make conclusion in Section IV.

In the following sections, (·) , (·)∗ and (·)*H* denote matrix transposition, matrix conjugate, and matrix conjugate transposition, respectively. 0*M* is the *M* × 1 zero column vector, while I*M* is the identity matrix of size *M*.

2. SYSTEM MODEL

V-BLAST system consists of M transmitting and N receiving antennas in a rich-scattering environment illustrated in Figure. 1 where a single data stream is de-multiplexed into M sub streams and each sub stream is then encoded into symbols and fed to its respective transmitter. The Transmitters 1 to M operate co-channel at symbol rate 1/ T symbols/sec, with synchronized symbol timing. Each transmitter is itself an ordinary QAM transmitter. The collection of transmitters comprises, in effect, a vector-valued transmitter, where components of each transmitted M-vector are symbols drawn from a QAM constellation. The power launched by each transmitter is proportional to 1/ M so that the total radiated power is constant and independent of M.

Let the Signal vector (s) transmitted from M antennas be s= [s1,s2,…..sM]T with the co-variance E(ssH)= σ_s^2.Then the received vector (x) is given by

x = H.s + v, (1)

here v is the N×1 zero-mean circular symmetric complex Gaussian (ZMCSCG) noise vector with the zero mean and the covariance σ_v^2 IN and H= [h1,h2 …..hM]=[h1 h2……hM] H is the N×M complex matrix hm and hm are the mth column and the nth row of H, respectively.

The Linear zero-forcing (ZF) estimate of s is

$\hat{s}$= H+ x = (HHH)-1HH x. (2)

Define α=σ_v^2/σ_s^2.The Linear minimum mean square error (MMSE) estimate of s is

$\hat{s}$= (HHH+ αIM)-1 HH x. (3)

Let R=(HHH+ αIM).Then the estimation error covariance matrix [4] P is given by

P=R-1= (HHH+ αIM)-1 (4)

The Ordered successive Interference Cancellation (OSIC) detection detects *M* entries of the transmit vector ‘s’ iteratively with the optimal ordering. In each iteration, the entry with the highest SNR among all the undetected entries is detected by a linear filter, and then its interference is cancelled from the received signal vector [5].

Suppose that the entries of ‘s’ are permuted such that the detected entry is sM ,the M-th entry. Then the Interference is cancelled by

xM-1 = xM – hMsM (5)

where sM is treated as the correctly detected entry and the initial xM = x. Then the reduced order problem is

xM-1 = hM-1 sM-1+ v (6)

where the deflated channel matrix HM-1= [h1,h2 …..hM-1] and the reduced transmit vector sM-1 = [s1,s2,.. sM-1] T.

The Linear estimate of sM-1 can be deduced from (6).The detection will proceed iteratively until all entries are detected.

3. DETECTION SCHEMES

Conventional detection and efficient square root algorithm for V-blast MIMO Wireless Communications are summarized as follows:

Conventional detection Scheme

Compute a linear transform matrix (P) for nulling. The most common criteria for nulling are zero-forcing (7) and minimum mean square error (8) for which the corresponding linear transform matrix are

P = H+= (HHH)-1HH (7)

P = (HHH+ αIM)-1 HH (8)

Where + denotes the Moore-Penrose pseudo-inverse and H denotes the Hermitian matrix.

2) Determine the optimal ordering for detection of the transmitted symbol by

k=argmin || (P) j ||2 (9)

Iterative Detection:

3) Obtain the kth nulling vector Wk by

Wk= (P) k

(10)

Where (P) k is the kth row of P

4) Using nulling vector Wk form decision statistic yk:

yk=Wk ri (11)

Where r is the received symbols which is a column vector

5) Slice yk to obtain $\hat{a}$k

$\hat{a}$k =Q (yk) (12)

Where Q (.) denotes the quantization (slicing) operation appropriate to the constellation in use

6) Interference Cancellation or the Reduced order problem: Assuming that $\hat{a}$k= ak, cancel ak from the received vector r resulting in modified received vector r1:

ri+1= ri - $\hat{a}$k (H)k (13)

Where (H)k denotes the kth column of H

7) Deflate H denoted by $H_{\bar{k}}$

H=$H_{\bar{k}}$ (14)

8) Form the linear transform matrix (P) utilizing the deflated H depending upon the criteria for nulling chosen, zero-forcing (7) and minimum mean square error (8).

9) Determine the optimal ordering for detection of the transmitted symbol by

k=argmin || (P)j ||2 (15)

10) If i > 1, let i=i-1 and go back to step 3

1. Simulation Results

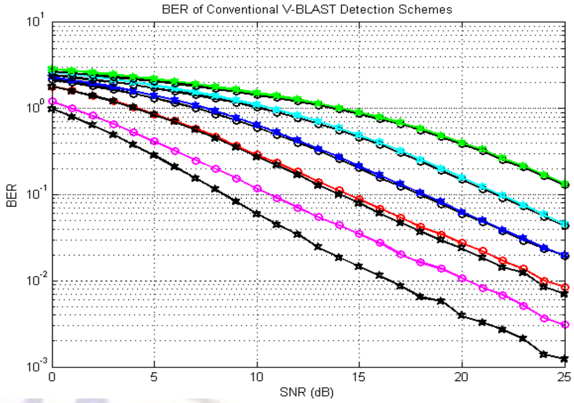
The simulation is performed using the following parameters:

TABLE I
SIMULATION PARAMETERS

Antenna Configurations (Transmitting X Receiving)	2x2;4x4;8x8;16x16
Input Image Dimension	320x300
SNR (db)	0 to 25
Compression Applied	None
Frame Size Assumed	4
Channel Characteristics	Rayleigh Flat Fading varying randomly with every frame
Modulation and Demodulation applied	4,16,64,256,1024 QAM

From Figure 2, 3 BER, SER comparison between zero-forcing (ZF) and minimum mean square error (MMSE) we observe the Bit Error Rate and Symbol Error Rate obtained for MMSE is lower than that of ZF due to the regularization (αIM) introduced in MMSE, which introduces a bias that leads to a much more reliable result than ZF when the matrix is ill-conditioned and when the estimation of the channel is noisy. From figure 2 (a),(b),(c),(d) and figure 3 (a),(b),(c),(d) we also observe MMSE outperforms ZF only when the Modulation scheme employed has lower constellation i.e. at lower data rates (4, 16, 64, 256 QAM), but at higher constellation i.e. at 1024 QAM Modulation scheme the BER obtained using MMSE and ZF are similar which is independent for given antenna configuration. Optimum BER and SER can also be achieved by increasing the number of transmitting and receiving antennas. The gaps observed in the graph indicate a BER of zero i.e. the transmitted image was received without any errors.

From Figure 4 we observe the difference in the quality of the image reconstructed at the receiver compared to the original image that was transmitted. The Quality of the Image Reconstructed i.e. the PSNR is higher at lower constellation i.e. at lower data rates (4, 16, 64, 256 QAM), but at higher constellation i.e. at 1024 QAM Modulation scheme the PSNR obtained using MMSE and ZF are similar. Improvement in PSNR is also observed when the number of transmitting and receiving antennas are increased (Figure 4(a),(b),(c),(d)). The gaps observed in the graph indicate a PSNR of infinity.



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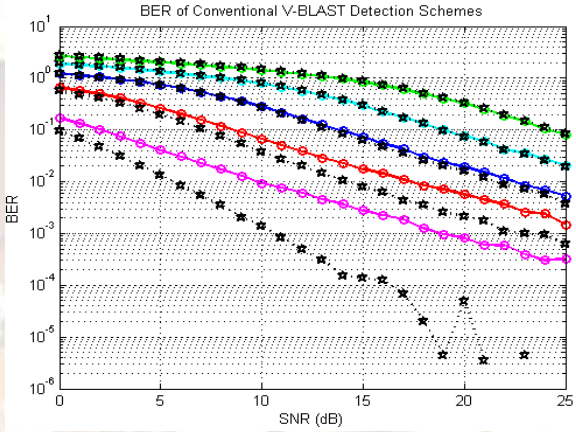

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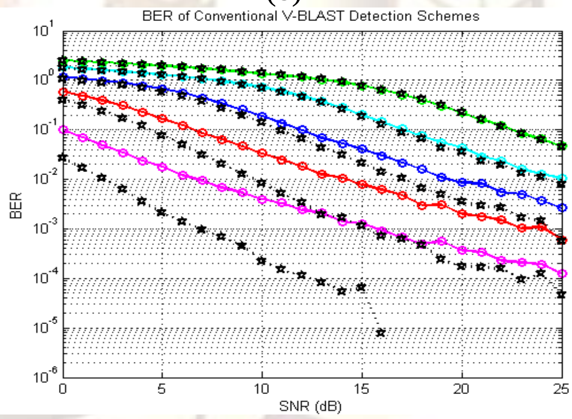

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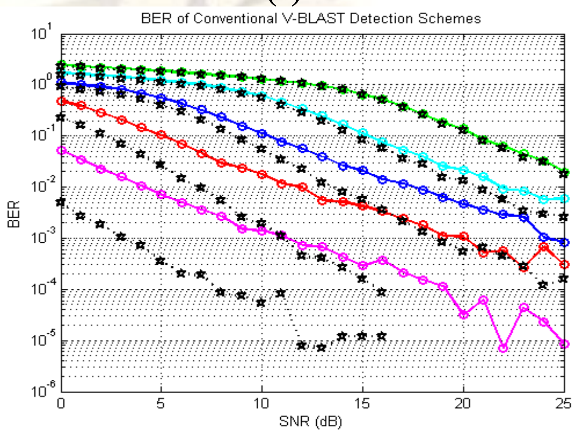

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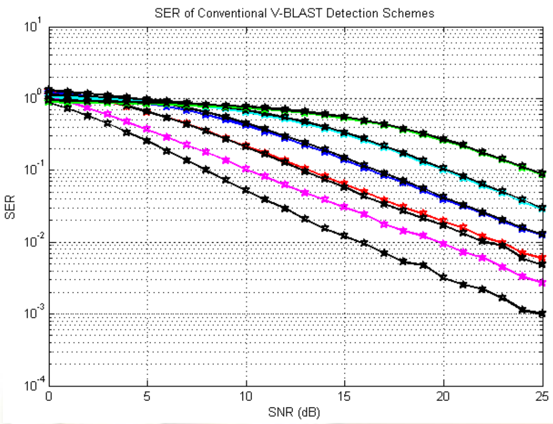
Figure 2: BER comparison between Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) a) BER observed for an 2x2 antenna configuration (b) BER for 4x4 antenna configuration (c) BER for 8x8 antenna configuration (d) BER for 16x16 antenna configuration.(1st (Lower most) black line, 2nd black line, 3rd black line, 4th black line, 5th (upper most) black line indicate BER observed employing MMSE detection scheme with 4-QAM modulation,16-QAM modulation,64-QAM modulation, 256-QAM modulation and 1024-QAM modulation respectively. The pink line, red line, blue line, cyan line, green line indicates BER observed employing ZF with 4-QAM modulation,16-QAM modulation, 64-QAM modulation,256-QAM modulation and 1024-QAM modulation respectively)



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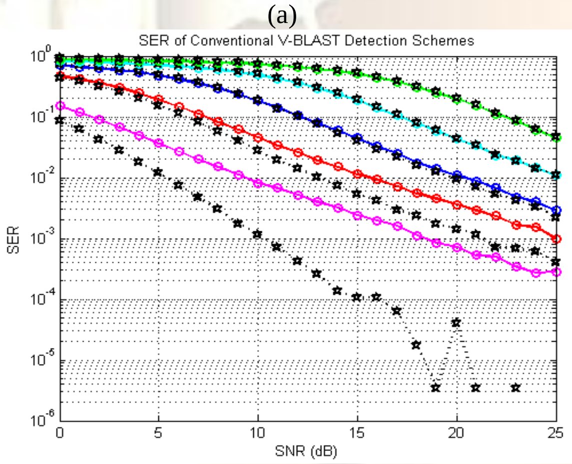

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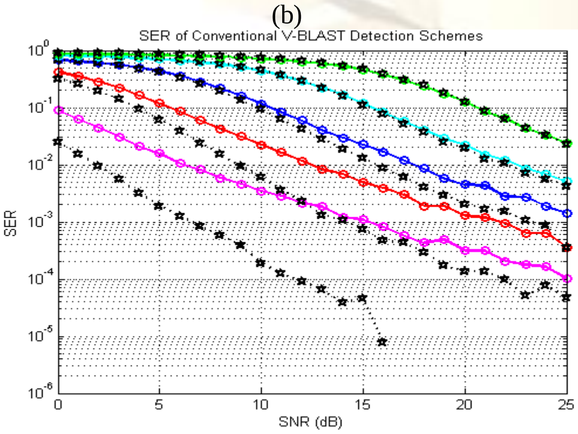

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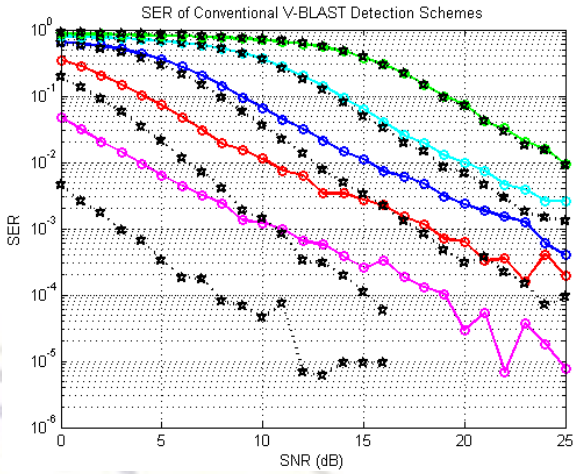

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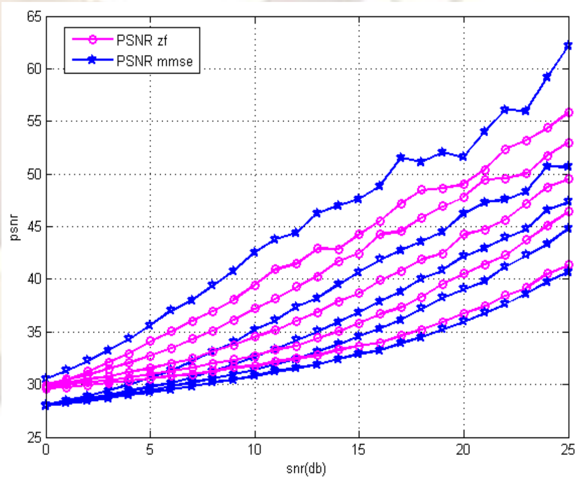
Figure 3: SER comparison between Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) (a) SER observed for an 2x2 antenna configuration (b) SER for 4x4 antenna configuration (c) SER for 8x8 antenna configuration (d) SER for 16x16 antenna configuration.(In the ascending order, 1st (Lower most) black line, 2nd black line, 3rd black line, 4th black line, 5th (upper most) black line indicate SER observed employing MMSE detection scheme with 4-QAM modulation,16-QAM modulation,64-QAM modulation, 256-QAM modulation and 1024 -QAM modulation respectively. The pink line, red line, blue line, cyan line, green line indicates SER observed employing ZF with 4 -QAM modulation,16-QAM modulation, 64-QAM modulation,256-QAM modulation and 1024-QAM modulation respectively)



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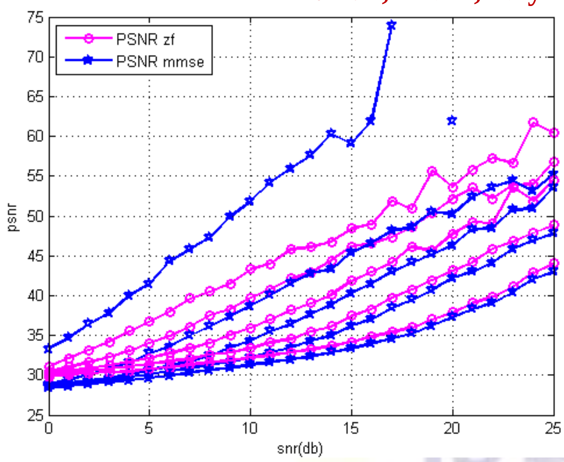

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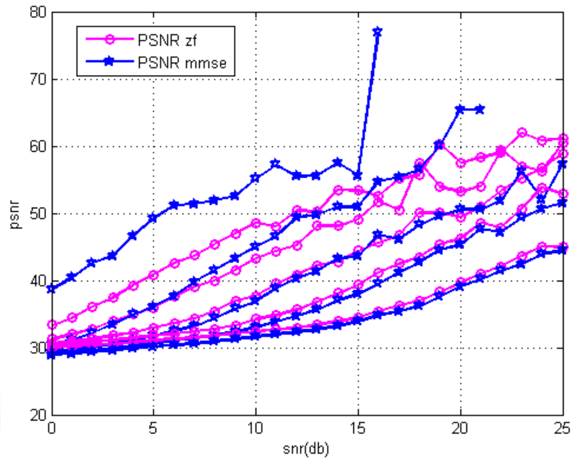

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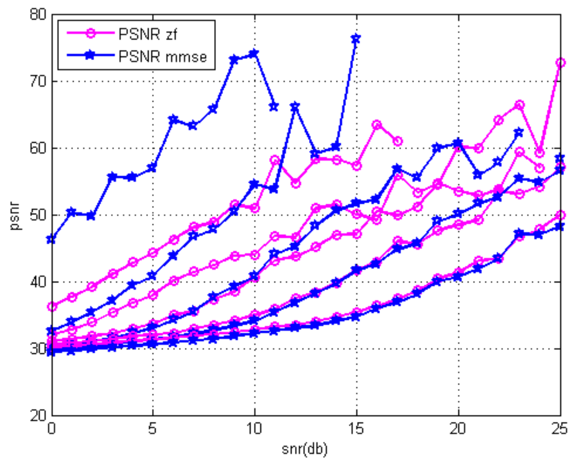

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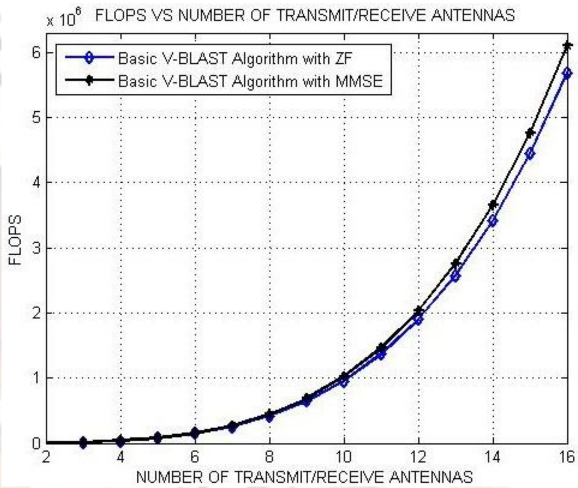
Figure 4: PSNR Comparison between the Reconstructed Output and the Original Image Transmitted For Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) (a) PSNR observed for an 2x2 antenna configuration (b) PSNR for 4x4 antenna configuration (c) PSNR for 8x8 antenna configuration (d) PSNR for 16x16 antenna configuration.(In the ascending order, 1st (Lower most) blue line, 2nd blue line, 3rd blue line, 4th blue line, 5th (upper most) blue line indicate PSNR observed employing MMSE detection scheme with 1024-QAM modulation ,with 256-QAM modulation , with 64-QAM modulation, with 16-QAM modulation and with 4-QAM modulation respectively. The 1st (Lower most) pink line, 2nd pink line, 3rd pink line, 4th pink line, 5th (upper most) pink line indicate PSNR observed employing ZF with 1024-QAM modulation, with 256 -QAM modulation, with 64-QAM modulation, with 16-QAM modulation , and with 4 -QAM modulation respectively)



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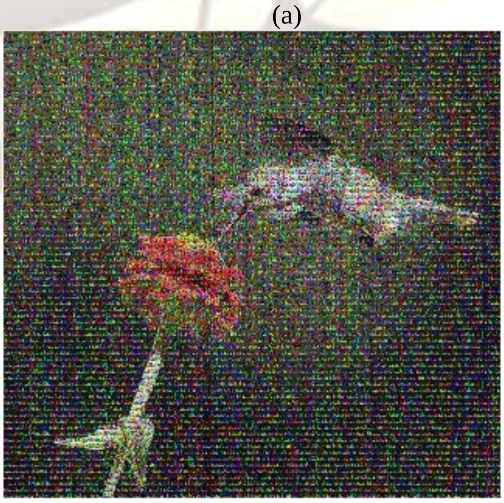

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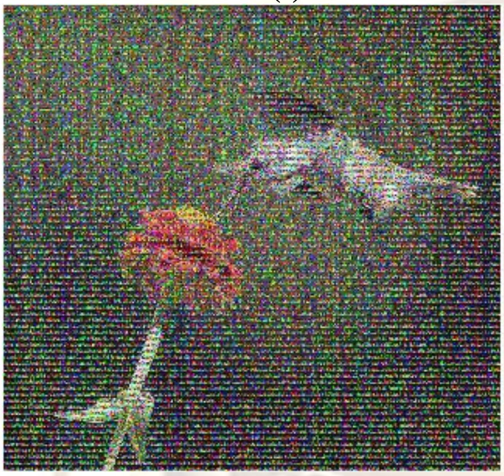

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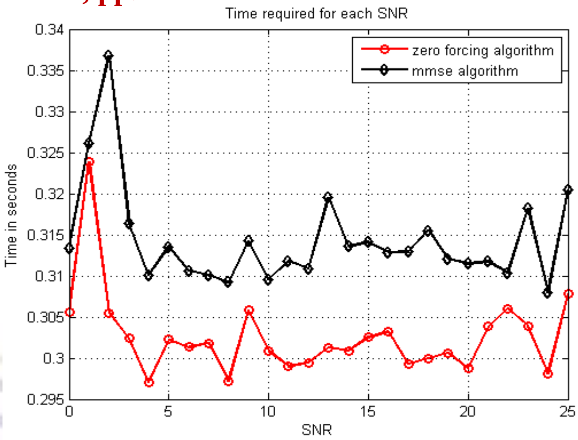
(e)

Figure 5: (a) TOTAL FLOPS required for ZF and MMSE (b) Original Transmitted Image (c) Reconstructed Output Image of ZF algorithm at SNR=0 (d) Reconstructed Output Image of ZF algorithm at SNR=20 (e) Reconstructed Output Image of MMSE algorithm at SNR=0 (f) Reconstructed Output Image of MMSE algorithm at SNR=20



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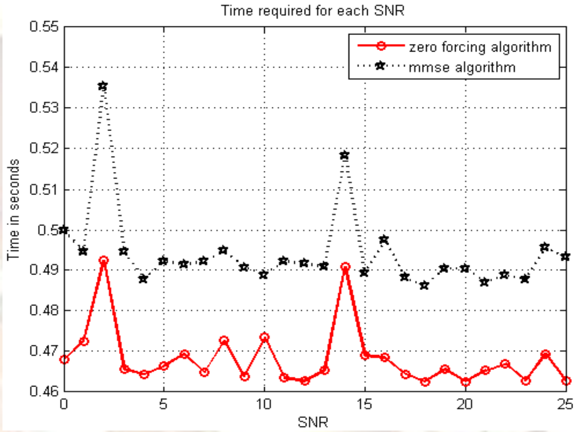

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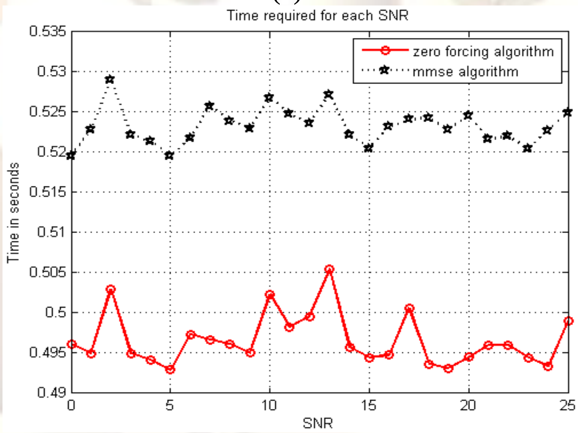

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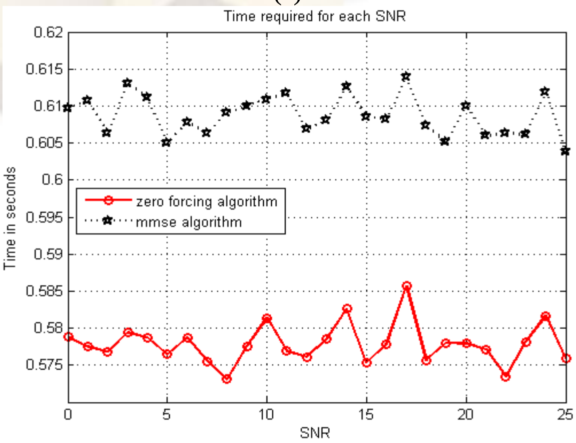

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Figure 6:(a) Time required for detection with 2x2 antenna configuration (b) Time required for 4x4 antenna configuration (c) Time required for 8x8 antenna configuration (d) Time required for 16x16 antenna configuration.(Black: conventional detection with MMSE; red conventional detection with ZF)

From Figure 5(a) we observe the Number of Floating Point Operations (FLOPS) required for MMSE and ZF increases monotonically for Number of Transmit and Receiving Antennas from 1 to 10, above which further increase in Number of Floating Point operations required for MMSE when compared to ZF is observed (one complex multiplication and addition requires six and two flops respectively). Figure 5:(b) is the original transmitted Image, Figure 5 (c),(d),(e),(f) are the Reconstructed Image at the receiver for SNR= 0 and 20 for MMSE and ZF algorithm. The Quality of the image is directly related to the BER observed, since the BER of MMSE outperforms ZF the quality of image obtained using MMSE is higher when compared to ZF.

From figure 6 compares the time required for detection for ZF and MMSE detection algorithms. The Time required is directly related to the Number of flops required for execution, as the number of transmitting and receiving antennas are increased an increase in time require for detection is observed as shown in figure 6 (a),(b),(c),(d) .

An Efficient square-root algorithm

An Efficient Square–Root Algorithm [8] for V-BLAST reduces the computational cost for the nulling and cancellation step. The algorithm is numerically stable since it is division free and uses only Orthogonal transformations such as Householders transformation or sequence of Givens Rotation[18][19] .The numerical stability of the algorithm also makes it attractive for implementation in fixed-point rather than floating-point, architectures.

Initialization:

1) Let m=M. Compute square root of P, i.e., P1/2and Qα

Form the so called (M+N+1) × (M+1) pre array

$$\Omega i\text{-}1= \begin{bmatrix} 1 & HP_{i-1}^{1/2}i \\ 0M & P_{i-1}^{1/2} \\ \text{-ei} & \text{Bi-1} \end{bmatrix}$$

and propagate the pre-array N times:

$$\Omega i\text{-}1\ .\ \Theta i = \begin{bmatrix} x & 0_M^T \\ x & P_{i-1}^{1/2} \\ x & \text{Bi} \end{bmatrix}$$

$$P_0^{1/2} = (1/\alpha v)\ IM\ ,\ B0=0N\times M \qquad (16)$$

where ei is an N×1 vector of all zeros except for the i-th entry which is unity, P1/2 is the square root of an M×M linear transform matrix P for MMSE is given by

P = (HHH+ IM)-1 HH (17)

Bi is an N×M sub-matrix of Ωi and BN = Qα , "×"denotes not relevant entries at this time, and Θi is any unitary transformation that block lower triangularize the pre-array Ωi

Iterative Detection:

2) Find the minimum length row of P1/2 and permute it to be the last M-th row.

3) Find a unitary transformation Σ such that P1/2 Σ is block Upper triangular

$$P1/2\ \Sigma= \begin{bmatrix} P(M\text{-}1)/2 & P_M^{(M-1)/2} \\ 0_{M-1}^T & P_M^{1/2} \end{bmatrix} \qquad (18)$$

Where $P_M^{(M-1)/2}$ and $P_M^{1/2}$ denote the last (M-1) ×1 sub-column and the (M, M)th scalar entry, respectively.

4) Update Qα to QαΣ

5) Form the linear MMSE estimate of am,

$$\hat{a}m = P_M^{1/2}\ q_{\alpha,M}^H r\ (m). \qquad (19)$$

Where qα,M is the M-th column of Qα.

6) Obtain am from âm via slicing.

7) Cancel the interference of am in r (m) to obtain the reduced- order problem by

r(M - 1) = r (M) − hMaM (20)

8) If m >1, let $m = m - 1$ and go back to step P2. With the corresponding r (m-1), Hm-1, P(M-1)/2 and Q_α^{M-1} instead of P1/2 and Qα .

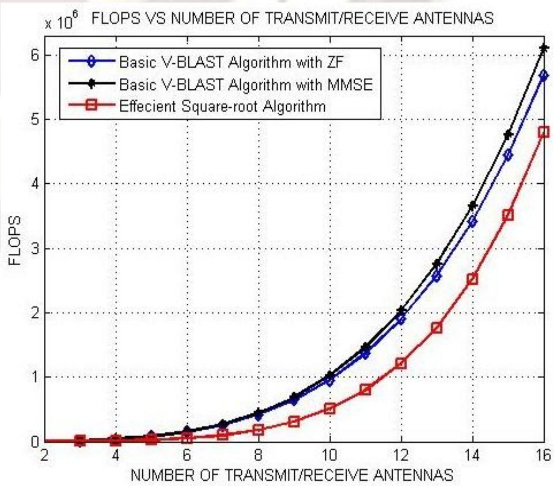
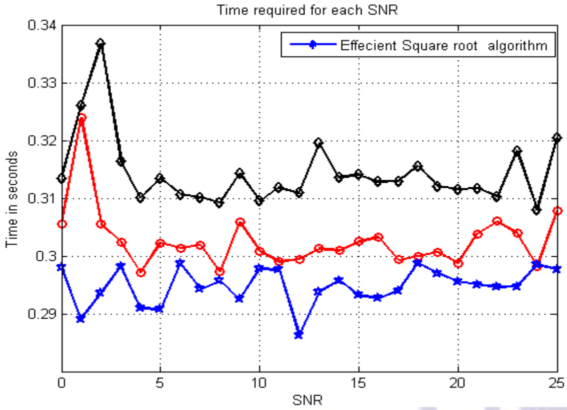


Figure 7: TOTAL FLOPS required for ZF and MMSE and an efficient square root algorithm employing MMSE



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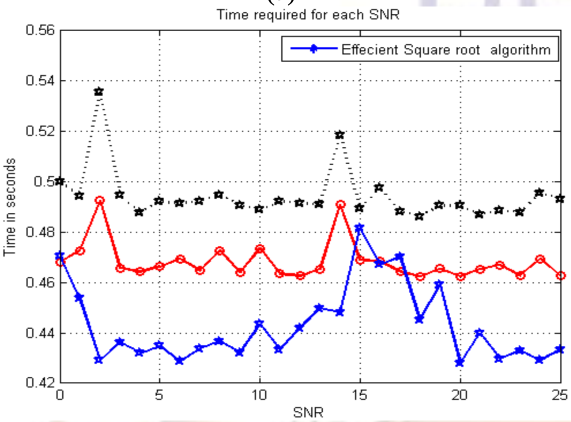

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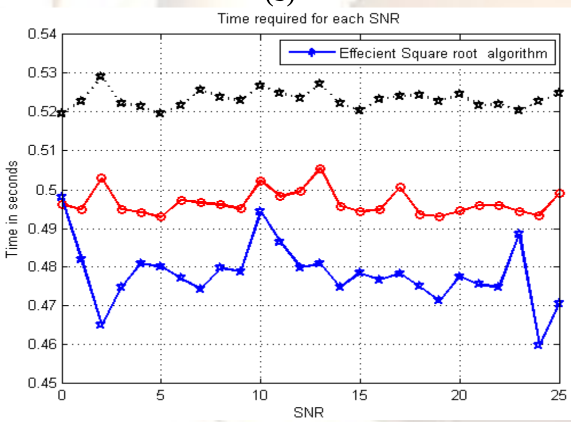

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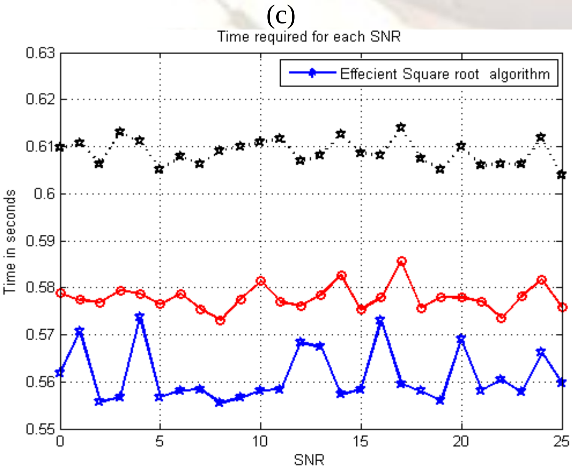

(d)

Figure 8:(a) Time required for detection for 2x2 antenna configuration (b) Time required for 4x4 antenna configuration (c) Time required for 8x8 antenna configuration (d) Time required for 16x16 antenna configuration. (Black: conventional detection with MMSE; red conventional detection with ZF; blue: efficient square root algorithm employing MMSE)

1. Simulation results

Simulation is performed using the parameters from Table I. The performance parameters such as BER, SER and PSNR are similar to the results obtained for conventional detection scheme employing MMSE. Figure 7 compares the Number of FLOPS required for the conventional Detection scheme employing MMSE and ZF and An Efficient Square-Root Algorithm for BLAST employing MMSE (one complex multiplication and addition requires six and two flops respectively). The efficient square root algorithm outperforms the conventional detection scheme in terms of Number FLOPS required for detecting the received symbols.

Admit of unitary or orthogonal transformation such as householder or givens rotation for detection reduced the computational cost for detection while sustaining the performance obtained with the conventional detection scheme employing MMSE.

Figure 8 compares the time required for detection between conventional detection employing ZF, MMSE and an efficient square root algorithm employing MMSE. Due to the achieved reduction in the Number of floating point operation, reduction in the time required for detection is observed when efficient square root algorithm is employed with MMSE.

4. CONCLUSION

This paper provides a performance comparison between different detection schemes employed in V-blast MIMO Wireless Communications which includes Conventional detection scheme employing MMSE and ZF Algorithm, where MMSE outperforms the ZF algorithm in terms of the BER, SER, and PSNR but at the cost of increase in the number of FLOPS required thereby increasing the time required for execution. The number of FLOPS required for MMSE and ZF monotonically increases from for 2 transmitting and receiving to 9 transmitting and receiving antennas, the number of FLOPS required for MMSE at 16 transmitting and 16 receiving antennas is 6.2x106, FLOPS of 5.8x106 for ZF is observed. Reduction in the number FLOPS required is achieved with an efficient square root algorithm with MMSE which employs orthogonal transformation to avoid squaring and inverse operations, thereby reducing the number of FLOPS required for detection while keeping the good performance when compared to the conventional detection scheme employing MMSE.The number of

FLOPS required for the efficient square root algorithm with MMSE employing 16 transmitting and 16 receiving antennas is 4.8x106 , A reduction of 1x106 FLOPS and 1.4x106 FLOPS is achieved compared to conventional detection scheme employing ZF and MMSE respectively.

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