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Dynamic Reanalysis of Beams Using Polynomial Regression Method

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ABSTRACT:

The paper focuses on dynamic reanalysis of simple beam structures using a polynomial regression method. The method deals with the stiffness and mass matrices of structures and can be used with a general finite element system. This method is applied to approximate dynamic reanalysis of cantilever simple beam structure and T-structure. Preliminary results for these example problems indicate the high quality approximation of natural frequencies can be obtained. The final results from regression method and Finite element method are compared.

Keywords: **mass matrix, stiffness matrix, natural frequency, dynamic reanalysis, polynomial regression.**

INTRODUCTION:

Reanalysis methods are intended to analyze efficiently new designs using information from previous ones. One of the many advantages of the substructure technique is the possibility of repeating the analysis for one or more of the substructures making use of the work done on the others. This represents a significant saving of time when modifications once are required. Modification is invariably required in iterative processes for optimum design never the less, in the case of large structures the expenses are still too high.[1]

Therefore, development of techniques which are themselves based on previous analysis, and which obtained the condensed matrices of the substructures under modification, with little extra calculation time, can be very useful. "General Reanalysis Techniques" are very useful in solving medium size problems and are totally essential in the design of large structures. Some steps in a dynamic condensation process are particularly characterized by their computational effort, as for instance:

- Stiffness matrix factorization
- Resolution of certain systems of linear equation
- Resolution of an eigen problem to obtain the normal vibration modes.

Reanalysis methods [2] are intended to analyze efficiently structures that are modified due to changes in the design. The object is to evaluate the structural response for such changes without solving the complete set of modified simultaneous equations. The solution procedures usually use the original response of the structure.

SOLUTION APPROACH- FINITE ELEMENT METHOD

Initially the beam is divided into smaller sections using successive levels of division. Analysis of each section is performed separately. Using the finite element technique, the dynamic analysis of beam structure is modeled.

$[K-\lambda M]\,[X]=0$ ----------------- (1)

Where k, m are the stiffness and mass matrix respectively.

The dynamic behavior of a damped structure [4] which is assumed to linear and discretized for n degrees of freedom can be described by the equation of motion.

$M\ddot{X}+C\dot{X}+Kx=f$-------------(2)

Where M, $C = \alpha M+\beta K$, and K are mass, damping and stiffness matrices, $\ddot{X}$, $\dot{X}$ and X are acceleration, velocity, displacement vectors of the structural points and "f" is force vector. Undamped homogeneous equation $M\ddot{X}+Kx=0$. Provides the Eigen value problem $(k-\lambda m)\,\phi = 0$.

Solution of above equation yields the matrices Eigen values λ and Eigen vectors ϕ

$$\lambda = \begin{bmatrix} w_1^2 & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & w_n^2 \end{bmatrix}, \quad \phi = [\phi 1, \phi 2 \ldots\ldots \phi n]$$

The eigenvector satisfy the orthonormal conditions $\phi^T M\,\phi=I$, $\phi^T K\,\phi=\lambda$, $\phi^T C\,\phi = \alpha I+\beta\lambda=\xi$, Using the transformation $X = \phi q$ in the equation of motion, and premultiplying by ϕ^T one obtains,

$\phi^T M\phi\ddot{q}+\phi^T C\phi\dot{q}+\phi^T K\phi q=\phi^T f$ -------------(3)

It is important note, that the matrices, $\hat{M} = \phi^T M\,\phi$, $\hat{C} = \phi^T C\phi$, $\hat{K} = \phi^T K\phi$ are not usually diagonalised by the eigenvectors of the original structure [3] Given an initial geometry $\hat{Y}$ and assuming a change ΔY in the design variables, the modified design is given by

$Y = \hat{Y}+\Delta Y$. ------------------ (4)

The geometric variables Y usually represent coordinates of joints, but other choice for these variables is sometimes preferred. The displacement analysis equations for the initial design are

$\hat{K}$r = R.

where $\hat{k}$ = stiffness matrix corresponding to the design $\hat{Y}$, R= load vector whose elements are usually assumed to be independent of the design variables and r= nodal displacements computed at $\hat{Y}$. The stiffness matrix and mass matrix of a typical plane truss element are

$$K=\frac{EA}{l^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ -6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \text{ and}$$

$$M=\frac{\rho A_e l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix}$$

'A' is the cross sectional area, 'l' is member length, of the beam ' ρ ' is density of the beam.

REANALYSIS OF REGRESSION METHOD

The statistical determination of the relationship between two or more dependent variables has been referred to as a correlation analysis, [6]whereas the determination of the relationship between dependent and independent variables has come to be known as a regression analysis.

1.1 Regression

The actual term "regression" is derived from the Latin word "*regredi*," and means "to go back to" or "to retreat." Thus, the term has come to be associated with those instances where one "retreats" or "resorts" to approximating a response variable with an estimated variable based on a functional relationship between the estimated variable and one or more input variables. In regression analysis, the input (independent) variables can also be referred to as "regressor" or "predictor" variables.

3.1.1 Linear Regression

Linear regression involves specification of a linear relationship between the dependent variable(s) and certain properties of the system under investigation. Linear regression deals with some curves as well as straight lines.

3.1.2 Ordinary Linear Regression

The simplest general model for a straight line includes a parameter that allows for inexact fits: an "error parameter" which we will denote as ε.

Thus we have the formula:

$Y = \alpha + \beta X + \varepsilon$ ---------------- (5)

The parameter, α, is a *constant*, often called the "intercept" while b is referred to as a *regression coefficient* that corresponds to the "slope" of the line. The additional parameter ε accounts for the type of error that is due to random variation caused by experimental imprecision. The regression procedure assumes that the scatter of the data points about the best-fit straight line reflects the effects of the error term,[12-15] and it is also implicitly assumed that ε follows a Gaussian distribution with a mean of 0. Now, however, we will assume that the error is Gaussian Figure 2 illustrates the output of the linear model with the inclusion of the error term.

3.1.3 Multiple Linear Regressions

The straight line equation is the simplest form of the linear regression given as

$Y=\alpha+\beta X+\varepsilon$

Where α+βX represents the deterministic part and ε is the stochastic component of the model.

The simple linear population model equation indicating the deterministic component of the model that is precisely determined by the parameters α and β, and the stochastic component of the model, ε that represents the contribution of random error to each determined value of Y. It only includes one independent variable. When the relationship of interest can be described in terms of more than one independent variable, the regression is then defined as "multiple linear regression." The general form of the linear regression model may thus be written as:

$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots + \beta_i X_i + \varepsilon$. --------------- (6)

Where, Y is the dependent variable, and X1, X2 … Xi are the (multiple) independent variables. Multiple linear regression models also encompass polynomial functions:

$Y = \alpha + \beta_1 X + \beta_2 X^2 + \ldots\ldots + \beta_i X^i + \varepsilon$, ------------- (7)

The equation for a straight line is a first-order polynomial. The quadratic equation,

$Y=\alpha + \beta_1 X + \beta_2 X^2$ ------------------ (8)

is a second-order polynomial whereas the cubic equation,

$Y = \alpha + \beta_1 X + \beta_2 X^2 + \beta_3 X^3$ -------------- (9)

is a third-order polynomial.

Taking first derivatives with respect to each of the parameters yields:

$\frac{\partial Y}{\partial \alpha} = 1$, $\frac{\partial Y}{\partial \beta_1} = X$, $\frac{\partial Y}{\partial \beta_2} = X^2$---------------- (10)

The model is linear because the first derivatives do not include the parameters. As a consequence, taking the second (or higher) order derivative of a linear function with respect to its parameters will always yield a value of zero. Thus, if the independent variables and all but one parameter are held constant, the relationship between the dependent variable and the remaining parameter will always be linear. It is important to note that linear regression does not actually test whether the data sampled from the population follow a linear relationship. It assumes linearity and attempts to find the best-fit straight line relationship based on the data sample. The dashed line shown in the figure (1) is the deterministic component, whereas the points represent the effect of random error.

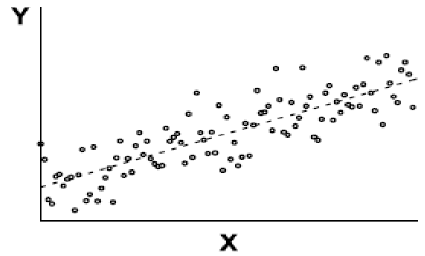

Figure 1: A linear model that incorporates a stochastic (random error) component.

3.1.4 Assumptions of Standard Regression Analyses

- The subjects are randomly selected from a larger population. The same caveats apply here as with correlation analyses. The observations are independent. The variability of values around the line is Gaussian.
- X and Y are not interchangeable. Regression models used in the vast majority of cases attempt to predict the dependent variable, Y, from the independent variable, X and assume that the error in X is negligible. In special cases where this is not the case, extensions of the standard regression techniques have been developed to account for non negligible error in X.
- The relationship between X and Y is of the correct form, i.e., the expectation function (linear or nonlinear model) is appropriate to the data being fitted.
- There are enough data points to provide a good sampling of the random error associated with the Experimental observations. In general, the minimum number of independent points can be no less than the number of parameters being estimated, and should ideally be significantly higher.

NUMERICAL EXAMPLES

The polynomial regression method is applied to a simple beam structures. In finite element method, **Discretization** means dividing the body into an equivalent system of finite elements with associated nodes. The element must be made small enough to view and give usable results and to be large enough to reduce computational efforts. Small elements are generally desirable where the results are changing rapidly such as where the changes in geometry occur. Large elements can be used where the results are relatively constant. The discretized body or mesh is often created with mesh generation program or preprocessor programs available to the user. Figure (2) shows an example of creating a finite element for a cantilever beam.

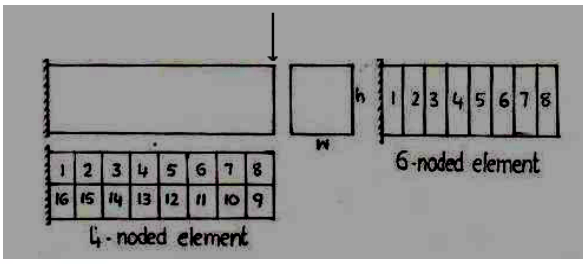

Figure 2: Descretized Element

The polynomial equation for regression method,
$f_n = C_1+C_2B+C_3H + C_4B^2 + C_5H^2 + C_6BH$.
These 3 values for both case studies

Young's modulus (E)	0.207×10^{12} N/m^2
Density (ρ)	7806 Kg/m^3
Cross section of area (A)	$0.029\times0.029 m^2$

1.2 Case Study 1

The Cantilever Beam of 1m length, shown in figure () is divided into 4 elements equally element Stiffness Matrix and Mass Matrix are extracted. Natural frequencies of the cantilever beam at each node are found from MATLAB program by considering two situations-
a) width alone is increased by 5% and
b) width and depth of the beam are increased by 5% each.

Reanalysis of the beam is done by Polynomial regression and the percentage errors are listed in the table.

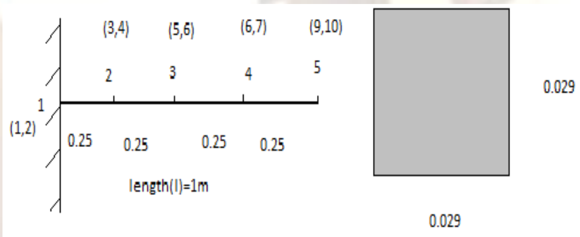

Figure 3: Cantilever beam with nodes and elements

First natural frequencies of cantilever beam by increasing depth of the beam by 5%. The polynomial regression equation is given by
$f_n = C_1+C_2B+C_3H + C_4B^2 + C_5H^2 + C_6BH$
Fitting target of lowest sum of squared absolute error = $8.7272727289506574E^{-05}$,
C_1 =-3.6333E^{-03}, C_2 = -1.053659E^{-04},
C_3 = 7.76275E^{02}, C_4 = -3.05561E^{-06},
C_5 = 8.07983E^{-11}, C_6 = 2.2512E^{+01}

First natural frequencies of cantilever beam by increasing width and depth of the beam by 5%.

Fitting target of lowest sum of squared absolute error $= 1.84703589743589 64E^{-01}$,

C_1 =-6.30489E^{-01}, C_2 = 4.028518E^{+02},
C_3 = 4.028518E^{+02}, C_4= - 1.0347691E^{+02},
C_5 = -1.0347691E^{+02}, C_6 = - 1.0347691E^{+02},

Table 1: Increasing the Depth of the Beam

Width	Height	Natural Frequency Fem(Hz)	Natural Frequency Regression(Hz)	%Error
0.029	0.029	22.53	22.52	-0.044
0.029	0.03045	23.65	23.64	-0.042
0.029	0.0319	24.78	24.69	-0.363
0.029	0.03335	25.91	25.88	-0.115
0.029	0.0348	27.03	27.01	-0.074
0.029	0.03625	28.16	28.13	-0.106
0.029	0.0377	29.29	29.27	-0.020
0.029	0.03915	30.41	30.40	-0.033
0.029	0.0406	31.54	31.53	-0.032
0.029	0.04205	32.67	32.66	-0.03
0.029	0.0435	33.79	33.76	-0.089

Table 2: Increasing Width and Depth of the Beam

Width	Height	Natural Frequency Fem(Hz)	Natural Frequency Regression(Hz)	%Error
0.029	0.029	22.47	22.47	0
0.03045	0.03045	23.60	23.61	0.04237
0.0319	0.0319	24.72	24.75	0.12135
0.03335	0.03335	26.08	26.89	3.10582
0.0348	0.0348	26.97	27.03	0.22246
0.03625	0.03625	28.09	28.16	0.24919
0.0377	0.0377	29.22	29.59	1.26625
0.03915	0.03915	30.34	30.38	0.13183
0.0406	0.0406	31.56	31.90	1.07731
0.04205	0.04205	32.59	32.70	0.33752
0.0435	0.0435	33.80	33.83	0.08875

4.2 Case Study 2

The dimensions of a T-structure are given in figure (4). The Cantilever Beam is divided into 6 elements. Then Element Stiffness Matrix, mass Matrix and natural frequencies are determined using MATLAB. Polynomial regression method is applied to this structure. The results from polynomial regression and FEM are compared for closeness.

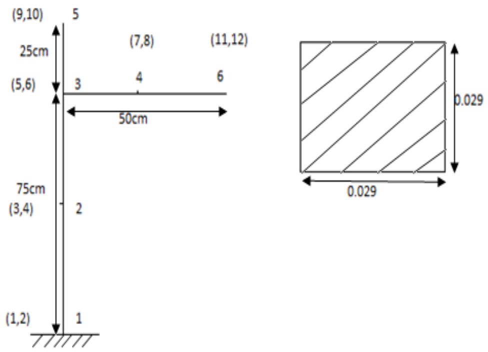

Figure 4: T-Structure with nodes and elements

The results are as follows: First natural frequencies of cantilever beam by increasing depth of the beam by 5%.
Fitting target of lowest sum of squared absolute error equal to 1.0240640782836770E^{-02},

C_1 =-8.043993E^{-02}, C_2 = - 1.35761E^{+01},
C_3 = 1.53493E^{+04}, C_4 = - 6.65185E^{-01}
C_5 = -3.99731E^{+02}, C_6 = 4.446583E^{+02}.

Natural frequencies of cantilever beam by increasing width and depth of the beam by 5%,
Fitting target of lowest sum of squared absolute error equal to 7.2417839254080818E^{+01},

C_1 =-3.483496E^{-00}, C_2 = 7.71712E^{+03},

C_3 = 7.71712E^{+03}, C_4 = - 1.8478E^{+01}

C_5 = -1.8478E^{+01}, C_6 = - 1.8478E^{+01}

Table 3:Increasing Depth of the Beam

Width	Height	Natural Frequency Fem (Hz)	Natural Frequency Regression (Hz)	%Error
0.029	0.029	444.69	444.71	4.4975×10^{-3}
0.029	0.03045	466.91	466.93	4.2834×10^{-3}
0.029	0.0319	489.16	489.17	2.0443×10^{-3}
0.029	0.03335	511.49	511.50	1.9550×10^{-3}
0.029	0.0348	533.63	533.65	3.7479×10^{-3}
0.029	0.03625	555.85	555.86	1.7990×10^{-3}
0.029	0.0377	578.02	578.04	3.4600×10^{-3}
0.029	0.03915	600.33	600.34	1.6657×10^{-3}
0.029	0.0406	622.57	622.57	0
0.029	0.04205	644.76	644.78	3.1019×10^{-3}
0.029	0.0435	667.02	667.04	2.0084×10^{-3}

Table 4: Increasing Width and Depth of the Beam(T-Structure)

Width	Height	Natural Frequency Fem(Hz)	Natural Frequency Regression(Hz)	%Error
0.029	0.029	444.36	444.36	0
0.03045	0.03045	465.79	466.43	0.137400
0.0319	0.0319	487.97	488.81	0.172141
0.03335	0.03335	514.76	515.18	0.17872
0.0348	0.0348	532.30	533.55	0.234741
0.03625	0.03625	554.51	555.93	0.256081
0.0377	0.0377	576.72	578.30	0.273963
0.03915	0.03915	598.87	600.68	0.302235
0.0406	0.0406	629.67	630.65	0.155635
0.04205	0.04205	643.26	645.43	0.181886
0.0435	0.0435	667.06	667.79	0.109435

CONCLUSION

From this work the following conclusions are drawn. The FEM method is applied for dynamic analysis of cantilever beam and T-structure using the MAT lab. Natural frequencies are obtained for cantilever and T-structure beams using FEM.the polynomial regression method is used for obtaining natural frequencies of cantilever beam and T-structure by varying width and depth for dynamic reanalysis.

The results obtained from reanalysis using regression method are close to results obtained using FEM. The minimum and maximum errors in regression method when compared with the results obtained by FEM are

	Minimum	Maximum
Cantilever beam	-0.36319 (with increasing width) 0 (with increasing width and depth)	-0.02000 3.10582
T-structure	0(with increasing width) 0(with increasing width and depth)	4.4975×10^{-3} 0.302235

REFERENCES:

[1]. M.M.Segura and J.T.celigilete, " a new dynamic reanalysis technique based on model synthesis" "Centro de studios e investigaciones tecnicas", de guipuzcoa(CEIT) apartado 1555, 20009 san Sebastian, spain (1994).
[2]. Z. Xie, W.S. Shepard Jr," Development of a single-layer finite element and a simplified finite element modelling approach for constrained layer damped structures", Finite Elements in Analysis and Design 45 (2009) pp530–537.
[3]. Uri kirsch , Michaelbogomolni , "analytical modification of structural natural frequencies"Department of civil and environmental engineering ,Technion- Israel institute of technology, Haifa 32000, Israel [2006].
[4]. Uri kirsch, G.Tolendano,"Finite Elements in Analysis and Design "Department of Civil and Environmental engineering , Technion Israel institute of technology, Haifa 32000, Israel.
[5]. M. Nad'a, "structural dynamic modification of vibrating system"* a Faculty of Materials Science and Technology, STU in Bratislava, Paulínska 16, 917 24 Trnava, Slovak Republic.
[6]. Tirupathi R.Chandrupatla , "Ashok D.Belegundu ,Rowan University Glassboro", new jersey and the Pennsylvania state university, university park, Pennsylvania.

[7]. Arthur Christopoulos and Michael J. Lew Critical Reviews in Biochemistry and Molecular Biology, 35(5):359–391 (2000).

[8]. Hill, A.V., The combinations of haemoglobinwith oxygen and with carbon monoxide. I, *Biochem. J.*, 7, 471 80, 1913.

[9]. Wells, J.W., "Analysis and interpretation of binding at equilibrium, in *Receptor-Ligand Interactions": A Practical Approach,* E.C. Hulme, Ed. Oxford University Press, Oxford, 1992, 289–395.

[10]. Hill, A.V., "The possible effects of the aggregation of the molecules of haemoglobin on its dissociation curves", *J . Physiol.* , 40, iv–vii,1910.

[11]. Hill, A.V., "The combinations of haemoglobin with oxygen and with carbon monoxide." I, *Biochem. J.*, 7, 471– 80,1913.

[12]. Motulsky, H.J., "*Intuitive Biostatistics"*, Oxford University Press, New York, 1995.

[13]. Motulsky, H.J., "*Analyzing Data with GraphPad Prism"*, GraphPad Software Inc., San Diego,CA, 1999.

[14]. Ludbrook, J., "Comparing methods of measurements", *Clin. Exp. Pharmacol. Physiol.*,24(2), 193–203, 1997.

[15]. Bates, D.M. and Watts, D.G., "*Nonlinear Regression Analysis and Its Applications"*, Wiley and Sons, New York, 1988.