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Rheotetrad And Spin Fluid

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ABSTRACT

Using the spin density fluid tensor, we studied torsion, bitorsion for spin module function σ and the proper density of the fluid ρ. It is shown that if spin and rotation are constant along velocity vector u^a then it is geodesic path. Also Rheotetrad used for central quadratic hyper surface.

Introduction :

Rheotetrad (u^a, P^a, Q^a, R^a) with signature (+, -, -, -) and Relativistic Serret Frenet formula are (Gursey F, 1957)

$u'^a = KP^a$
$P'^a = Ku^a + TQ^a$
$Q'^a = -TP^a + BR^a$
$R'^a = -BQ^a$

There was lot of research on geodesic path but for non-geodesic path some attension is necessary. In Einstein-Cartan spaces (with torsion) there are stationary solutions (Soleng H.H.) containing a perfect fluid with spin density, a spin fluid. In this paper the spin density in the fluid S_{ij} (J.P.Krisch) used to discuss torsion and bitorsion. Rotation tensor Ω_{ij} (Gursey F, 1957) also taken into consideration. On using equation of central quadratic hypersurface (Book on Tensor calculus and Riemannian Geometry by P.C. Agarwal publication Krishna Prakashan Mandir, Meerut (U.P.) India.) it is observed that torsion is infinite for flat surface.

Spin and Rheotetrad :

The spin density in the fluid Sij is (J.P. Krisch)

$$S_{ij} = \rho\sigma (P_iR_j – R_iP_j)$$

where σ is spin module function and ρ is the proper density of the fluid. On taking covariant derivative along vector u^a and using Serret Frenet formulae

$$S'_{ij} = (\sigma'\rho+\rho'\sigma) (P_iR_j - R_iP_j) + k\rho\sigma(u_iR_j - R_iu_j) + T\rho\sigma(Q_iR_j - R_iQ_j) + B\rho\sigma(Q_iP_j - P_iQ_j)$$

It is antisymmetrical tensor and if $\rho\sigma$ are non zero constants then spin is constant along velocity vector u^a iff it is geodesic path.

Following Carter and Mclenaghan, the spin tensor Sij is Penrose-Floyd tensor (Penrose and Word RS. 1980) if

$$\Delta_m S_{ij} - \Delta_j S_{im} = 0$$

It gives S_{ij} is penrose-floyd tensor iff ρ,σ are constants and

$$(P_iR_j), m - (P_iR_m),j = 0,$$
$$(R_iP_j), m- (R_iP_m),j = 0.$$

Antisymmetrical Rotation tensor :

The Anitisymmetrical Rotation tensor defined by (Gursey F, 1957) is

$$\Omega_{ab} = K (P_au_b - u_aP_b) + T (PaQ_b - Q_aP_b) + B(Q_aR_b-R_aQ_b)$$

On using relativistic Serret Frenet formulae, the covariant derivative along velocity vector u^a is

$$\Omega'_{ab} = K' (P_au_b - u_aP_b) + T' (PaQ_b - Q_aP_b) + B' (Q_aR_b - R_aQ_b)$$

It is observed that

i) Rotation along velocity vector u^a is constant Iff K, T, B are constants.
ii) Spin density fluid Sij is Perpendicular to rotation of the proper frame.

Killing Tensor and central quadratic Hyper surface :

Killing tensor :

$$K_{ij} = \Omega_i^{\ l}\Omega_{lj}$$

$$\therefore K_{ij} = K^2(P_iP_j - u_iu_j) -T^2(P_iP_j + Q_iQ_j) – B^2(Q_iQ_j+R_iR_j) - K(u_iQ_j+u_iQ_i) + TB (P_iR_j - R_iP_j).$$

It is symmetrical killing tensor.

If a fixed point O is taken as a pole and 's' is the distance of any point P the Riemannian coordinates u^i at P with pole at origin O are given by (Book. Tensor calculus and Riemannian Geometry by P.C. Agarval Publication krishna Prakashan mandir, Meerut (U.P.) India.)

$u^i = s\xi^i$

where ξ^i being unit tangent in the direction of OP. The equation of central quadratic hypersurface (Book. Tensor calculus and Riemannian Geometry by P.C. Agarwal Publication krishna Prakashan mandir, Meerut (U.P.) India.) given by

$u^i K_{ij} u^j = 1$

which implies $K^2 = 1$ or $K = \pm I$

Similarly, at $K^2 = 1$

$$P^i K_{ij} P^j = 1 \Rightarrow T=0$$

and $$Q^i K_{ij} Q^j =1 \Rightarrow B=0$$

Theorem : If $u^i = s\xi^i$ then the following are true

i) $K = \pm s$

ii) $T = \pm\sqrt{s^2 - 1}$

iii) $B^2 = -s^2$

Proof : If $u^i K_{ij} u^j = 1 \Rightarrow K^2 = s^2$ or $K = \pm s$

$P^i K_{ij} P^j = 1 \Rightarrow K^2 - T^2 = 1$ or

$T = \pm\sqrt{K^2 - 1} = \pm\sqrt{s^2 - 1}$

$Q^i K_{ij} Q^{ij} = 1 \Rightarrow -T^2 - B^2 = 1$

$: B^2 = -(1+T^2)$

$= -(1+s^2-1)$

$= -s^2$

Remark : a) as $s \to \infty$, $K \to \infty$. $\therefore$ curvature is infinite for flat space

b) for real value of T, $s \geq 1$

c) Bitorsion is imaginary

Conformal killing vector on S_{ij} :

The killing vector A_{nm} is

$$A_{nm} = S_{ni} S^i_m$$

$\therefore$ $A_{nm} = (\rho\sigma)^2 \ (R_n R_m - P_n P_m)$ (On simplification)

It is skew symmetric tensor and if

$K_{nm} = A_{nm}$

then $(\rho\sigma)^2 = -B^2$ and

$-(\rho\sigma)^2 = K^2 - T^2$

$\therefore K^2 - T^2 = B^2$

It follows that bitorsion B is dependent on T and K

There is trivial result, when at $T=K \Rightarrow B=0 \Rightarrow \rho\sigma=0$.

It is observed that for spin $\rho\sigma \neq 0$ and hence it is non-geodesic flow of the fluid.

Conclusion :

While searching the results on non-geodesic flow, it is seen that very few research is seen on accelerated motion or non-geodesic flow. For non geodesic flow it is necessary to concentrate on curvature, torsion and bitorsion for the worldline of a particle. If distance from origin to any point P is infinite then curvature if infinite. For central quadratic hypersurface, curvature and distance from origin to any point P are equal. For non trivial solutions (non geodesic flow), there will be non zero spin module function σ and the proper density ρ of the fluid's must be non-zero. The study of Geometry like curvature, torsion and bitorsion can be extended for the effects of chemical potential, Heat transfer, entropy on the worldline of a particle and non-geodesic models.

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