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Jitter Noise Power Reduction In Ofdm By Oversampling

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Abstract

The OFDM system has multi sub carriers to send the high speed data.At high data rates there will be chance of timing jitter in OFDM system due to mismatch of the sampling clock at the receiver with the transmission speed. The effects caused by timing jitter is a significant limiting factor in the performance of very high data rate OFDM systems. Oversampling can reduce the noise caused by timing jitter. Both fractional oversampling achieved by leaving some band-edge OFDM subcarriers unused and integral oversampling are considered. Oversampling results in a 3 dB reduction in jitter noise power for every doubling of the sampling rate. We can also observes that the total channel capacity of OFDM can be improved by oversampling.

Keywords: Timing jitter, over sampling, jitter noise power, OFDM

I. INTRODUCTION

Orthogonal frequency division multiplexing(OFDM) is becoming widely applied in wireless communications systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay. It has been used in digital audio broadcasting (DAB) systems, digital video broadcasting (DVB) systems, digital subscriber line (DSL) standards, and wireless LAN. OFDM is used in many wireless broadband communication systems because it is a simple and scalable solution to inter symbol interference caused by a multipath channel. Data rates in optical fiber systems are typically much higher than in RF wireless systems. At these very high data rates, timing jitter is emerging as an important limitation to the performance of OFDM systems. A major source of jitter is the sampling clock in the very high speed analog-to-digital converters (ADCs) which are required in these systems. Timing jitter is also emerging as a problem in high frequency band pass sampling OFDM radios. In OFDM, fractional oversampling can be achieved by leaving some band-edge subcarriers unused. Very high speed ADCs typically use a parallel pipeline architecture not a PLL for this model.

II.OVERVIEW OF OFDM SYSTEM

An OFDM signal is a superposition of N sinusoidal carriers with frequency separation F_N, each sub-carrier is modulated by complex symbols with period T_N equal to the inverse of the frequency separation, i.e. $T_N=1/F_N$. The modulated carriers overlap spectrally but, since they are orthogonal within a symbol duration (the kth carrier frequency is $f_k = f_o + k\, F_N$ where f_o is some reference frequency and $0< k < N$) , the signal associated with each sinusoid can be recovered as long as the channel does not destroy the orthogonality. In practice, the samples of the OFDM signal are generated by taking the inverse discrete Fourier –transform (IDFT) of a discrete-time input sequence and passing the transform samples through a pulse shaping filter. At the receiver dual transformations are implemented.

Two periodic signals are orthogonal when the integral of their product, over one period, is equal to zero. Definition of Orthogonal:

Continuous Time:

$$\int_0^T \cos(2\pi n f_0 t)\times\cos(2\pi m f_0 t)dt = 0 \quad (\mathrm{n} \neq \mathrm{m})$$

Discrete Time:

$$\sum_{k=0}^{N-1} \cos\left(\frac{2\pi k n}{N}\right)\times\cos\left(\frac{2\pi k m}{N}\right) = 0 \quad (\mathrm{n} \neq \mathrm{m})$$

The carriers of an OFDM system are sinusoids that meet this requirement because each one is a multiple of a fundamental frequency. Each one has an integer number of cycles in the fundamental period.

III.SYSTEM MODEL

Consider the high-speed OFDM system .The OFDM symbol period, not including the cyclic prefix, is T. At the transmitter, in each symbol period, up to N complex values representing the constellation points are used to modulate up to N subcarriers. Timing jitter can be introduced at a number of points in a practical OFDM system but in this letter we consider only jitter introduced at the sampler block of the receiver ADC. Ideally the received OFDM signals sampled at uniform intervals of T/N. The effect of timing jitter is to cause deviation τn

between the actual sampling times and the uniform sampling intervals. In OFDM systems while timing jitter degrades system performance, a constant time offset from the 'ideal' sampling instants is automatically corrected without penalty by the equalizer in the receiver.

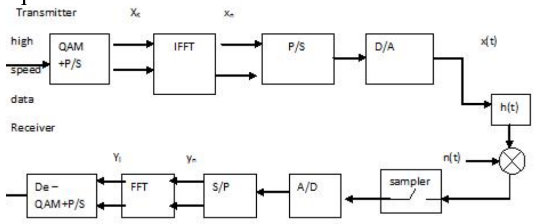


Fig: OFDM block diagram

In OFDM system there is no chance for inter carrier interference(ICI) because of the sub carriers are orthogonal to each other. where as CDMA and GSM technologies are single carrier systems .At high data rates there will be chance of timing jitter in OFDM system due to mismatch of the sampling clock at the receiver with the transmission speed.

IV.TIMING JITTER IN OFDM:

Timing jitter is τ_n often modeled as a wide sense stationary (WSS) Gaussian process with zero-mean and variance σn^2.

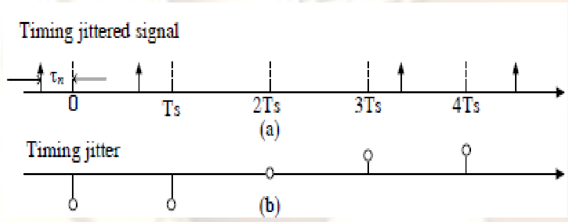


Fig: Definition of timing jitter

The effect of timing jitter can be described by a timing jitter matrix. The compact matrix form for OFDM systems with timing jitter is

$$Y = WHX^T + N \qquad (1)$$

where **X**, **Y** and **N** are the transmitted, received and additive white Gaussian noise (AWGN) vectors respectively, **H** is the channel response matrix and **W** is the timing jitter matrix.

where

$$Y = [Y_{-N/2+1} \ldots . \; Y_0 \ldots \; Y_{N/2}]^T$$
$$H = \mathrm{diag}(H_{-N/2+1} \ldots .. H_0 \ldots . \; H_{N/2})$$
$$X^T = [X_{-N/2+1} \ldots .. X_0 \ldots \ldots \; X_{N/2}]$$

Timing jitter causes an added noise like component in the received signal.

$$Y = HX^T + (W - I)HX^T + N \qquad (2)$$

where **I** is the $N \times N$ identity matrix. The first term in (2) is the wanted component while the second term gives the jitter noise .The elements of the timing jitter matrix **W** are given by

$$W_{l,k} = \frac{1}{N} \sum_{-\frac{N}{2}+1}^{\frac{N}{2}} e^{j2\pi k \frac{\tau_n}{T}} e^{j\frac{2\pi(k-l)n}{N}} \qquad (3)$$

where n is the time index, k is the index of the transmitted subcarrier and l is the index of the received subcarrier.

The timing jitter matrix is given by

$$\mathrm{W} = \begin{bmatrix} w_{-\frac{N}{2}+1,-\frac{N}{2}+1} & \cdots & w_{-\frac{N}{2}+1,0} & \cdots & w_{-\frac{N}{2}+1,\frac{N}{2}} \\ \vdots & \ddots & \vdots & ⋰ & \vdots \\ w_{0,-\frac{N}{2}+1} & \cdots & w_{0,0} & \cdots & w_{0,N/2} \\ \vdots & ⋰ & \vdots & \ddots & \vdots \\ w_{\frac{N}{2},-\frac{N}{2}+1} & \cdots & w_{\frac{N}{2},0} & \cdots & w_{\frac{N}{2},\frac{N}{2}} \end{bmatrix} \qquad (4)$$

V. OVERSAMPLING TYPES:

The effect of both fractional and integral oversampling in OFDM can be used to reduce the degradation caused by timing jitter. To achieve integral oversampling, the received signal is sampled at a rate of MN/T, where M is an integer. For fractional oversampling some band-edge subcarriers are unused in the transmitted signal. When all N subcarriers are modulated, the bandwidth of the baseband OFDM signal is $N/2T$, so sampling at intervals of T/N is Nyquist rate sampling. If instead, only the subcarriers with indices between $-N_L$ and $+N_U$ are non zero, the bandwidth of the signal is $(N_L + N_U)/2$. in this case sampling at intervals of T/N is above the Nyquist rate. The degree of oversampling is given by $(N_L + N_U)/N$.

VI.EFFCT OF OVERSAMPLING ON JITTER NOISE POWER:

In the general case, where both integral and fractional oversampling are applied, the signal samples after the ADC in the receiver are given by

$$y_{n_M} = y\left(\frac{n_M T}{NM}\right) = \frac{1}{\sqrt{N}} \sum_{k=-N_L}^{N_U} H_k X_k e^{\left(\frac{j2\pi k}{T} \times \frac{n_M T}{NM}\right)} + \eta\left(\frac{n_M T}{NM}\right) \qquad (5)$$

where n_M is the oversampled discrete time index and η is the AWGN. With integral oversampling, the N-point FFT in the receiver is replaced by an 'oversized' NM-point FFT. The output of this FFT is a vector of length NM with elements.

$$Y_{l_M} = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{NM}} \sum_{n_M=-NM/2+1}^{NM/2} y_{n_M} e^{\left(\frac{-j2\pi n_M l_M}{NM}\right)} \qquad (6)$$

where l_M is the index at the output of the NM point FFT. The modified weighting coefficients for the oversampling case,

$$w_{l_M,k} = \frac{1}{NM} \sum_{n_M=-NM/2+1}^{NM/2} e^{j2\pi k \frac{\tau_{n_M}}{T}} e^{j\frac{2\pi}{NM}(k-l_M)n_M} \quad (7)$$

By using the approximation $e^{j\theta} = 1 + j\theta$ for small θ, as

$$w_{l_M,k} \approx \frac{1}{NM} \sum_{n_M=-NM/2+1}^{NM/2} \left(1 + \frac{j2\pi k \tau_{n_M}}{T}\right) e^{j\frac{2\pi}{NM}(k-l_M)n_M}$$

in(5) and (7)

So for $k \neq l_M$ the variance of the weighting coefficients is given by

$$E\left\{\left|w_{l_M,k}\right|^2\right\} \approx \left(\frac{2\pi k}{MNT}\right)^2 \sum_{n_m} \sum_{d_m} E\{\tau_{n_M} \tau_{p_M}\} e^{j\frac{2\pi}{MN}(k-l)d_M} \quad (8)$$

Where $n_M - p_M = d_M$. When the timing jitter is white, then $\{\tau_{nM} \tau_{nM-d_M}\} = 0$ for $d_M \neq 0$ so

$$E\left\{\left|W_{l_M,k}\right|\right\} \approx \left(\frac{1}{NM}\right)\left(\frac{2\pi k}{T}\right)^2 E\{\tau^2{}_{n_M}\} \; k \neq l_M \quad (9)$$

From (9) it can be seen that white timing jitter $E\{|w_{lM},k|^2\}$ is inversely proportional to M so increasing the integer oversampling factor reduces the inter carrier interference (ICI) due to timing jitter. It is also that $E\{|wlM,k|^2\}$ depends on k^2 but not on l_M, so higher frequency subcarriers cause more ICI, but the ICI affects all subcarriers equally.

AVERAGE JITTER NOISE POWER FOR EACH SUBCARRIER

$$Y_{l_M} = H_{l_M} X_{l_M} + \sum_{k=-N_L}^{N_v} \left(W_{l_M,k} - I_{l_M,k}\right) H_k X_k + N(l) \quad (10)$$

where the second term represents the jitter noise. we consider a flat channel with, $H_k = 1$, and assume that the transmitted signal power is distributed equally across the used subcarriers so that for each used subcarrier $E\{X^2{}_k\} = \sigma^2{}_s$. Then the average jitter noise power, $Pj\,(l)$ to received signal power of lth subcarrier is given by

$$\frac{P_j(l)}{\sigma_s^2} = \frac{E\{|\sum_{k=-N_l}^{N_v}(W_{l,k} - I_{l,k})X_k|^2\}}{\sigma_s^2}$$

$$= \sum_{k=-N_L}^{N_v} E\{|W_{l,k} - I_{l,k}|^2\}$$

Rearranging the terms in (11)equation

$$\frac{P_j(l)}{\sigma_s^2} = \frac{\pi^2}{3M}\left(\frac{N_v N}{T_N^2}\right) E\{\tau_N^2\} \quad (12)$$

If there is no integral oversampling or fractional oversampling ,M = 1 and $Nv = N$,

$$\frac{P_j(l)}{\sigma_s^2} = \frac{\pi^2}{3}\left(\frac{N^2}{T_N^2}\right) E\{\tau_N^2\} \quad (13)$$

Comparing (12) and (13) it can be seen that the combination of integral oversampling and fractional oversampling reduces the jitter noise power by a factor of Nv/NM.

CHANNEL CAPACITY

OFDM system can be considered as a number of parallel subchannels, one for each subcarrier. The total channel capacity can be found by summing the capacity of each sub carrier channel .By applying Shannon's capacity formula,the capacity of the lth subcarrier is given by

$$C_l = \log_2(1 + SNR_l) \quad (14)$$

Where SNR_l is the SNR on lth subcarrier.

The total channel capacity of the OFDM system is given as follows.

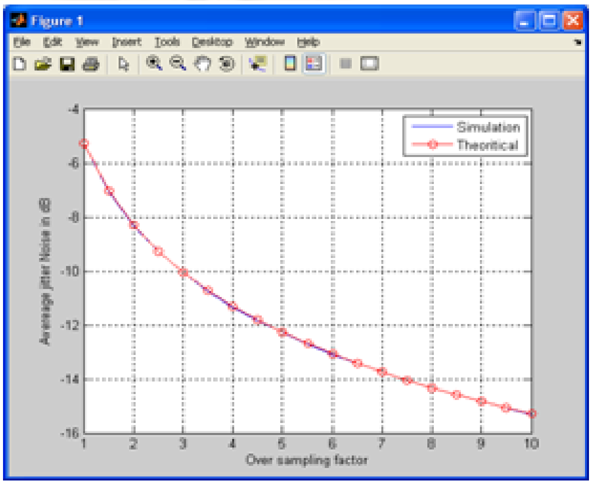
$$C_T = \sum_{l=-\frac{N}{2}+1}^{N/2} \log_2(1 + SNR_l) \quad (15)$$

So from the equations (12) and (13) we can observe that the total channel capacity of OFDM system will increases after oversampling compare with the total channel capacity before oversampling.

VII.SIMULATION RESULTS:

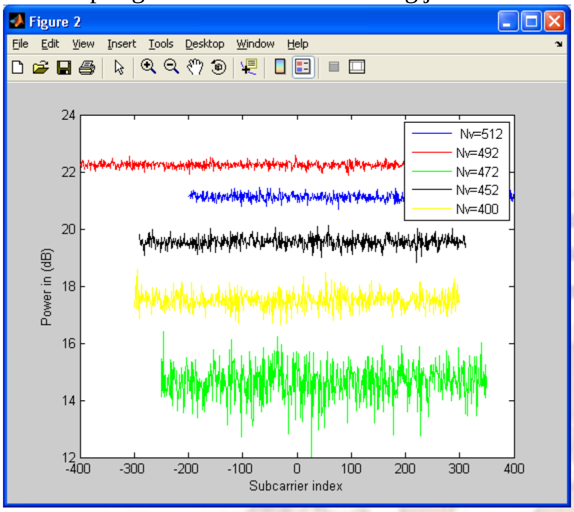
By using matlab software ,the simulation results are shown below.

Figure 1: The graph is drawn between the oversampling factor and jitter noise power .



Result: due to oversampling ,whenever increases the sampling factor by two ,there was the decreases in jitter noise power by 3db .

Figure2: The graph is shown below for the fractional oversampling how to reduce the timing jitter.



Result: The jitter variance is not changed when oversampling is applied, so the jitter represents a larger fraction of the sampling period for the oversampled systems. The variance of the noise due to jitter as a function of received subcarrier index when band-edge subcarriers are unused. It shows that the power of the jitter noise is not a function of subcarrier index and that removing the band-edge subcarriers reduces the noise equally across all subcarriers. Average jitter noise power as a function of the oversampling factor. There is close agreement between theory and simulation.

Increasing the sampling factor gives a reduction of jitter noise power, so every doubling of the sampling rate reduces the jitter noise power by 3 dB.

VIII. CONCLUSIONS

It has been shown both theoretically and by simulation that oversampling can reduce the degradation caused by timing jitter in OFDM systems. Two methods of oversampling were used: fractional oversampling achieved by leaving some of the band-edge subcarriers unused, and integral oversampling implemented by increasing the sampling rate at the receiver. For the case of white timing jitter both techniques result in a linear reduction in jitter noise power as a function of oversampling rate. Thus oversampling gives a 3 dB reduction in jitter noise power for every doubling of sampling rate. It was also shown that in the presence of timing jitter, high frequency subcarriers cause more ICI than lower frequency subcarriers, but that the resulting ICI is spread equally across all subcarriers.

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