

## **Assessment of load-carrying capacity of bored pile in clay soil using different methods**

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### **Abstract**

It is very difficult to predict the load carrying capacity of bored piles because of the complications that may arise such as difficult ground conditions, presence of ground water, method of boring, method of concreting, quality of concrete, expertise of the construction staff, the ground conditions and the pile geometry. Therefore the Pile design must be accompanied by in situ load testing. Many geotechnical codes emphasize that pile design must be based on static load tests or on calculations that have been validated by three tests.

This paper describes the simulation of four piles with different lengths carrying different loads embedded in the clay soil in Khartoum town-Sudan using BS 8004, Monte Carlo simulation using Matlab software and Finite Element Code -Plaxis software. The results from these methods were compared together and recommendation are made to estimate the pile capacity in these soils. The design parameters of pile are estimated, and back calculation of safety factors are made.

Keyword: BS code, Monte Carlo simulation, Finite element method

### **Introduction**

Bored pile is type of reinforced concrete pile which is used to support high building that has heavy vertical load. Normally bored piling has to be carried on those tall buildings or massive industrial complexes, which require foundations which can bear the load of thousands of tons, most probably in unstable or difficult soil conditions.

Bored piles resist the uplift load by skin friction forces, and the formulae used to determine the magnitude of these forces seems to be an area where various codes and standards produce completely different results, the German code DIN 1054 (DIN 1996), which is widely used in Germany and other European countries, the uplift capacities strongly depend on the strength of the soil, i.e., the angle of friction normally depend determined by indirect methods such as standard penetration tests (Krabbenhoft et al. 2008); whereas the methods proposed by Fleming et al.(1992), mainly used in UK, take strength of the soil into account. whereas

the method proposed by Reese and O'Neill(1994), mainly used in USA, almost ignore the strength of the soil as long as the soil can be characterized as being frictional soil (Krabbenhoft. 2008).

### **Geological setting**

The study area is the part of Khartoum basin, which is the one of the major central Sudan rift basin. The sedimentary sub-basin is elongated in NW-SE trend, where the Pan-African Basement complex bounds it on the northeast and southwest, and forms its bottom limit at a depth 500m. The sub-surface geology belongs to three Formations, which are regionally interconnected. These Formations are the (upper recent) superficial deposits and river alluvium, which rest unconformable on the Gezira Formation (quaternary-tertiary) and the upper part of Omdurman Formation (upper cretaceous) (Awad, 1994). Most of the surface is covered by clay soils, which varies in thickness. consists of unconsolidated clay, silts, sand and gravel. Its rests unconformable on the Cretaceous sandstone formation and is overlain by blown sand and other superficial. Awad (1994) suggested that it comprise the area between white and blue Nile. Abdelsalam (1966) divided Gezira formation into three members lower Mungata Member, Lower sandy Member and upper clay Member. Awad (1994) considered Wad Medani Member as part of Gezira formation.

### **Engineering properties of subsurface soil**

The boreholes revealed existence of alternating layers of very stiff low to high plasticity silty clays (CL to CH) and very stiff low to high plasticity silts (ML to MH) in the upper 10 meters. This is underlain by medium dense sand (SM or SP-SM) layer extended down to 20 meter and this layer overlain a very dense sand layer extend 25 depth. The alternating layers of weak mud-stone and weak sand stone extended down to the bottom of the boreholes at about 35 meters. These weak mud-stone and weak sandstone are belong to Omdurman formation which are extended to deepest depth. Data from the various exploration methods were used as a basis for typical sections to illustrate the more significant geological conditions. The objectives is to illustrate clearly the problems of the geologic environment influencing design and

construction. Three dimensional sections and fence diagram were also plotted to help for sites with

complex geology, Fig 1.

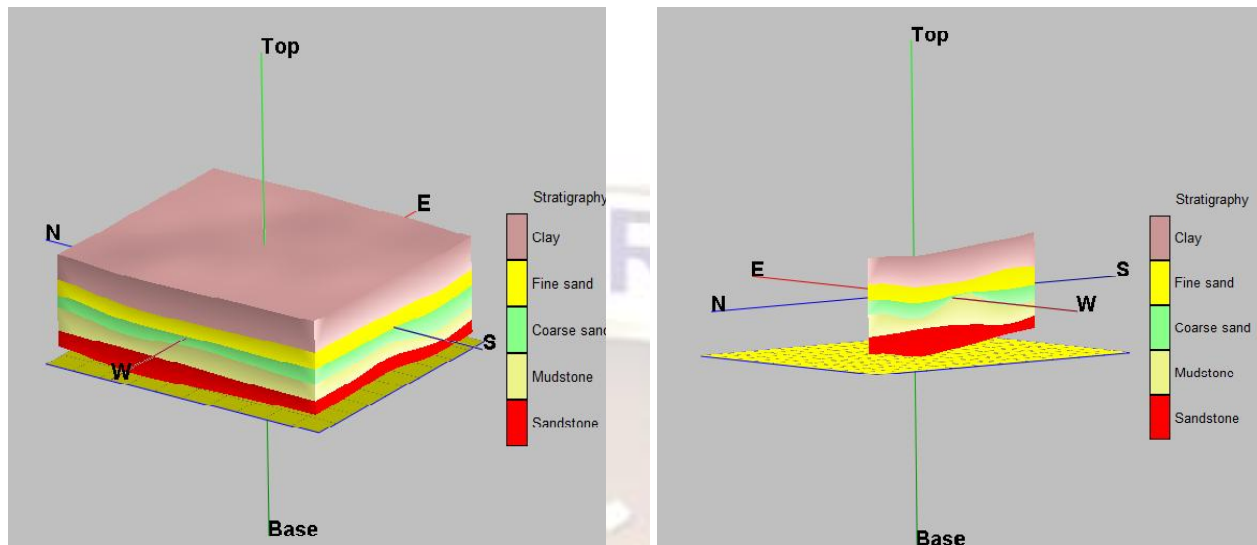


Fig 1 Three dimension and fence diagram models for the subsurface soil in the study area

### Axial response of deep foundation

The axial load-displacement response of driven piles and drilled shafts may be expressed in terms of elastic continuum theory. Solutions have been developed using boundary element formulations (Poulos and Davis, 1980; Poulos, 1994), finite elements (Jardine et al. 1986), and approximate closed form solutions by Randolph (1979). The generalized method characterizes the soil by two elastic parameters: soil modulus ( $E_s$ ) and Poisson's ratio ( $\nu_s$ ). Soil modulus may be either uniform with depth (constant  $E_s$ ) or a Gibson-profile (linearly increasing  $E_s$  with depth). The pile may either be a floating-type or end-bearing type where the tip is underlain by a stratum of stiffer material. The elastic theory solution for the vertical displacement ( $\delta$ ) of a pile foundation subjected to axial compression loading is expressed by

$$\delta = \frac{Q * I_p}{E_{sL} * d}$$

where  $Q$  = applied axial load at the top of the shaft,  $E_{sL}$  = soil modulus at the top the pile tip or foundation base,  $d$  = foundation diameter, and  $I_p$  = influence factor. Solution for  $I_p$  depend on the pile slenderness ratio ( $L/d$ ), pile modulus, and soil modulus (Randolph & Wroth, 1979; Poulos & Davis, 1980; Poulos, 1989). The modified form of the expression to account for nonlinear modulus degradation is:

$$\delta = \frac{Q * I_p}{E_{max} * d} * (1 - Q/Qu)^{0.3}$$

**Settlement of a single pile**

The settlement of a single rigid pile in homogeneous elastic half space can be determined based on theory of elasticity (Randolph & Worth 1978) (in(EI-Mossallary & Lutz 2006) as follows:

$$\frac{Q}{G r_0 S_{single}} = \frac{4}{E(1-\nu)} + \frac{2\pi L}{\xi r_0} \quad (8.30)$$

$$\xi = li(r_m/ro)$$

where:

$Q$  = Applied load

$S_{single}$  = Settlement of single pile

$G$  = shear modulus of the soil

$r_0$  = pile radius

$L$  = length of the pile

$r_m$  = influence radius at which shear stresses become negligible. Randolph (1977) has suggested

$r_0 = 2.5 L(1-\nu)$  based on a parametric study using an axi-symmetrical finite element analysis.

The above-mentioned equation may be modified to consider approximately the pile stiffens, the soil in homogeneity in vertical direction, the thickness of the compressible layer (EI-Mossallamy et. al 2006) and nonlinear soil stress/strain behavior adjacent to the pile shaft (Randolph 1977 and Randolph & Worth 1978). Another possibility to determine the settlement of a single pile is to use the chart of Poulos (Poulos & Davis 1980) or to apply the recommendation values as give by standards (DIN 1054-100)

### Pile Design Parameters

Fieldwork and laboratory tests have been carried out during the geotechnical investigations to determine pile parameters and to obtain a full overview of subsurface soils to minimize uncertainties. Determination of undrained cohesion was carried out by unconfined compression tests on undisturbed samples, Figure 2 shows the variation

of undrained cohesion ( $C_u$ ) with no trend with depths, This large scatter of the undrained cohesion values at different depths is due to the variation of seasonal depositional materials. Adhesion factors were calculated according to different sources. The data used to calculate soil parameters and bearing capacities are shown in Table 1.

**Table 1 data used in calculations**

Depth (meter)	Dry density (kN/m <sup>3</sup> )	Friction angle ( $\phi$ ) degree	Cohesion (C) kN/m <sup>2</sup>	Undrained shear strength $C_u$ (kN/m <sup>2</sup> )
2	12.56	3	44.9	46.216
4	12.9	6	17	23.054
6	13.4	11	15	30.6282
8	13.6	9	50	71.459
10	13	8	18.8	37.0703
12	13.1	8	40	62.093
14	13.3	17	33	89.9271
16	12.6	14	20	67.123

The estimation of adhesion factors obtained by using different sources is shown in Table 2

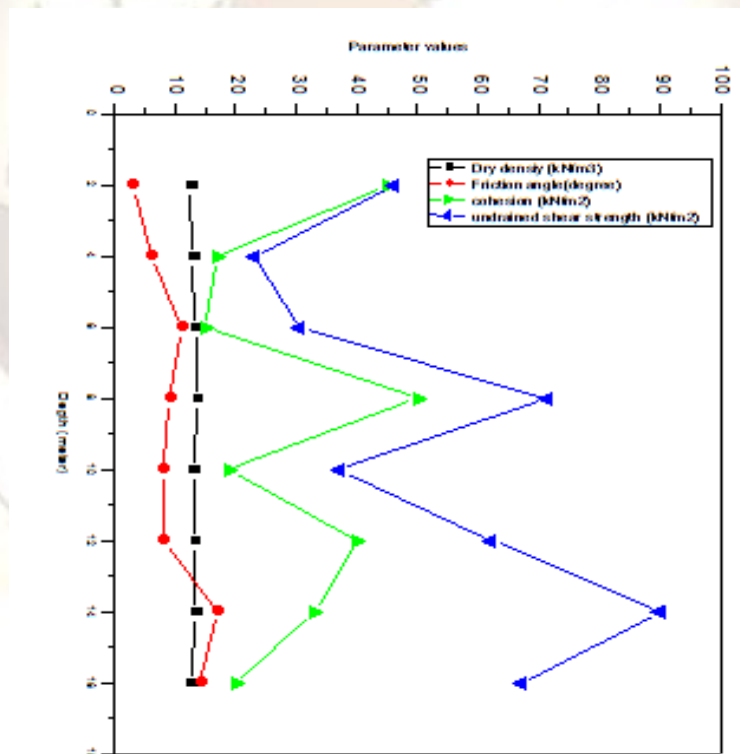


Figure 2 variation of undrained shear strength, no trend with depth

Table 2 Calculating of adhesion factor for different sources ( $\alpha$ )

Depth (meter)	Undrained shear strength kN/m <sup>2</sup>	Adhesion factor (Bowel)	Peck et al. (1974)	Adhesion factor (EM 1110-2-2906)	(Das 1995)
2	46.216	0.98	0.93	0.82	0.97
4	23.054	1.05	1.0	1.1	0.98
6	30.6282	1.07	0.98	0.95	0.91
8	71.459	0.85	0.89	0.83	0.68
10	37.0703	1	0.96	0.87	0.87
12	62.093	0.92	0.92	0.65	0.75
14	89.9271	0.78	0.85	0.5	0.6
16	67.123	0.89	0.9	0.66	0.7

For better visualization of the differences of adhesion factor obtained from different sources using undrained shear strength Figure 3 is drawn.

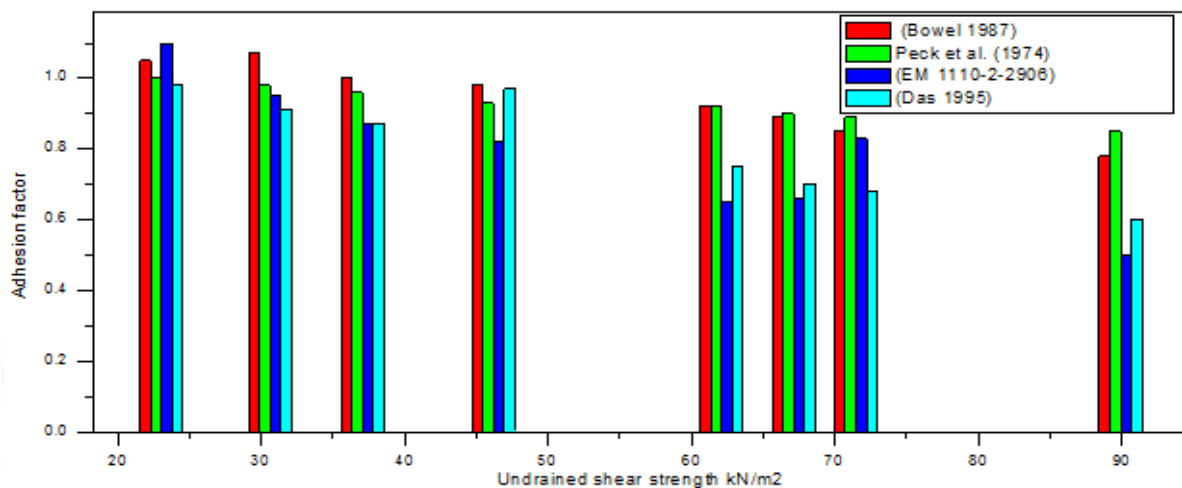


Figure 3 Adhesion factor variations according to different sources

### Determination of bearing capacity of pile

For circular cross sectional pile, ultimate load capacity of pile using shear strength parameters may calculated using the following formulae (Tomlinson 1957)

$$q_{ult} = 9 * C_U * \pi * (d/2)^2 + \alpha * c_{uav} * d * L$$

$$\text{pile base resistance} = 9 * C_U * \pi * (d/2)^2$$

$$\text{pile Shaft resistance} = \alpha * c_{uav} * d * L$$

$q_{ult}$  = Ultimate load capacity of pile

$c_u$  = undrained shear strength (low value around the pile tip)

$$\pi = 3.14159265$$

d = diameter of the pile

L = length of the pile (imbedded in the soil)

$\alpha$  = adhesion factor

$c_{uav}$  = average undrained shear strength along the pile length

### Calculation the bearing capacity of pile

For hypothetical calculation the lengths and diameter of the pile are 10, 12, 14, 0.5, meter respectively. The soil parameters and adhesion factor are taken from Table 2. The low value of undrained shear strength (37.0703 kN/m<sup>2</sup>) around the pile tip is used to determine the base resistance, where as the average undrained shear strength along the pile shaft (41.6855) is used to determine the shaft resistance using the above equation for different adhesion factors. The mean adhesion factor is used to calculate the average undrained shear strength along the pile length Table 3. For easy calculation to obtain bearing capacities a software code using Matlab was written. To calculate the net bearing capacity a partial factors of 3, 1.5 were applied to the base and shaft resistance respectively. The result of calculations are shown in Table 4.

Table 3. The result of mean adhesion factors

Mean adhesion factor (Bowel 1987)	Mean adhesion factor Peck et al. (1974)	Mean adhesion factor (EM 1110-2-2906)	Mean adhesion factor (Das 1995)
0.99	0.952	0.914	0.882

Table 4: Results of bearing capacities for bore pile with different pile lengths and adhesion factors

Pile length (m)	Pile diameter(m)	$\alpha$	Base resistance kN/m <sup>2</sup>	Shaft resistance kN/m <sup>2</sup>	Ultimate bearing capacity kN/m <sup>2</sup>	Design strength value of pile kN/m <sup>2</sup>
10	0.5	0.99	65.51	1297	1362	886.3
		0.952	65.51	1247	1312	853.1
		0.914	65.51	1197	1262.6	819.9
		0.885	65.51	1159	1225	794.6
12	0.5	0.99	109.7	1683	1793	1159
		0.952	109.7	1619	1728	1116
		0.914	109.7	1554	1664	1073
		0.885	109.7	1505	1614.7	1040
14	0.5	0.99	158.9	2242	2401	1548
		0.952	158.9	2156	2315	1491
		0.914	158.9	2070	2229	1433
		0.885	158.9	2005	2164	1390
16	0.5	0.99	124.2	2662	2787	1816
		0.952	124.2	2560	2684	1748
		0.914	124.2	2480	2582	1680
		0.885	124.2	2380	2504	1628

## Result

The above analysis of bearing capacity of piles using BS code indicates that the net allowable load capacity of 0.5 diameter piles, with a length of about 10 m embedded into clay soil is estimated about 886.3 kN and 794.6 kN, respectively using different adhesion factors, where as for the pile with, 12 meter in length and 0.5 meter diameter is 1159 kN/m<sup>2</sup> and 1040 kN/m<sup>2</sup>, and for 14 meter length the results show that the bearing capacity is 1548 to 1390. The bearing capacity for 16 meter length pile it around 1816 kN/m<sup>2</sup> and 2628 kN/m<sup>2</sup> Even the estimation of the ultimate pile capacity by various sources seems to be close to each other, Figure 4 shows that the comparison of bearing capacities obtained using various adhesion factors reveal that the pile bearing capacity using adhesion factor suggested by Bowel (1987) gave high value(951 kN/m<sup>2</sup>), where as the most conservative result is given by Das 1995 with ( 856 kN/m<sup>2</sup>).

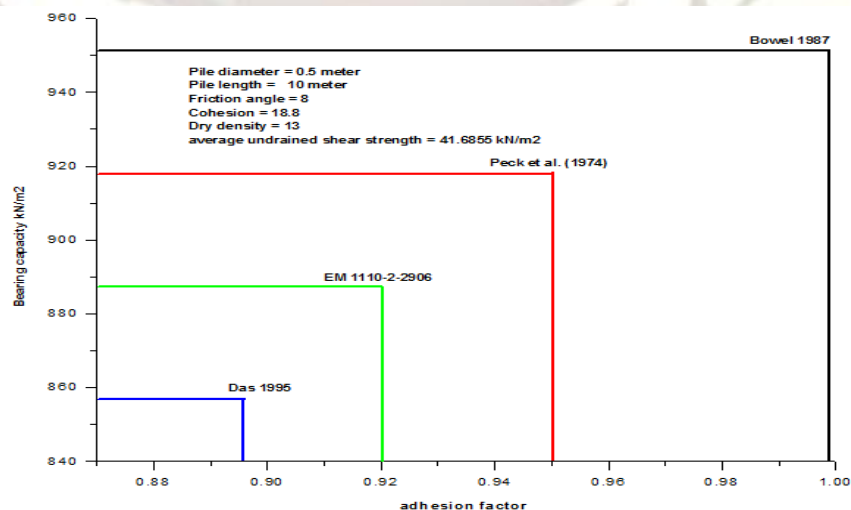


Figure 4 comparison of bearing capacities obtained using various adhesion factors

**Bearing capacity of pile using Monte Carlo simulation**

Monte Carlo is a class of computational algorithms that rely on repeated random sampling to compute their results (Silver et al 2010). Monte Carlo methods are often used in computer simulations of physical and mathematical systems. Monte Carlo methods vary, but tend to follow a particular pattern:

1. Define a domain of possible inputs.
2. Generate inputs randomly from a probability distribution over the domain.
3. Perform a deterministic computation on the inputs.
4. Aggregate the results.

Different pile lengths imbedded in clay and silty soil are consider for Monte Carlo simulation using Matlab software. software program to calculate bearing capacity of pile was written, all the values used are the same as those used to calculate the bearing capacities using BS 8004 except random values have been taken to undrained shear strength as an important parameter for calculating the design value of bearing capacity of piles.

This code is optimize the time calculations with factor of (200) less than that if loop command is used. The calculation for one operation is only took three seconds compare to 10 minutes when loop command is used.

The results of calculation using Monte Carlo simulation is shown in Table 5.

Table 5 results from Monte Carlo simulation

Base resistance kN/m <sup>2</sup>			Shaft resistance kN/m <sup>2</sup>			Ultimate resistance kN/m <sup>2</sup>			Design value kN/m <sup>2</sup>			$\alpha$	Pile length & diameter
mean	min	max	mean	min	max	mean	min	max	mean	min	max		
66.8	65.5	68	1312	129	132	1379	136	139	897	886	907.	0.99	L= 10 m D = 0.5
	1			7	0		2	6			8		
66.8	65.5	68	1262	124	127	1329	131	134	863.	853	873.	0.95	
	1			7	7		2	5	5		8	2	
66.8	65.5	68	1211	119	122	1278	126	129	829.	819	839.	0.91	L= 12 m D = 0.5
	1			7	6		3	4	9		9	4	
66.8	65.5	68	1173	115	118	1240	122	125	804.	794.	813	0.88	
	1			9	7		5	5	3	6		5	
111.	109.	112.	1702	168	172	1813	179	183	1172	1159	1185	0.99	L= 12 m D = 0.5
	7	7		3	1		3	3					
111.	109.	112.	1637	161	165	1748	172	176	1128	1116	1141	0.95	
	7	7		9	5		8	7				2	
111.	109.	112.	1571	155	158	1682	166	170	1084	1073	1097	0.91	L= 12 m D = 0.5
	7	7		4	9		4	1				4	
111.	109.	112.	1521	150	153	1633	161	165	1051	1040	1063	0.88	
	7	7		5	8		4	1				5	

Table 5 continue

Base resistance kN/m <sup>2</sup>			Shaft resistance kN/m <sup>2</sup>			Ultimate resistance kN/m <sup>2</sup>			Design value kN/m <sup>2</sup>			$\alpha$	Pile length & diameter
mean	min	max	mean	min	max	mean	min	max	mean	min	max		
162.7	158.9	166.5	2264	2242	2286	2427	2401	2452	1564	1548	1580	0.99	L= 14 m D = 0.5
162.7	158.9	166.5	2177	2156	2198	2340	2315	2365	1506	1491	1521	0.952	
162.7	158.9	166.5	2090	2070	2111	2253	2229	2277	1448	1433	1463	0.914	
162.7	158.9	166.5	2024	2005	2044	2187	2164	2210	1404	1390	1418	0.885	
127.7	124.2	131.2	2588	2563	2613	2715	2687	2744	1767	1750	1785	0.99	L= 16 m D = 0.5
127.7	124.2	131.2	2488	2464	2512	2616	2589	2644	1702	1684	1719	0.952	
127.7	124.2	131.2	2389	2366	2412	2517	2490	2543	1635	1619	1652	0.914	
127.7	124.2	131.2	2313	2291	2336	2440	2415	2467	1585	1569	1601	0.885	

The above table shows increasing of base resistance (66.8 to 162.7 kN/m<sup>2</sup>) until depth of fourteen meters and then value decrease sharply (127.7 kN/m<sup>2</sup>) Figure 5.

resistance, shaft resistance, ultimate pile resistance and design value respectively.

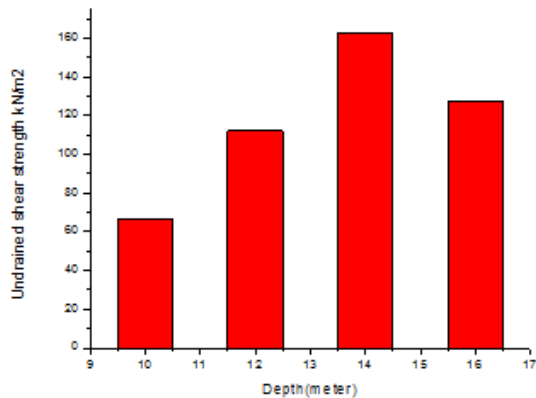


Figure 5 increasing the base resistance up to limit depth

The undrained shear strength ( $C_u$ , for cohesion soils) is the product of depth, effective unit weight of the soil and the cohesion ( $c_u = h * \gamma * \tan(\alpha) + c$ ), it is obvious increasing the depth if (unit weight of the soil and the cohesion) are constant increases the overall value of undrained shear strength which increase the product of bearing capacity. The decreasing of bearing capacity after limit depth is due to change of the soil strata to become more weaker.

The increase of bearing capacity with increase of adhesion factor is noted, for 10 meter pile length the adhesion factor suggested by Bowel (1987) gave high value (897 kN/m<sup>2</sup>), where as the most conservative result is given by Das 1995 with (804.3 kN/m<sup>2</sup>) Figure 6.

The comparison between the bearing capacity design value obtained by theoretical calculations and Monte Carlo simulation show that the results from Monte Carlo method is conservative, the correlation is shown in figure6 . Figures 7, and 8 show perfect distribution to the results of base

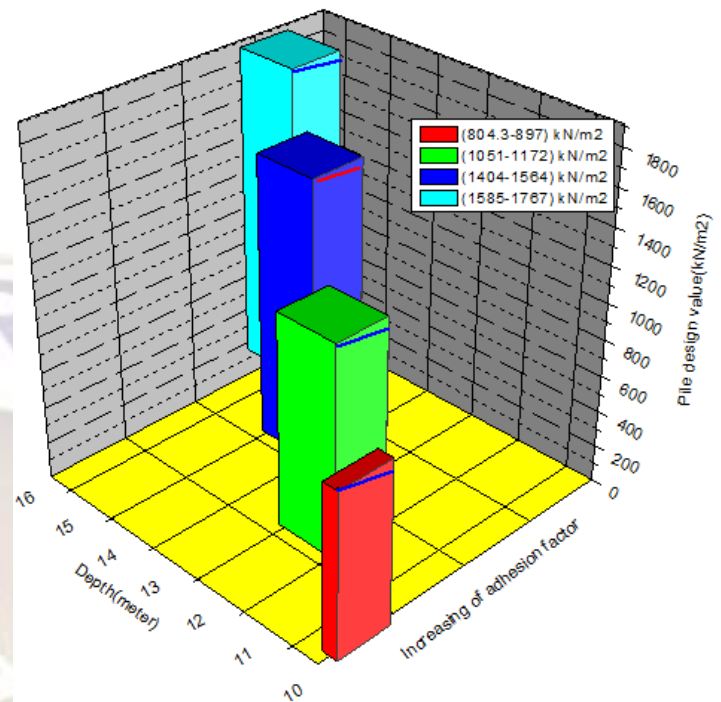


Figure 6 increasing bearing capacity with increasing of depth and adhesion factor

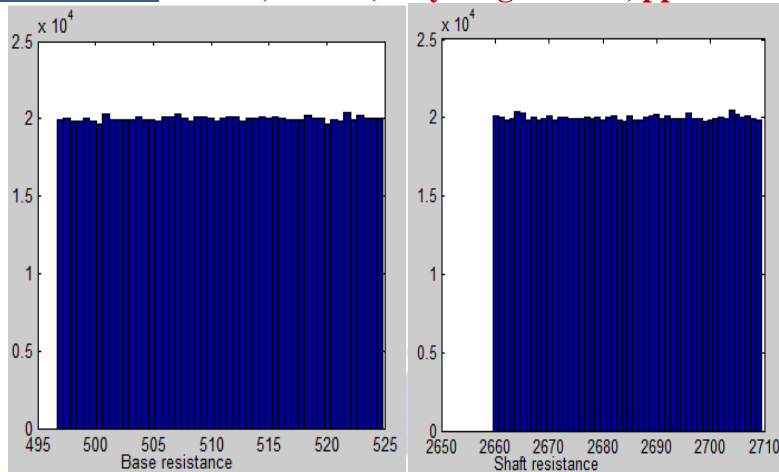


Figure 7 Base resistance and shaft resistance (kN/m<sup>2</sup>) using Monte Carlo simulation

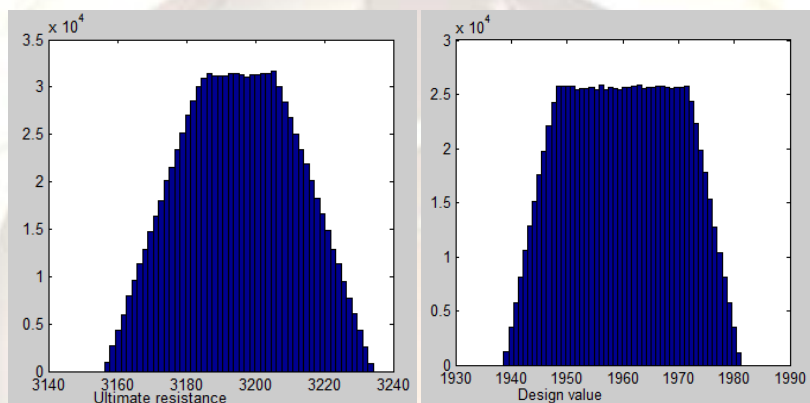


Figure 8 Ultimate resistance and design value (kN/m<sup>2</sup>) using Monte Carlo simulation

### Finite element Simulation

Finite element techniques are popular in recent years in the field of foundation engineering. To date, a variety of finite element computer programs have been developed with a number of useful facilities and to suit different needs E.Y. N OH et al., (2008) . The behavior of soil is also incorporated with appropriate stress-strain laws as applied to discrete elements. The finite element method provides a valuable analytical tool for the analysis and design of foundations.

The pile imbedded 16 meter depth and 0.5 diameter is gained high bearing capacity. This pile will consider in finite element analysis using Plaxis software.

### Create the model

Model is usually used as a first approximation of soil behavior. Due to its simplicity, it is highly popular and gives reasonable results. The model involves five parameters, i.e. Young's modulus, E, Poisson's ratio,  $\nu$ , cohesion, c, internal friction angle,  $\phi$ , and dilatation angle,  $\psi$ .

### Pile model

The piles were model as Linear Elastic Model This model represents Hooke's law of isotropic linear elasticity. The model involves two elastic stiffness parameters, i.e. Young's modulus, E, and Poisson's ratio,  $\nu$ . The linear elastic model is seldom used to simulate soil behavior. It is primarily. used for stiff massive structural systems install in the soil, such

as the test piles in this paper.

The model was drawn sufficiently large so that the boundaries do not influence the results of the problem to be studied Figure 9.



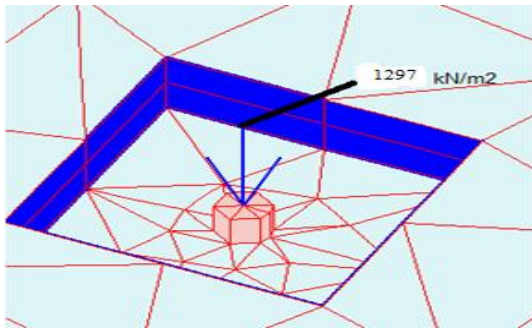


Figure 9 3D model of the pile showing point load 1297 kN/m<sup>2</sup>

Table 6 shows the results of allowable pile bearing capacity estimated from finite element analysis. Compare with the results obtained from calculations using BS code and Monte Carlo simulation, finite element results are the most conservative. Back calculations of factors of safety show that finite element gained highest factor of safety for pile embedded to 10 meters (1.7) depth and the value decrease with the increasing of pile length to reach (1.41) for 14 meters pile length. Whereas the factor of safety is constant for calculation using BS standard and Monte Carlo simulation for all piles lengths Table 7.

Table 6 the result of finite element method

Pile length (meter)	Ultimate bearing capacity kN/m <sup>2</sup>	Design value kN/m <sup>2</sup>	Safety factor
10	1362	817.2	1.7
	1312	817.4	1.61
	1262.6	824.5	1.53
	1225	815.9	1.5
12	1793	1185.2	1.51
	1728	1188.9	1.45
	1664	1186.4	1.4
	1614.7	1185.2	1.36
14	2401	1537.2	1.56
	2315	1537.2	1.51
	2229	1533.6	1.45
	2164	1530	1.41

Table 7 Safety factors from back calculations for all three methods used in this study

Length of pile (m)	Factor of safety		
	BS 8004	Monte Carlo	Finite element
10	1.54	1.54	1.5-1.7
12	1.55	1.55	1.36-1.51
14	1.55	1.55	1.41-1.56
16	1.53	1.54	

Load/settlement curves for all piles are shown in figure 10.

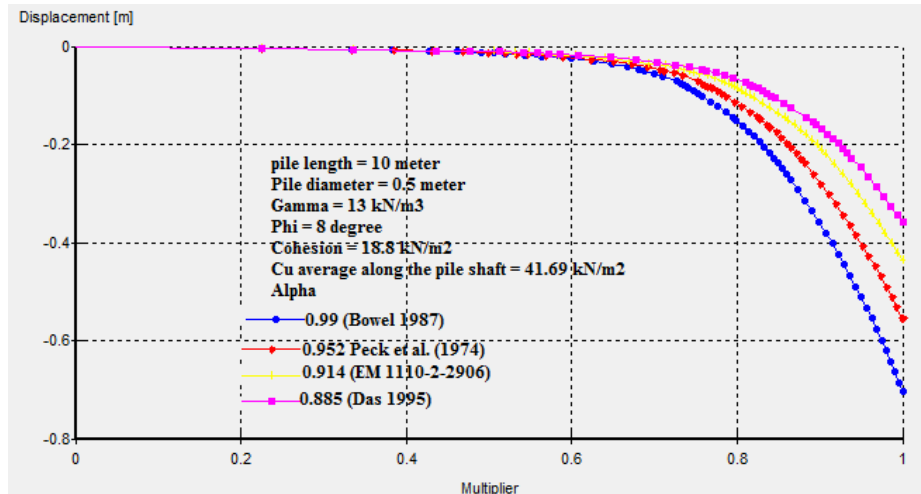


Figure 10 load/displacement for 10 meters pile.

The shear stresses and vertical displacement are shown in figures 11

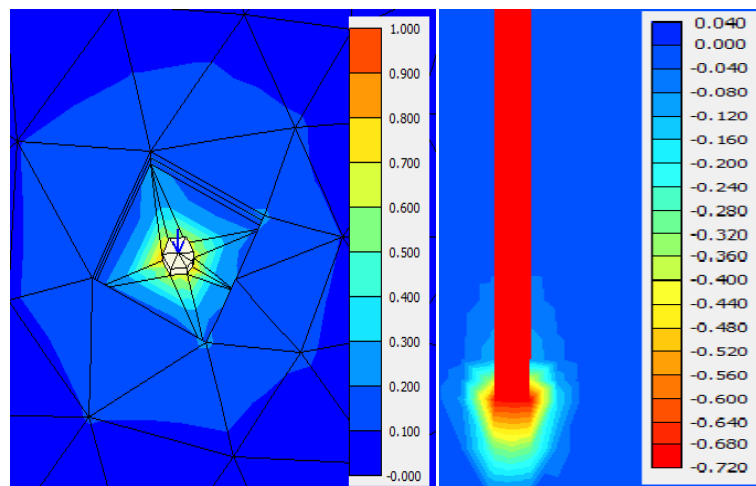


Figure 11 Relative shear stresses and vertical displacement

### Recommendations

Proper determination of adhesion factor is vital when calculating the bearing capacities for clay soils using undrained shear strength. Pile net allowable load determined from load/settlement curves by using finite element method are lower than theoretically computed capacities for all the piles using BS code and Monte Carlo simulation, whereas Monte Carlo simulation gave higher values.

For all piles and all methods used in this study more than 94% of the load is carried along the pile shaft and less than 6% is resist by pile base.

The bearing capacities obtained when using various adhesion factors for the same pile lengths, diameters and parameters embedded in the same soil strata have shown high differences, no or few differences is obtained when finite element methods is used.

The factors of safety obtained from back calculations using Monte Carlo method and BS code have the same value for all different adhesion factors for all pile lengths, whereas the factors of safety determined from finite element code decreases with increase of pile length.

The combinations of finite element method, Monte Carlo simulation and calculations of bearing capacities using empirical equations reduce the uncertainties associated with the determination of allowable design value for pile foundation

Monte Carlo method accurately characterizes, the values of base and shaft bearing capacities this method is recommended to be used in pile foundation modeling

The behavior and distribution of the load along the pile shaft and base is clearly noted in the 3D models obtained from finite element code, this

method is also recommended for pile foundation design.

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