

A COMPLETE APPROACH OF OPTIMIZATION AND SIMULATION OF JOSEPHSON JUNCTION AS A SWITCHING ELEMENT

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ABSTRACT

A systematic theoretical-approach has been made to estimate various parameters, characteristics and their optimization methods for a Josephson junction to be useful as a high-speed switching element in Josephson digital logic electronics. The dynamic response of Josephson junction has been obtained using computer simulation. Our concept of turn-on delay of Josephson junction is applied in order critically assign the switching speed of Josephson junction as a switching element. This paper gives complete understanding of switching dynamics of a Josephson junction and device parameters optimization techniques for scientists and researchers before they are fabricated experimentally.

1. INTRODUCTION

Eventhough the conventional Josephson technology is much superior to semiconductor technology concerning speed and power consumption, it is yet necessary to further increase the speed and develop optimization techniques in obtaining better features of the junction over the present day Josephson technology. Since the present thesis deals with realization of Josephson logic and memory circuits needed for the supercomputer, it is very much necessary to have a thorough investigation of Josephson junction parameters, characteristics and their optimization in order to get highest switchig speed. Further, it is essential to have a thorough understanding of switching dynamics of a Josephson junction. Due to this in section-2 of the present chapter we have made an attempt to give a detailed theoretical approach regarding Josephson function properties, characteristics and their optimizations which are necessary in designing logic and memory circuits based on the Josephson technology. A brief review has been made which will enable to give a better theoretical understanding of physics of a Josephson junction. A systematic theoretical approach has been made in order to estimate junction parameters of various Josephson junctions. It is also necessary to see the effect of thermal noise on the junction characteristics and parameters, since these affect the switching speed of the Josephson junction. Due to this, in the present chapter we have shown the effect of thermal noise on the junction switching performance. Recently from 1986 [1], there is a great trend in discovering new superconducting materials of critical temperatures (Tcs) above liquid nitrogen temperature(77⁰K)[2]. In the present paper we have made

an attempt to show the impact of new high-Tc superconductors in the Josephson junction electronics.

Further, a detailed study on dynamics response of the Josephson junction is essential since there is a lack of analytical expressions in order to obtain the nature of Josephson junction. In the present chapter we have made an attempt to study the dynamic response of a Josephson junction by solving the current equation of a Josephson junction on a computer. The details are given in section5.

2. BRIEF REVIEW OF JOSEPHSON EFFECT

It is well-known that metals like Sn, In, Pb, Nb, etc. become superconductors when they are cooled to very low temperatures (a few degrees Kelvin). Superconductors exhibit zero resistance for dc current and exclude magnetic flux penetration from their interiors (Meissner effect) [3]. Microscopically, the transition for a metal to a superconductor is interpreted as the pairing of otherwise-free electrons near the Fermi surface to form Cooper pairs. The pairing action is attributed to an electron-lattice-electron interaction. The BCS theory indicates that this is attractive. When this attractive force is larger than the Coulomb repulsive force between electrons, the electrons near the Fermi surface associate in pairs. They do this even though the kinetic energy of an electron pair is greater than two unpaired electrons [4]. The paired electrons are coupled within a certain distance, known as the coherence length of the superconductor. As a result of the pairing process, electrons condense into a lower total energy state, and are separated from the excited state by an energy gap 2Δ . This energy gap is centered at the Fermi energy level and represents the amount of energy required to break one of the electron pairs. The energy gap is temperature dependent and is approximately equal to $3.52k_B T_c$, where k_B is the Boltzmann constant and T_c is the critical temperature of the superconductor.

The presence of the superconducting energy gap 2Δ can be easily observed by looking at the tunneling behaviour of a superconductor-insulator-superconductor sandwich structure. For tunneling between two identical superconductors, the quasiparticle tunneling current is given [5] by

$$I_q = \frac{1}{R_n} \int_{-\infty}^{+\infty} \frac{E + ev}{[(E + ev)^2 - \Delta^2]^{1/2}} \frac{E}{[E^2 - \Delta^2]^{1/2}} X [f(E + ev) - f(E)] dt \quad (1)$$

where Δ is the bias voltage applied to the junction, 2Δ is the energy gap of the superconductor, E is the quasi-

particle excitation energy, $f(E+eV)$ and $f(E)$ are the quasi-particle Fermi distribution functions of the two superconducting films forming the junction, and R_n is the normal tunneling resistance of the junction.

The current-voltage relation as described by Eqn.(1) and shown in Fig. 1 is markedly different from that of a metal-insulator-metal sandwich structure. The presence of the superconducting energy gap modifies the density states available for tunneling as indicated by $(F+ev)/[(E+ev)^2 - \Delta^2]^{1/2}$ and $E/(E^2-\Delta^2)^{1/2}$ in Eqn.(1). Only a small current flows for the junction voltage smaller than $2\Delta/e$ because of a lack of available states for tunneling, while larger rise of current is observed for a junction voltage larger than $2\Delta/e$ since energy states outside the gap become now available for tunneling. For a larger junction voltage, the normal tunneling resistance curve is approached asymptotically.

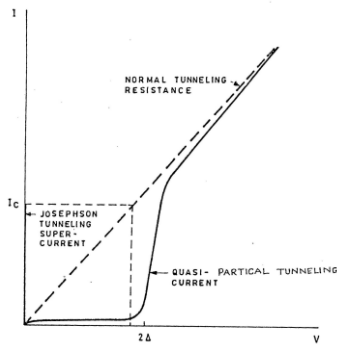


Fig. 1 Current – Voltage characteristics of a sandwich – type Josephson tunnel junction.

In a superconductor, all the Cooper pairs are locked in phase, and the behaviour of superconducting state described by a single wave function is of the form [6]

$$\varphi(r,t) = \varphi(r,t) e^{i\theta(r,t)} \text{----- (2)}$$

where θ is the phase of the electron pairs and $\varphi(r,t)\varphi(r,t)$ represents the electron pair density at position r at time t .

When two superconductors are separated by a large distance (five times greater than the coherence length of the superconductor), the phase angle of one superconductor bears no relation with the other.

But as the separation between the superconductor becomes very small (such as 10 to 30 Å for a Lead - Leadoxide-Lead sandwich junction), the two superconductors can be weakly coupled in phase. In this case, the electron pairs can tunnel through the insulating barrier, resulting in a zero-voltage current flow. This is known as the dc Josephson effect. The current-voltage characteristic of a sandwich-type Josephson tunnel density is given by

$$J = J_o \sin \theta \text{----- (3)}$$

where θ is the phase difference across the junction [7], and J_o is the maximum current density that the barrier can support [8].

$$J_o = \frac{\pi \Delta(T)}{2 e R_n} \tanh \frac{\Delta(T)}{2 k_B T} \text{---- (4)}$$

where T is the temperature, and R_n and $\Delta(T)$ are the normal resistance and temperature dependent energy gap of the junction, respectively. The phase difference across the junction is dependent upon the junction bias voltage V given by

$$\frac{d\theta}{dt} = \frac{2ev}{\hbar} \text{----- (5)}$$

Thus, at a constant voltage bias V , an ac Josephson current of frequency $2eV/\hbar$ is generated in the junction, with the $2eV/\hbar$ term approximately 483MHz/ μ v. Eqn. (5) describes the ac Josephson effect which is a phenomenon often exploited for the high frequency mixing detection and other rf applications.

The phase difference across the junction is also dependent on the magnetic field H passing through the junction barrier [9],

$$\Delta\theta = \frac{2 \mu_o e d}{\hbar} (\vec{H} \times \vec{n}) \text{----- (6)}$$

where \vec{n} is the normal to the plane of the junction and $d = \lambda_1 + \lambda_2 + T_{ox}$, in which λ_1 and λ_2 are the London penetration depths in the two superconductors and T_{ox} is the thickness of the insulating barrier.

For a small junction having a width less than the Josephson penetration depth $\lambda_j = (\hbar/2\mu_o e d J_o)^{1/2}$, the current density at zero magnetic field is uniform. It can be shown that the magnetic field dependence of the total supercurrent in this junction obeys the relation,

$$I_c = I_{c_o} \left[\frac{\sin(\pi I/I_o)}{(\pi I/I_o)} \right] \text{----- (7)}$$

where $I_{c_o} = J_o A$, with A being the area of the junction. The quantity I_{c_o} is the maximum supercurrent that the junction can carry in the absence of the magnetic field, $I_o = h/2e = 2.07 \times 10^{-15}$ weber is the magnetic flux, and I is the total magnetic flux enclosed in the junction. This Fraunhofer shape dependence of the critical current on the magnetic flux is shown in Fig. 2.

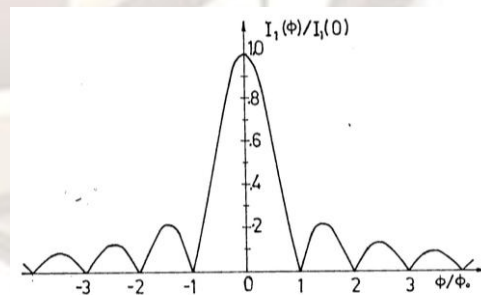


Fig. 2 Magnetic field dependence of the maximum Josephson current I_1 .

The ratio I_o/I_{c_o} [10] is important when using the junction as a switching element in logic circuits. As I_{c_o} represents the control current value for the supercurrent that is suppressed, the I_o/I_{c_o} finally determines the current gain with $I_o = J_o W L$ and the ratio becomes,

$$\frac{I_o}{Ic_o} = \frac{1}{2\pi} L \frac{1}{\lambda_j} j^2 \text{ ----- (8)}$$

Evidently, for a large I_o/Ic_o , the junction should have a large L/λ_j ratio. However, for long junctions ($L/\lambda_j > 1$) and/or large current density, the self-field created by the junction current alters the field dependence drastically. The junction exhibits the Meissner effect, screening the external magnetic field within the Josephson penetration depth from the edges by creating circulating currents in the junction (Fig.3). The distributed or long junction can be viewed as an interferometer with many distributed loops, where the base- and counter-electrodes of the Josephson junction form a set of small discrete inductors. The result is an asymmetric ($I_g - I_c$ characteristic) threshold characteristic. Further, the long junctions are having long switching delays. Thus, the long junctions ($L/\lambda_j > 1$) are not preferable for logic applications.

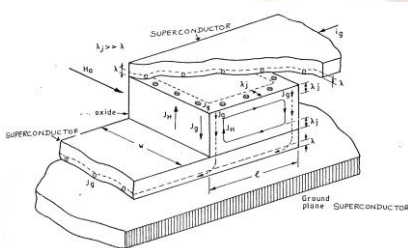


Fig. 3

Applied and induced currents in Josephson junction showing Josephson penetration depth.

The idea of using the Josephson junction as switching device was first discussed by Rowell, designed and then measured by Matisoo [11]. Since then, Josephson tunneling devices have been used in many experiments on high-speed digital circuits. With improved measurement techniques and junction miniaturization, very fast switching times have been recorded by the IBM and Japanese research groups.

The oxide-barrier junction (Fig.4a) is not only the type of device that exhibits the Josephson effect (Fig.4b-f). There are also point-contact device [12], the Dayem bridge [13], the coplanar proximity-effect device [14], normal-metal-barrier junction [15], semiconductor-barrier junction, etc. Each of these Josephson devices will carry zero-voltage supercurrent upto a certain value.

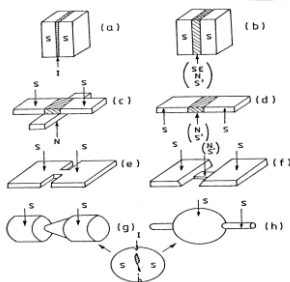


Fig. 4 Different types of structures where the Josephson effect can take place. (a) tunnel junction, i.e., S-I-S sandwich, (b) sandwich, S-N-S or S-S'-S, (c) proximity effect bridge, (d) ion-implanted bridge, (e) Dayem bridge, (f) variable thickness bridge, (g) point contact, (h) blob-type junction. Here S- stands for superconductor, S' for

superconductor with reduced critical parameters, N normal metal, SE for semiconductor and I-stands for insulator.

When the critical currents are exceeded, the devices switch into the non-zero voltage characteristic. These have been explained in terms of the lumped circuit model having the distributed internal resistance and capacitance of a device in parallel with an idealized Josephson element by McCumber [16] and Stewart [17] (shown in Fig.5). The current I flowing through the junction is the sum of the individual current flowing through the resistance R_n , Capacitance C_j and the idealized Josephson element

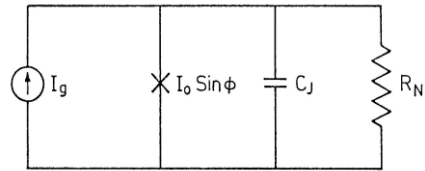


Fig.5

Equivalent circuit of a Josephson Junction.

$$I = I_o \sin \theta + C_j \frac{dv}{dt} + \frac{V}{R_n} \text{ ----- (9)}$$

The normalized I-V characteristic of the junction as a function of the dimensionless admittance ratio, β (also called McCumber parameter) is shown in Fig.6. Further, the hysteretic and non-hysteretic behaviour of a Josephson junction is decided by the McCumber parameter and is given by,

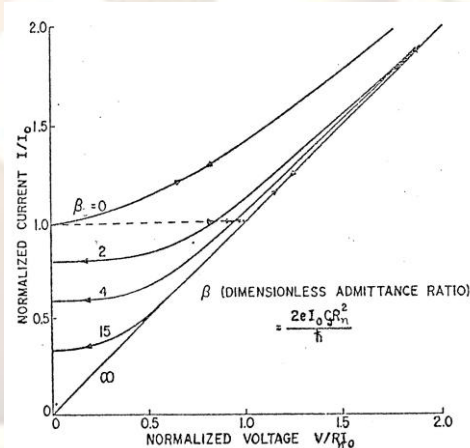


Fig. 6

The normalized I-V characteristic of a Josephson Junction as a function of β

$$\beta = \frac{2e}{h} I_o R_n^2 C_j \text{ ----- (10)}$$

where I_o , R_n and C_j are the critical current, normal resistance and capacitance of the Josephson junction respectively [18]. If $\beta > 1$, the device I-V characteristic (shown in Fig.7) becomes hysteretic whereas for $\beta = 1$, it becomes moderate hysteretic and for $\beta < 1$, non-hysteretic in nature. The device with a highly hysteretic characteristic is useful for a latching logic circuit [19], whereas the devices with moderate hysteresis may be used for non-latching circuits [20]. Further, the Josephson device with a non-hysteretic characteristic is desirable for designing

detectors[21], SQUID magnetometers [22] and Vortex transistor logic gates [23].

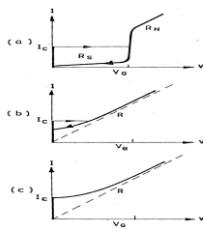


Fig. 7 Current-Voltage characteristics of Josephson devices for different values of β (see text). (a) Highly hysteretic device ($\beta > 1$) with R (V) of a tunnel junction. (b) Moderately hysteretic device ($\beta = 1$) with R voltage dependent. (c) Non-hysteretic device ($\beta < 1$) with R voltage independent.

Josephson tunnel junctions are being investigated as potential switching elements for the ultra-fast computers. Their advantage is that they switch very fast, at extremely low power levels. Among the different Josephson device configurations the tunnel junction (Fig.4a) is presently preferred for circuit applications. The reason is that it has a high resistance R_j upto the gap voltage $2\lambda/e$ which allows the transfer of most of the junction current into an external resistive load.

The I-V curve of a Josephson junction is shown in Fig.8. A current I_g smaller than the Josephson threshold current (critical current) I_0 is fed into the device. The junction remains in the zero-voltage ($V = 0$) and effectively shorts the load. If $I_g > I_0$ then the junction immediately switches to a non-zero voltage state and reaches the intersection of the load R_1 with single particle tunneling curve as shown in Fig.8. The resulting junction voltage produces a current $i = V_g/R_1$ in the load. Normally, the device remains in the voltage state even after the control current or gate current is removed and I_g has to be momentarily reduced until v_g reaches a characteristic voltage, $V_{min} = (2I_0 / K/C_j)^{1/2}$ where $K = I_0 / 2 \pi$, at which the junction switches back to the zero-voltage state [24]. This is called latching operation that requires resetting after each logic cycle. However, if a load R_2 which intersects near V_{min} is connected across the device it becomes non-latching and switches back to $V = 0$ upon the removal of the control current I_g .

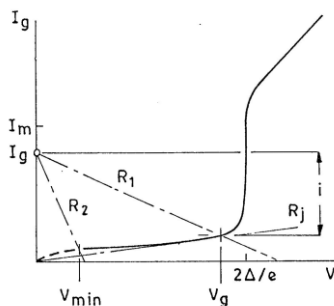


Fig. 8 I-V characteristic of a Josephson tunneling device. Two external load lines are shown. R_1 produces

latching operation. With R_2 , intersecting near the resetting voltage V_{min} , the device becomes non-latching.

From the above brief review of physics of the Josephson junction, it can be concluded that a Josephson junction to be chosen as a switching element for high-speed Josephson logics and memory circuits, and the following conditions have to be satisfied : (i) The Josephson junction should be a tunnel type and small in size ($L/\lambda_j < 1$). (ii) The I-V characteristic should be hysteretic in nature ($\beta > 1$) so that it can provide the latching operation.

3. JUNCTION PARAMETERS AND THEIR OPTIMIZATION

For two identical superconductors, the pair or supercurrent of the Josephson junction at any temperature T (below critical temperature, T_c) is given by [8] as

$$I_c = J_c A = \frac{\pi \Delta(T)}{2eR_n} \tanh\left(\frac{\Delta(T)}{2k_B T}\right) \quad \text{----}$$

(4)

where J_c and A are the Josephson critical current density and junction area respectively. Further, $\Delta(T)$ is the temperature dependent energy gap of the superconductor and k_B is the Boltzmann constant. Eqn.(1) has simplified versions as

$$I_c = \frac{\pi \Delta(0)}{2eR_n} \quad \text{for } 0 < T \leq T_c/2 \quad \text{-----(4a)}$$

$$= \frac{\pi \Delta^2(T)}{4ek_B I_c R_n} \quad \text{for } T \leq T_c \quad \text{----- (4b)}$$

For a low applied voltage, the tunneling current density of a metal-oxide-metal or a metal-semiconductor-metal structure (Simon [25]) is represented by

$$J = 3.1 \times 10^{10} \phi^{1/2} V/S \exp(-S \phi^{1/2}) \quad \text{-- (11)}$$

where J is the normal current density, S , the thickness of the barrier, V , the applied voltage and ϕ , the average barrier height in electron volts.

From Eqns. (4) and (11), one can obtain the Josephson current density in terms of barrier parameters, ϕ and S by taking the superconductor energy gap as V . It is clear from the above equation that the critical current density depends on the factor $\exp(-\phi^{1/2} S)$. For an oxide barrier, the barrier height is large and to obtain Josephson tunneling the thickness should be very small. Generally the oxide

thickness is in the range of 30 \AA . Usually it is very difficult to control such an oxide thickness by an ordinary thermal oxidation technique. Therefore, K.F. and glow discharge oxidation techniques are used in preparing the oxide barrier. Also the barrier has a large dielectric constant which results in a large junction capacitance. This limits the use of an oxide barrier junction in millimeter-waves and SQUID magnetometry.

We to the above tact the use of a (deposited) semiconductor film as a junction barrier has become attractive. Because the latter has a small barrier height compared to an oxide one, which gives a better tolerance for a slight variation in thickness. Semiconductor barriers

may potentially be thicker than oxide barriers for the same current density. The semiconductor barrier has a relatively low dielectric constant (generally less than 10) compared to an oxide barrier. The Josephson junction capacitance is represented by the conventional expression,

$$C_j = \frac{\epsilon_o \epsilon_r A}{d} \quad \text{----- (12)}$$

where ϵ_r is the relative dielectric constant of the barrier material, A, the junction area and d, the thickness of barrier material. For a given area of the junction, the junction capacitance depends on the dielectric constant of the barrier material. By choosing a different dielectric material, the value of the junction capacitance can be changed.

After the discovery of Josephson effect in 1962 [7], many superconducting materials have been used for Josephson junctions, e.g., Sn, Tn, Pb, Nb, etc. The materials like Sn, In have low critical temperatures, therefore junctions based on them cannot be operated at liquid helium temperature. Lead was found to be suitable for fabrication of Josephson junctions since its critical temperature is high above the liquid helium temperature ($T_c = 7.2^\circ \text{K}$). But Lead has a greatest problem of thermal cycling. It cannot withstand a large number of thermal cyclings from the room temperature to liquid helium temperature. Lead alloy based Josephson junctions have been investigated to overcome the thermal cycling problem. Since there are still a small thermal cycling and the thermal annealing instability problems in Lead alloy Josephson junctions, robust hard Niobium junctions have been therefore studied. The Niobium junctions were verified to withstand thermal stress, undergone in fabrication usage, and to have high reliability and reproducibility in regard to electronic performance.

Further, the Josephson junctions that are candidates for Integrated circuits processing [26] must possess (i) a large superconducting energy gap Δ (and consequently a high transition temperature, T_c) to ensure well developed superconductivity at practical cryogenic temperatures, (ii) a high quality metal-oxide tunneling barrier with a low dielectric constant to provide good, controllable tunneling characteristics and a low device capacitance, (iii) and good mechanical properties in order to withstand the thermally induced stress associated with thermal cycling between room temperature and liquid helium temperature. (iv) Also desirable are superconducting coherence and magnetic penetration lengths that are large and small, respectively. The former governs the distance scale over which the properties of superconducting material must be maintained homogeneous to avoid degrading the superconducting properties, and the latter (magnetic penetration length) governs the extent to which magnetic fields penetrate the superconductor and put a lower limit on the physical thickness of the various superconducting circuit elements. (v) Finally, to minimize difficulties in processing, one desires materials that are not sensitive to impurities or damage and that can be processed at the room temperature.

Basaviah et al [27] have adopted a theoretical procedure (using eqn.(4) and Eqn.(11)) in order to obtain junction parameters of a Pb/PbO/Pb Josephson junction.

Here we have adopted the same technique in order to obtain a number of junction parameters of Josephson junctions using different superconductors (Table -I) and dielectric barrier materials. The average barrier height and the thickness of dielectric barriers (Table-II) have been taken from the experimental data [28]. The theoretically estimated junction parameters are given Table III. It is found that the junction parameters estimated by this method are almost coinciding with experimentally obtained junction parameters [29]. It seems from the

results (Table-III) that Nb/A10 x/Nb Josephson junctions are well suited for the Josephson junction electronics since it has a low dielectric constant which leads to a low junction capacitance. Further, it has all the excellent features which are necessary for a Josephson junction to be used as a candidate for integrated circuit processing [26]. Due to this we have selected the

Nb/A10 x/Nb Josephson technology [29] in order to realize theoretically the

Josephson logic and memory circuits, However, we have used Pb-alloy technology at some places since it has been used extensively by IBM [30] and Japanese [31] for their supercomputer projects. The parameters of Pb-alloy and Nb/A10 x/Nb Josephson technology [29] that are chosen for simulation throughout the thesis are given Table-V.

The I-V characteristic of a Josephson junction can be theoretically verified by an empirical relation [32] by $I = G(V) V$ ----- (13a)

where $G(V) = \frac{1}{R_j + \left(\frac{1}{R_n} - \frac{1}{R_j} \right) \frac{1}{1 + \exp\left(\frac{-V - V_g}{V_d} \right)}}$ -----(13b)

where R_n is the normal resistance of the junction, R_j , the subgap resistance of the junction, V_g , the energy gap of the junction and V_d is the energy gap depth of the junction.

Fig.9 shows the I-V characteristics of the theoretically obtained various Josephson Junctions (Table -IV), Fig.9 (a) – (f) shows the the order of junctions (1-6) given in Table – IV, Further, in fig 10 we have drawn the I-V characteristics of Josephson Junctions using two. Technologies (according to Table-V). The solid curve indicates the I-V characteristic of a Pb-alloy Josephson junction whereas the dotted curve is that of a Nb/A10 x/Nb Josephson junction.

Thermal noise effect:

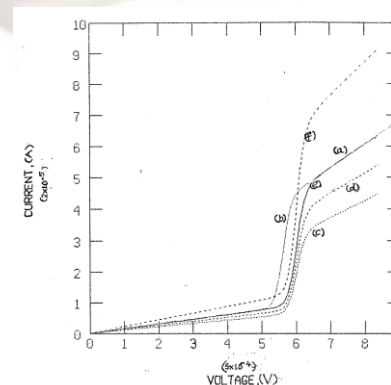


Fig.9 The I-V characteristics of theoretically obtained Josephson junctions. The parameters used for plotting are given Table – III.

According to Klien and Mukherjee [33], the thermal noise problem is minimum when the Josephson junction is used as switching element in logic devices.

However, to establish immunity against thermal noise [34], minimum values of I_c and L should be

$$\left. \begin{aligned} I_c &\gg K_B T / I_o \quad 3 \times 10^8 \text{ A} \\ L &\gg \phi_o^2 / 2K_B T \quad 4 \times 10^{-8} \text{ H} \end{aligned} \right\} \text{ at } 4.2^\circ \text{ K}$$

If the junction parameters are considered below this limit, the thermal noise may not a problem for Josephson devices when used for logic and memory circuits.

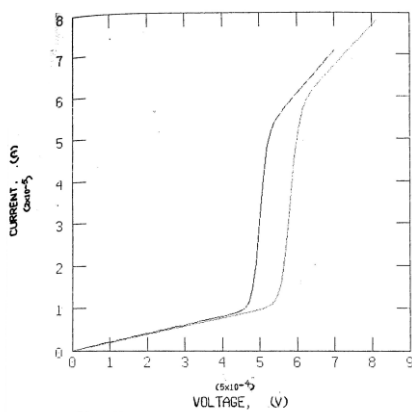


Fig.10 The I-V characteristics of Josephson junctions using Pb – alloy and Nb/A10x/Nb technologies. The parameters used for plotting are given in Table – V.

High temperature superconductivity impact

The high temperature superconductor features [35] are the high critical temperature T_c , high band gap energy Δ , and short coherent length for electronic device applications. The high transition temperature superconductor improves Josephson junction device performance in switching time T_o and junction gap voltage V_g . The intrinsic switching time T_o is limited by the uncertainty principle. The product of energy uncertainty ΔE and time uncertainty Δt is in the order of Planck's constant h ,

$$\Delta E \cdot \Delta t = \hbar$$

The intrinsic switching time T_o is estimated as being equal to Δt . therefore, $T_o = \Delta t = \hbar / \Delta E = h / e V_g$

The Josephson junction gap voltage V_g is almost linearly dependent on the critical temperature. Therefore, if the critical temperature T_c increases λ_g times, the junction gap voltage V_g increases λ_g times also.

It is expected that the new high- T_c superconductor based Josephson junctions may have advantages over low- T_c junctions because of the high energy gap which leads to high gap frequency thereby the frequency of operation can be more for high- T_c superconductor Josephson junctions [35]. Further, the new high- T_c based Josephson junctions may permit high switching speeds which will be an excellent feature for Josephson electronic applications [36] [37] [38] [39]. A serious obstacle to digital electronics with

high- T_c superconductors is the difficulty of making the two film junction when the second film must be heated to 650°C or even higher temperature in the presence of oxygen atmosphere.

4. DYNAMIC ANALYSIS OF A JOSEPHSON JUNCTION

The dynamic response of a Josephson junction is obtained by solving the current equation of the Josephson junction. The equations are solved by using the computer [41] since there is a lack of analytical expressions to see the dynamic response of the Josephson junction.

The current equation of the Josephson junction is given by

$$I = I_o \sin \theta + C_j \frac{dV}{dt} + \frac{V}{R_n} \quad \text{---(9)}$$

$$\text{and } v = K \frac{d\theta}{dt} \quad \text{----(5)}$$

Eqn. (9) can also be written as

$$I = i_J + i_C + i_G$$

Where $i_J = I_o \sin \theta$ (Josephson current or supercurrent)

$$i_C = C_j dV/dt \text{ (Displacement current)}$$

and $i_G = G(V) V$ (Quasi-particle current)

Suppose an inductive load, L is connected to the Josephson junction, then the load current (i_L) is as follows:

$$\frac{di_L}{dt} = \frac{1}{L} (V - i_L R_L) \quad \text{-----(14)}$$

where R_L is the load resistor.

The dynamic response of a Josephson junction can be obtained by solving the Eqns.(4),(9) and (14) using the Runge-Kutta method on a computer. For the convenience of numerical simulation, the Eqns.(4), (9) and (14) can be written as follows:

$$\frac{d^2 \theta}{dt^2} = \frac{\pi}{\phi_o C_j} [I - I_o \sin \theta - \frac{\phi_o}{2\pi R_n} \frac{d\theta}{dt} - i_L] \quad \text{---(15)}$$

$$\frac{di_L}{dt} = \left(\frac{\phi_o}{2\pi} \frac{d\theta}{dt} - i_L r_L \right) / L \quad \text{----(16)}$$

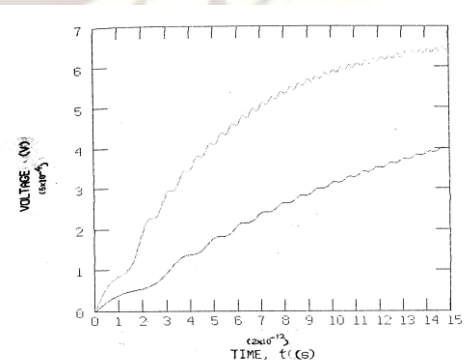


Fig.11 The dynamic response of a Josephson junction. The solid curve shows the output voltage variation with time for a Pb-alloy Josephson junction whereas the dotted curve indicates the output voltage variation with time for Nb/A10x/Nb Josephson junction. The parameters used for plotting are given in Table – V.

The computer simulation results of the above equations are shown in Fig.11 and Fig.12. The voltage variation with time of a Josephson junction for different technologies, Pb-alloy and Nb/A10 x/Nb are plotted in Fig.11. The solid curve indicates the variation of junction voltage with time for a Pb-alloy based Josephson junction (parameters given Table-V) whereas the dotted line is for a Nb/A10 x/Nb based Josephson junction. It can be observed from the two curves that the rise time is small for the dotted curve than in the case of the solid curve. This is due to a lower junction capacitance (0.37 ps) selected in the case of the Nb/A10 x/Nb Josephson junction. In Fig.12 we have shown the output current (load current) variation with time. The solid curve is obtained from Pb alloy based Josephson junction whereas the dotted curve is from the Nb/A10 x/Nb Josephson junction.

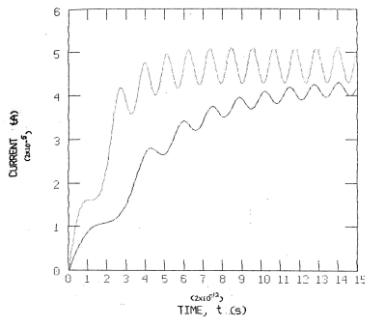


Fig.12 The dynamic response of a Josephson junction. The solid curve shows the output current variation with time for a Pb-alloy Josephson junction whereas the dotted curve indicates the output current variation with time for Nb/A10x/Nb Josephson junction. The parameters used for plotting are given in Table-V.

Further, in Fig.13, the dynamic response of each current of a Josephson junction, i_C , i_G , i_J , and i_L is shown. This will give an exact phenomenon involved in switching dynamics of a Josephson junction. It can be observed from Fig.15 that through the capacitance, the current is initially high and later on it starts discharging. The voltage is initially zero and starts increasing upto the gap voltage. The Josephson current creates a self-magnetic flux which is equal to $L_o I_o$, where L_o is the self-inductance of the junction. This inductance plays a predominant role in switching dynamics of the Josephson junction. Thus the Josephson junction equivalent to a circuit consists of an inductor, a capacitance and a resistor. When a current is applied to a Josephson junction, inductor acts as an open circuit whereas capacitance acts as a short circuit. In the next instant when the capacitor starts discharging current and simultaneously there will be the development of voltage across the junction. Due to this fact, there will be some considerable delay for the initiation of voltage $0.0+V$. This delay is called the turn-on delay of a Josephson junction. More detailed analysis of the turn-on delay of a Josephson junction is given paper [41].

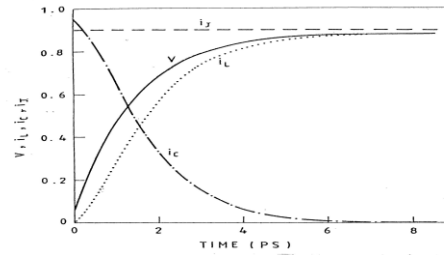


Fig.13 The dynamic response of a Josephson junction at each stage, where i_C represents the charging current, i_J represents the Josephson current, V represents the output voltage and i_L represents the load current.

5. CONCLUSIONS

It can be concluded from the various sections of this chapter that the Josephson junction to be used as a high-speed switching element must have the following : one, Josephson junction be of tunnel type and small in size ($L/\lambda_j < 1$), second, the I-V characteristics should be hysteretic in nature ($\beta > 1$) so that it can provide latching operation, third, the junction critical current should be high and the junction capacitance should be as small as possible.

The systematic procedure adopted to estimate the various junction parameters will give the better idea of the Josephson junction characteristics and will be useful for experimentalists before making Josephson junctions. It is observed that the Nb/Al - oxide/Nb Josephson junction may be the ideal junction for logic and memory circuits. Due to this tact we have selected Nb/Al-oxide/Nb Josephson technology [29] in order to simulated Josephson logic memory circuits which will be discussed in the future chapters. However, we have also used at certain places, the Pb-alloy technology [30], since it has been extensively used by IBM [30] and Japanese [31] scientists.

Further, we have seen the thermal noise effect on the Josephson junction characteristics and it is observed that the thermal noise is not at all a problem. No doubt the high-Tc superconducting based Josephson junctions have advantages over low-Tc based Josephson junctions, but difficulty lies is the fabrication process of high-Tc Josephson junction. It is expected that the dynamic response of the Josephson junction discussed in section-4 will be able to give a better understanding of switching dynamics of Josephson junction.

The properties and optimizing techniques discussed in various section of the chapter are used throughout the thesis in designing logic and memory circuits.

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TABLE - I
SOME IMPORTANT PARAMETERS OF SUPERCONDUCTORS

| Superconductor | Critical Temperature (OK) | Energy gap Δ at 0° K (eV) | Penetration depth $\lambda(0)$ Tc (nm) |
|--------------------|---------------------------|----------------------------------|--|
| Pb | 7.2 | 1.2 | 39 |
| Pb-In-Au | 7.0 | 1.2 | 135 |
| Pb-Bi | 8.3 | 1.7 | 20.2 |
| Nb | 9.2 | 1.52 | 85 |
| NbN | 16.0 | 2.4 | 295 |
| Nb ₃ Sn | 17.8 | 3.3 | 170 |
| Nb ₃ Ge | 23.6 | 3.9 | 150 |

TABLE - II
THE PARAMETERS OF JOSEPHSON JUNCTION BARRIERS

| Josephson Junction | Average barrier height in eV θ | Dielectric costant ϵ_r |
|----------------------------------|---------------------------------------|---------------------------------|
| Pb/Pb0/Pb | 1.05 | 10.0 |
| Pb-In-Au/oxide/Pb-Bi | 1.7 | 12.7 |
| Nb/a-Si (Phosphorous doped) / Nb | 0.75 | 7.0 |
| Nb/a-Si (undoped) / Nb | 1.2 | 8.8 |
| Nb/Yb-oxide/Nb | 3.4 | 5.0 |
| Nb/ Al-oxide/Nb | 1.7 | 9.0 |

TABLE - III
THE JUNCTION PARAMETER OF VARIOUS THEORETICALLY OBTAINED JOSEPHSON JUNCTIONS

| | Josephson Junction | J _c (A/m ²) | I ₀ (mA) | R _n (Ω) | C _j (pF) | λ_j (m) | Vg (mV) |
|---|--------------------------------|------------------------------------|---------------------|-----------------------------|---------------------|------------------------|---------|
| 1 | Pb/Pb0/Pb | 1.4 x 10 ⁷ | 0.085 | 42 | 0.26 | 9.6 x 10 ⁻⁵ | 2.3 |
| 2 | Pb-In-Au/oxide/Pb-Bi | 2.1x10 ⁷ | 0.13 | 33 | 0.33 | 7.7x 10 ⁻⁵ | 2.8 |
| 3 | Nb/a-Si (Phosphorous doped/Nb) | 1.6x10 ⁷ | 0.10 | 47 | 0.19 | 9.1x 10 ⁻⁵ | 3.0 |
| 4 | Nb/a-Si (undoped)/Nb | 1.9x10 ⁷ | 0.12 | 39 | 0.23 | 8.1x 10 ⁻⁵ | 3.0 |
| 5 | Nb/Yb-oxide/Nb | 3.2x10 ⁷ | 0.20 | 23 | 0.13 | 6.2x 10 ⁻⁵ | 3.0 |
| 6 | Nb/Al-oxide/Nb | 2.3x10 ⁷ | 0.14 | 33 | 0.24 | 7.4x 10 ⁻⁵ | 3.0 |

TABLE - IV
THE JUNCTION PARAMETERS OF VARIOUS THEORETICALLY OBTAINED JOSEPHSON JUNCTIONS.

| Josephson Junction | I ₀ (A) | C _j (pF) | R _n (Ω) | V _{min} (mV) | β | W _j (H ₂) |
|--------------------------------|--------------------|---------------------|-----------------------------|-----------------------|---------|----------------------------------|
| Pb/Pb0/Pb | 0.085 | 0.26 | 42 | 0.33 | 19 | 4.0x10 ¹¹ |
| Pb-In-Au/oxide/Pb-Bi doped/Nb) | 0.13 | 0.33 | 33 | 0.36 | 24 | 4.4x10 ¹¹ |
| Nb/a-Si (Phosphorous doped/Nb) | 0.10 | 0.19 | 47 | 0.42 | 20 | 5.0x10 ¹¹ |
| Nb/a-Si (undoped)/Nb | 0.12 | 0.23 | 39 | 0.41 | 21 | 5.0 x10 ¹¹ |
| Nb/Yb-oxide/Nb | 0.20 | 0.13 | 23 | 0.71 | 7 | 8.6 x10 ¹¹ |
| Nb/Al-oxide/Nb | 0.14 | 0.24 | 33 | 0.44 | 18 | 5.3 x10 ¹¹ |

Table – V
THE JOSEPHSON JUNCTION PARAMETERS
USED FOR COMPUTER – SIMULATION

| Josephson Junction | I_0 (mA) | C_j (pF) | R_n (Ω) | V_{min} (mV) | β | W_j ($\times 10^{11}$ Hz) |
|----------------------|------------|------------|--------------------|----------------|---------|------------------------------|
| Pb-In-Au/oxide/Pb-Bi | 0.0796 | 1.0 | 24.7 | 0.16 | 23.0 | 2.0 |
| Nb/Al-oxide/Nb | 0.0870 | 0.37 | 26.0 | 0.28 | 11.0 | 3.4 |

TABLE - VI
Comparison of transition temperatures of a superconductor with the energy gap expressed in millivolts and frequency

| Transition temperature ($^{\circ}$ K) | 1 | 10 | 15 | 30 | 90 | 120 |
|--|-------|------|-----|-----|------|------|
| Energy gap (mV) | 0.03 | 4.3 | 6.6 | 8.6 | 26 | 35 |
| Frequency (T GHz) | 0.068 | 1.03 | 2.1 | 4.1 | 12.3 | 16.4 |