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M/M/c queueing model for bed-occupancy management

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Abstract:
The present paper describes the movement of patients in a hospital by using queuing model with exponential arrival and service time distributions. A queuing model is used to determine the optimal number of beds in order to improve hospital care. Also, we describe a way of optimising the average cost per day by balancing costs of empty beds against costs of delayed patients.

Keywords: bed occupancy; Poisson distribution; queuing theory

The theoretical model

We consider a M/M/c queue in which the number of beds is fixed. We assume that, patient arrivals follow a Poisson process with arrival rate λ and the service time follows Poisson distribution with mean τ. Here, τ is the average length of stay per patient.

The average number a of arrivals during an average length of stay τ is $a = \frac{\lambda}{\tau}$, known as the offered load. From queuing theory, we see that the probability that, j beds are occupied is given by

$$P(j\ occupied\ beds) = P_j = \frac{\frac{a^j}{j!}}{\sum_{k=0}^{c}\frac{a^k}{k!}}, \quad j = 0,1,2,\ldots\ldots,c. \qquad (1)$$

and $P_j = 0$, for $j > c$.

The probability that, all c beds are occupied is given by

$$P_c = \frac{\frac{a^c}{c!}}{\sum_{k=0}^{c}\frac{a^k}{k!}}$$

From the above formula, we deduce that the probability that, all c beds are occupied or the fraction of arrivals that is lost is given by

$$B(c,a) = \frac{\frac{a^c}{c!}}{\sum_{k=0}^{c}\frac{a^k}{k!}}, \qquad (2)$$

called the Erlang's loss formula.

The mean number of occupied beds is given by

$$a' = a[1 - B(c,a)] \qquad (3)$$

which is also known as the carried load. The carried load a' equals that portion of the offered load a that is not lost from the system. The offered load a is the carried load a', if the number of beds are infinite i.e. when $c = \infty$, $a = a'$. The lost load (LL), that is, the offered load a that is lost from the system and is given by

$$LL = aB(c,a). \qquad (4)$$

We see that, the proportion of arrivals that is lost, is the ratio of the lost load to the offered load, that is

$$B(c,a) = \frac{aB(c,a)}{a}.$$

We define the bed occupancy as

$$\rho = \frac{a'}{c}. \qquad (5)$$

Assuming that, the system is in steady state, we must have $\rho \leq 1$.

Application of the model

The loss model (optimising the number of beds)

We consider the model where the arrival rate is $\lambda = 25$ patients per day and the mean length of stay $\tau = 10$ days. In table-1, we present a table of various system characteristics as a function of the number of beds c for a hospital. When we have 100 beds, the probability that, the fraction of arrivals that is lost i.e. lost demand probability, is near to zero and the bed occupancy is 2.5% i.e. on average there are 2.5 patients in a hospital. We can see from this table that, the bed occupancy is decreased as the number of beds increase. So, we can conclude that, the more number of patients can be admitted to the hospital with maximum of 100 beds.

Table-1 The main service features ($\lambda = 25$ patients per day, $\tau = 10$ days and the offered load $a = \dfrac{\lambda}{\tau} = 2.5$)

Number of beds c	Lost demand probability $B(c,a)(\%)$	Mean number of patients (L)	Bed occupancy ρ (%) $\rho = \dfrac{a'}{c}$
100	$5.5*10^{-118}$	2.5	2.5
105	$4.6*10^{-126}$	2.5	2.38
110	$3.1*10^{-134}$	2.5	2.27
115	$1.6*10^{-142}$	2.5	2.17
120	$6.9*10^{-151}$	2.5	2.08
125	$2.4*10^{-159}$	2.5	2
130	$6.9*10^{-168}$	2.5	1.92
135	$1.6*10^{-176}$	2.5	1.85
140	$3.1*10^{-185}$	2.5	1.78
145	$5.1*10^{-194}$	2.5	1.72
150	$7.1*10^{-203}$	2.5	1.67

From the table, we wish to have at most 100 beds in a hospital, because the lost demand probabilities approach zero rapidly.

The cost model (optimising the average cost per unit time)

Here, beds correspond to the inventory where idle beds are the on-hand inventory and occupied beds are unfilled orders. A patient arrival corresponds to a demand and the subsequent length of stay of the patient corresponds to waiting time for replenishment. Patients who are turned away (i.e. lost demand) because there are no empty beds, corresponds to unsatisfied demands.

We consider h as a holding cost per day for each empty bed, h>0 and π as a fixed penalty cost incurred for each patient that is turned away (i.e. lost demand), π>0. We also consider p as a profit per patient per day. Therefore, the total cost to the customer is the sum of variable costs (treatment), fixed costs (holding costs), profit and penalty cost (of lost demand). We observe that, the average demand that is lost per unit time is equal to $\lambda B(c,a)$, the average idle-bed inventory is equal to $c - a[1 - B(c,a)]$(by (3)) and therefore, the average service provider revenue per day is given by

$$r(c) = -\pi\lambda B(c,a) - h\{c - a[1 - B(c,a)]\} + pa[1 - B(c,a)] \quad (6)$$

Now, we want to find c to maximise $r(c)$. This is the optimal number of beds where we balance the number of empty beds against the number of delayed patients. We note that, for a public health service, $p = 0$. We are interested in maximising revenue $r(c)$ or minimising cost which is given by

$$g(c) = \pi\lambda B(c,a) + h\{c - a[1 - B(c,a)]\} \quad (7)$$

Here, the cost minimisation means the optimal balance between holding cost h and penalty cost π.

Now, we say that, we are indifferent between the use of the consecutive c and $c+1$ when $g(c) = g(c+1)$. From this equation, we deduce that this equation is equivalent to

$$\frac{\lambda}{\tau}[B(c,a) - B(c+1,a)] = \left(1 + \frac{\pi\tau}{h}\right)^{-1} \quad (8)$$

This equation shows that, the optimal choice of c depends only upon the parameters $\dfrac{\lambda}{\tau}$ and $\dfrac{\pi\tau}{h}$. It means that, for a given value of c, we may evaluate the indifference by means of a graph of $\dfrac{\pi\tau}{h}$ versus $\dfrac{\lambda}{\tau}$. This graph is known as the indifference curve, which describes equation (6). When the curves for different values of c are close together, we say that, we are indifferent to the choice of $\dfrac{\pi}{h}$ for given values of λ and τ. This means that, for such choices of c, the optimal number of beds is not very dependent on the ratio $\dfrac{\pi}{h}$.

Here, we assume that the total cost per patient per day is $Rs.200$, where $Rs.100$ are incurred with respect to the bed and $Rs.100$ with respect to the treatment. We also assume that, the holding cost $h = Rs.100$ per day and the penalty cost as o % of the total cost of turning away the patient. (Because the lost demand probabilities are near to zero.)

Table-2 The values of the average cost per unit time $Rs.g(c)$

Beds c	$\frac{\pi}{h} = 20$ $g_1(c)$	$\frac{\pi}{h} = 30$ $g_2(c)$	$\frac{\pi}{h} = 40$ $g_3(c)$
100	9750	9750	9750
105	10250	10250	10250
110	10750	10750	10750
115	11250	11250	11250
120	11750	11750	11750
125	12250	12250	12250
130	12750	12750	12750
135	13250	13250	13250
140	13750	13750	13750
145	14250	14250	14250
150	14750	14750	14750

This table shows that, if we increase the penalty cost to holding cost ratio $\dfrac{\pi}{h}$ three times from 20 to 40, then the number of beds needed for the minimal average cost is at most 100. Here, the average cost remains constant as the ratio $\frac{\pi}{h}$ is increased from 20 to 40. This means that the ratio $\frac{\pi}{h}$ has no significant influence on the optimal number of beds. Here, we note that we do not draw the indifference curves. We consider the tabular data.

Conclusions

From table-1, we can see that, for 100 beds, the lost demand probability is near to zero and the bed occupancy is 2.5%. Also, the bed occupancy is decreased as the number of beds increase.

From table-2, we can see that, for 100 beds, the average cost per unit time is minimum and it increases rapidly as the number of beds increase.

Hence, finally we conclude that, there should be at most 100 beds in a hospital with arrival rate $\lambda = 25$ patients per day and service rate $\tau = 10$ days per patient.

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