Ariani Budi Safarina / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue 4, June-July 2012, pp.649-654

Modified Nakayasu Synthetic Unit Hydrograph Method For Meso Scale Ungauge Watersheds

Ariani Budi Safarina*

*(Department of Civil Engineering, Jenderal Achmad Yani University, Jalan Terusan Jenderal Sudirman Cimahi 40532 Indonesia)

ABSTRACT

The aim of this study is to modify of Nakayasu synthetic unit hydrograph method for use in ungauge watershed that defined characteristics such as area, length of the main river and slope. The research was conducted in 20 meso scale watersheds in Java Indonesia with hydrology map-based on geographic information systems, and real time data of rainfall and river water at the outlet of each watershed. Observation unit hydrograph was calculated using the convolution method and then compared with Nakayasu synthetic unit. Hydrograph. Comparison parameters between these two hydrograph are determined then be optimized to minimize error. The result of this study is nine coefficients modified of Nakayasu method based on optimization results to be used in calculating the modified Nakayasu synthetic unit hydrograph in the watershed with a certain area, length of main rivers and slope. The coefficient is a coefficient in the time of concentration equation, time to peak equation, peak discharge equation, rising curve equation and decreasing curve equation.

Keywords **– Comparison Parameter, Convolution Method, Peak Discharge, Time to Peak, Ungauge Watershed**.

INTRODUCTION

Ungauge watershed is the watershed where there is no river water level and rainfall gauge in the watershed area. To analyze the magnitude of river discharge and other hydrological parameter in this such watershed, required a specific method as a synthetic unit hydrograph method. Synthetic unit hydrograph started by Snyder (1938) and Gray (1961) from Chow [5] which gives some characteristics such as peak discharge, time to peak and time base. Then the addition of the characteristic parameters of the reservoir of watershed, which was pioneered by Clark, (1943), from Chow [5]. In 1972, the Soil Conservation Service (SCS) results in a dimensionless synthetic unit hydrograph.. .Nakayasu in 1940 promoted the synthetic unit hydrograph based on research rivers in Japan, which until recently widely used and known as Nakayasu synthetic unit hydrograph.Nakayasu synthetic unit hidrograph method in the application at 32 watersheds in Java, Indonesia, shows the average 22 % error of hydrograph's shape and 9% error of peak discharge [1]. The deviation can lead to errors in design and should be minimized.

The aim of this study is to modify of Nakayasu synthetic unit hydrograph method so that the deviation of considerable hidrograph can be reduced. Modifications carried out on the coefficient of the Nakayasu equation such as the time of concentration equation, time to peak equation, peak discharge equation, rising curve equation and decreasing curve equation**.** By doing this modification, the analysis results in a rainfall runoff model in an ungauge watershed can be more accurately

I. PHYSICAL CHARACTERISTIC OF WATERSHED

The main physical characteristics of a watershed is the area, shape, elevation, slope, orientation, soil type, channel networks, water storage capacity and land cover. Effect of type of characteristics are different. Soil type can control l infiltration, surface water capacity, and groundwater. The combined influence of all factors is the classification for small and large watersheds [6]. Large watershed is a watershed with the dominant influence of storage capacity, so the effect of rainfall on the reservoir is small. Large watershed is insensitive to variations in rainfal intensity and land use. Generally, a large watershed has a large size with the main river.Small watershed is controlled by overland flow, land use, slope, etc., have a peak flow variation is very large. Influence of the storage capacity is small, and watersheds are very sensitive to rainfall, so the response to it quickly. Watershed in a swamp which the area is slightly smaller, has the characteristics of the watershed as a large watershed.

Area of the watershed is a physical characteristics that is always used in the analysis of hydrological basin. The area represent a watershed storage capacity.Uhlenbrook Stefan [2] in his paper defines that, according to the area, watershed is divided into small watershed (Area < 1 km^2), meso scale watershed (10 km^2 < Area < 1000 km^2) and macro watersheds (> 1000 km^2).

Sosrodarsono and Kensaku [7] and Hundecha [3] gives the shape coefficient of the watershed, F, as a comparison between area (A) and square of main river length (L), with the equation:

$$F = A/L^2 \qquad (1)$$

According to Sharma [8], the shape of a watershed can be represented by a river network The slope of the watershed is generally represented by the slope of the main river. The slope of the major rivers in a watershed affects the water drainage in the watershed and the magnitude of time of concentration. Sharma define the slope (S) of watershed as the function of contour interval (D), number of contours that cut watershed boundary (N) and watershed perimeter (P) with the equation,

$$S = DN/P \quad (2)$$

II. HYDROGRAPH ANALYSIS

Hydrograph is a graph that represents the stream discharge versus time [9] Flow hydrograph is the result of the runoff , which consists of the overland flow, interflow and base flow which is generated from rainfall.

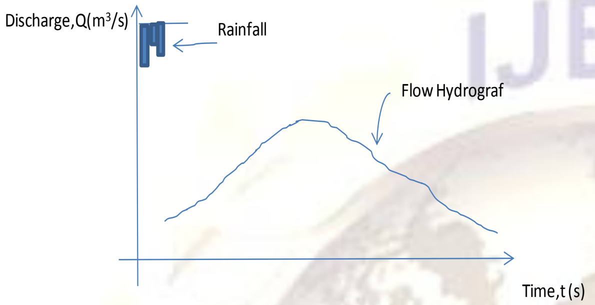

Fig 1 Flow Hydrograph

Unit hydrograph is a simple linear model that can be used to obtain hydrograph which can be determined from any effective rainfall [4,5] The basic assumption used in this linear model is

- Effective rainfall has a constant intensity over the effective duration.
- Effective rainfall is distributed uniformly at every point in the watershed.
- Hidrograph time base direct runoff from a rainfall effectively with a specific duration is constant.
- Ordinate of direct runoff hidrograph from a basic flow is proportional to the total amount of effective rainfall for each hidrograph.
- For a watershed, hidrograph generated for each specific effective rainfall, describe the same watershed characteristics

III. CONVOLUTION METHOD

An impulse, either step or pulse response function, is defined to have a continuous time domain. If the domain of time discrete with Δt duration interval, then there are two ways to describe the function of continuous time in discrete time domain, the system of pulse data and sampled data systems. Pulse data systems used for precipitation and the value of discrete input function according to Chow [5] are:

$$Pm = \int_{(m-1)\Delta t}^{m\Delta t} I(\tau)d t \qquad m=1,2,3... \quad (3)$$

Pm is the depth of precipitation during the interval of time (in inches or centimeters). Sampled data system used for flow and direct runoff, so the value of output of the system at time intervals to n (t = nΔt) is

$$Qn = Q(n\Delta t) \qquad n = 1,2,3,\dots \quad (4)$$

Qn is the instantaneous value of flow at the end of time interval to n (in cfs or m^3/s). Thus the input and output variables for watershed system recorded with different dimensions and uses a different representation of discrete data as well. Effect of pulse input of duration Δt starting at time (m-1) Δt and the output at time t = n Δt measured by the value of the unit pulse response function h [t-(m-1) Δt] = h [nΔt-(m-1) Δt] = h [(n-m +1) Δt], then equation (3) becomes

$$h[(n-m+1)\Delta t] = \frac{1}{\Delta t}\int_{(n-m)\Delta t}^{(n-m+1)\Delta t} u(l)dl \quad (5)$$

By discretizing convolution integral at t = n Δt and subtitute, the importance of the convolution equation with the input and output pulse Pm in Qn as a function of time sampled data:

$$Qn= P_1h[(n\Delta t)]+P_2h[(n-1)\Delta t]+...$$
$$+P_mh[(n-m+1)\Delta t]+...$$
$$+P_Mh[(n-M+1)\Delta t] \quad (6)$$

Continuous pulse response function h(t) can be represented into discrete time domain as a function of sample data U. Thus the discrete convolution equation for the linear system:

$$Qn = \sum_{m=1}^{n\le m} P_m U_{n-m+1} \quad (7)$$

IV. NAKAYASU SYNTHETIC UNIT HYDROGRAPH METHOD

Dr. Nakayasu doing research on rivers in Japan and he produces a synthetic unit hydrograph [10] The synthetic unit hydrograph are as follows,

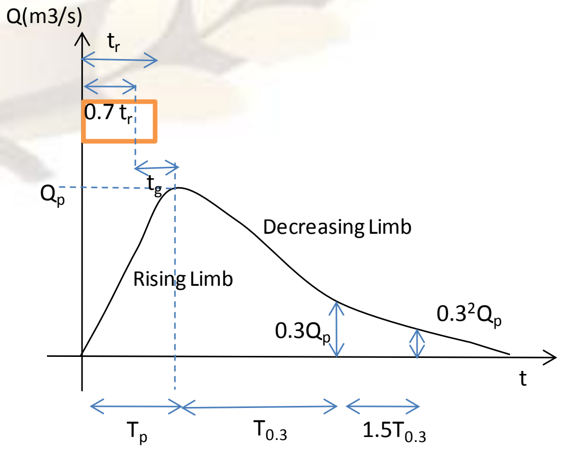

Fig 2 Nakayasu Synthetic Unit Hydrograph

The Equation of Nakayasu Method

1) Time lag (T_p) is the function of the time of concentration (t_g) and duration of effective rainfall (t_r). The equation of T_p,is showed below,

$$T_p = t_g + 0{,}8\, t_r \quad (8)$$

2) Peak discharge is the funtion of watershed area (A), watershed characteristic coefficient (C), unit rainfall (R_o), tame lag (T_p) and time required to discharge reduction up to 30% peak discharge ($T_{0.3}$). The equation is,

$$Q_p = \frac{CAR_o}{3{,}6(0{,}3T_p + T_{0,3})} \quad (9)$$

3) Rising limb curve,Q_a, is the function of peak discharge (Q_p), time (t) and time lag (T_p). The equation is,

$$Q_a = Q_p\left(\frac{t}{T_p}\right)^{2,4} \quad (10)$$

4) Decreasing limb curve, Q_d, is the function of peak discharge (Q_p), time(t), time lag (T_p) and time required to discharge reduction up to 30% peak discharge ($T_{0.3}$).The equation is,

$Q_d > 0{,}3\, Q_p$:

$$Q_d = Q_p . 0{,}3^{\frac{t-T_p}{T_{0,3}}} \quad (11)$$

$0{,}3Q_p > Q_d > 0{,}3^2 Q_p$:

$$Q_d = Q_p . 0{,}3^{\frac{t-T_p+0{,}5T_{0,3}}{1{,}5T_{0,3}}} \quad (12)$$

$Q_d > 0{,}3^2 Q_p$

$$Q_d = Q_p . 0{,}3^{\frac{t-T_p+1{,}5T_{0,3}}{2T_{0,3}}} \quad (13)$$

V. VALIDATION METHOD

Validation of the model is the justification that a model, in the domain of application, appropriate and consistent with the objective of the Model and Simulation. Validation of the model makes the true model [13]. In this research, synthetic methods are validated against observation unit hydrograph by using the comparison parameters. Comparison parameters consists of the error shape of hydrograph (E) and discrepanty ratio (d) of the peak discharge, peak time and time base [11].

Error magnitude is meant to measure the the similarity of the synthetic hydrograph curve and observation curve[Ariani, IJCEE]. The equation developed from the method of Sum Square Error as follows:

$$E = \frac{\sqrt{\frac{\sum (qobs - q\sin)^2}{N}}}{Qp_{obs}}$$

(14)

The discrepanty ratio of hydrograph unit parameters is,

$$d(Qp) = \frac{Qp_{\sin}}{Qp_{0bs}}$$

(15)

$$d(Tp) = \frac{Tp_{\sin}}{Tp_{0bs}}$$

(16)

$$d(Tb) = \frac{Tb_{\sin}}{Tb_{0bs}}$$

(17)

VI. GENERALIZED REDUCED GRADIENT METHOD

Generalized Reduced Gradient method is used as optimization method in this research. Basic concept of *Reduced Gradient* method is that at each stage, the problem is only on the independent variables [12]. If y can be obtained from z, then the objective function f, can be a function of z alone. Gradient of a function f which depends on z is as follows,

$$_{\Gamma}T = \nabla_z f(y,z) - \nabla_y f(y,z) B^{-1} C$$

(18)

where B and C respectively are the coefficients of y and z

Generalized Reduced Gradient method can provide a solution of non Linear Programming which is the general form :

Min f(x)

Subject to h(x) = 0 a < x < b

Generalized reduced gradient is,

$$_{\Gamma}T = \nabla_z f(y,z) - \nabla y f(y,z)[\nabla_y h(y,z)]^{-1} \nabla_z h(y,z) \quad (19)$$

VII. RESULT AND DISCUSSION

There are 26 watersheds studied in this research. The number and the name of those watersheds is shown below. The studied watersheds are shown below,

TABLE I

THE STUDIED WATERSHEDS

Number	Watershed
1	Cisangkuy-Kamasan
2	Cikapundung-Pasirluyu
3	Ciliw ung-Sugutamu
4	Ciliw ung-Katulampa
5	Cimanuk-Leuw idaun
6	Cikeruh-Jatiw angi
7	Cilutung-Damkamun
8	Cilutung-Bantarmerak
9	Cokroyasan-Winong
10	Bogow onto-Pungangan
11	Progohulu-Badran
12	Klaw ing-Pegandegan
13	Gajahw ong-Papringan
14	Bedog-Guw osari
15	Tambakbayan-Pulodadi
16	Code-Pogung
17	Progo-Duw et
18	Gajahw ong-Wonokromo
19	Code-Koloran
20	Winongo-Padokan
21	Winongo-Sinduadi
22	Amprong-Mahdyopuro
23	Lesti-Taw angrejeni
24	Brantas-Gadang
25	Paritraya-Bendo
26	Brangkal-Sooko

The first name of watershed indicate the name of the main river, and the second is the name of the outlet.

Watershed characteristics such as the watershed area, the main river length and the slope of each watershed, are shown below

TABLE II

WATERSHEDS CHARACTERISTICS

Watershed	Area (Km^2)	Slope (%)	L (Km)
1	203.38	4.850	34.00
2	112.13	0.500	33.09
3	254.00	3.650	78.00
4	151.00	11.150	23.00
5	450.68	0.030	42.00
6	115.76	0.060	30.00
7	628.86	0.295	55.17
8	324.38	0.020	33.00
9	131.00	1.400	65.17
10	347.00	6.400	51.29
11	506.00	7.400	38.95
12	1099.00	0.600	42.59
13	22.00	4.000	23.00
14	113.00	0.300	47.45
15	66.00	9.000	22.52
16	29.00	7.000	26.00
17	162.00	0.700	33.27
18	19.00	3.500	12.00
19	58.63	0.400	33.27
20	47.00	0.700	31.59
21	13.06	8.000	14.00
22	236.00	4.000	34.00
23	440.00	5.900	27.36
24	572.00	3.200	48.00
25	547.00	2.380	44.80
26	981.00	0.200	64.00

The Scenario of Optimization

The equations of Nakayasu Method that will be modified are as follow,

TABLE III

THE NAKAYASU EQUATION THAT WILL BE MODIFIED

Nakayasu Equations	
$t_g=$	$C_1 + C_2 *L$
$T_p=$	$C_3 * T_r + t_g$
$T_{0.3}=$	$C_4 * t_g$
$Q_p=$	$C_5 * A / C_6 * (C_7 T_p + T_{0.3})$
$Q_{raising}=$	$Q_p * (t/T_p)^{C8}$
$Q_{decreasing}=$	$Q_p * C_9^{((t-Tp)/T0.3)}$

Optimization Scenario

Min $\sum e_i^2 = \sum (q_{iobs} - (q_{nkysdmod})_i)^2$

Subject to $Qp = Qp_{obs}$

$Tp = Tp_{obs}$

$\sum (q_{nkysmodi} + q_{nkysmodi+1})*(t_{i+1} - t_i)/2 = \sum (q_{obsi} + q_{obsi+1})*(t_{i+1} - t_i)/2$

$q_{nkysmod} = Q_{raising} + Q_{decreasing}$

$$Qr = Qp\left(\frac{t}{Tp}\right)^{C_8}$$

$$Qd = Qp * C_9^{((t-Tp)/T_{0.3})}$$

$$Qp = \frac{C_5 A}{C_6 * (C_7 * Tp + T_{0.3})}$$

$$Tp = C_3 * tr + t_g$$

$$t_g = C_1 + C_2 * L$$

$$T_{0.3} = C_4 * t_g$$

Modified Coefficient

Modified coefficient of Nakayasu equations are shown below,

TABLE IV

NAKAYASU MODIFIED COEFFICIENT

Watershed	Nakayasu Modified Coefficient								
	0.4*	0.058*	0.8*	2*	1*	3.6*	0.3*	2.4*	0.3*
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
1	0.40	0.11	0.80	2.04	0.91	3.57	0.22	0.72	0.33
2	0.35	0.06	0.94	2.00	0.72	4.52	0.10	0.91	0.30
3	0.52	0.12	0.69	2.79	1.37	3.54	0.13	1.08	0.08
4	0.99	0.12	0.30	1.01	1.14	2.58	0.80	2.40	0.30
5	0.39	0.01	2.27	2.02	0.85	3.63	0.30	2.23	0.76
6	0.52	0.07	2.47	1.95	0.94	3.61	0.15	0.95	0.24
7	0.52	0.06	2.74	1.27	1.72	3.24	0.30	1.00	0.76
8	0.52	0.06	0.60	2.89	1.16	3.62	0.23	2.40	0.30
9	0.59	0.01	0.80	4.68	0.93	2.88	0.30	2.40	0.30
10	0.59	0.03	0.80	2.90	0.94	2.90	0.30	2.40	0.30
11	0.52	0.02	0.80	5.40	1.11	3.58	0.13	2.40	0.30
12	0.52	0.11	0.80	2.04	1.15	3.49	0.30	2.40	0.30
13	0.40	0.03	0.80	2.87	1.12	3.55	0.30	2.40	0.30
14	0.57	0.06	0.80	1.68	1.10	3.47	0.30	2.40	0.30
15	0.53	0.12	0.80	1.93	1.13	3.51	0.30	2.40	0.30
16	0.62	0.10	0.69	2.15	0.99	3.37	0.10	2.40	0.30
17	0.62	0.14	0.80	1.36	1.21	3.22	0.30	2.40	0.30
18	0.62	0.05	0.80	2.77	1.08	3.45	0.30	2.40	0.30
19	-0.73	0.06	0.80	3.22	0.94	2.95	0.30	2.40	0.30
20	0.52	0.08	0.80	1.50	1.11	3.50	0.30	2.40	0.30
21	0.52	0.12	0.80	3.38	1.14	3.50	0.30	2.40	0.30
22	-0.50	0.05	0.80	2.24	0.97	3.14	0.30	2.40	0.30
23	0.72	0.09	0.80	1.36	1.07	3.35	0.30	2.40	0.30
24	0.72	0.01	0.80	2.65	0.80	2.57	0.30	2.40	0.30
25	1.04	0.03	0.80	2.52	1.01	3.14	0.30	2.40	0.30
26	0.61	0.02	0.80	2.25	1.02	3.17	0.30	2.40	0.30
Average	**0.49**	**0.07**	**0.96**	**2.42**	**1.06**	**3.35**	**0.28**	**2.11**	**0.33**
Max	**1.04**	**0.14**	**2.74**	**5.40**	**1.72**	**4.52**	**0.80**	**2.40**	**0.76**
Min	**-0.73**	**0.01**	**0.30**	**1.01**	**0.72**	**2.57**	**0.10**	**0.72**	**0.08**
S	**0.36**	**0.04**	**0.58**	**0.99**	**0.19**	**0.39**	**0.13**	**0.59**	**0.14**

* Original coefficient

Nine modified coefficients above can be used as a new modified Nakayasu based on the characteristics of its watershed. As an example for the watershed with characteristics such as a watershed number one can use it's modified coefficient to obtain accurate results.

Average , maximum and minimum modified coefficients as in the above table can be used also as a new coefficient. Maximum and minimum modified coefficient are upper and lower limits of each new coefficient so that the user can choose. Using these coefficients of statistical analysis is of course have an error larger then if it using the characteristics of the watershed. This is due to the statistical analysis because is always there is a standard deviation of the data as can be seen in the table above.

Error in hydrograph's shape and discrepanty ratio of the hydrograph parameter are shown as follow,

TABLE V

ERROR AND DISCREPANTY RATIO OF NAKAYASU MODIFIED

Watershed	Nakayasu Modified				
	Enkys	d(Qp)	d(Tp)	d(Tb)	Vol DRO(mm)
1	0.05	1.00	1.00	1.68	1.00
2	0.07	1.00	1.00	2.18	1.00
3	0.26	1.00	1.00	1.24	1.00
4	0.18	1.00	1.00	1.00	1.00
5	0.03	1.00	1.00	1.75	1.00
6	0.04	1.00	1.00	1.19	1.00
7	0.10	1.00	1.00	0.75	1.00
8	0.15	1.00	1.00	1.20	1.00
9	0.21	1.00	1.00	1.00	1.00
10	0.15	1.00	1.00	1.11	1.00
11	0.13	1.00	1.00	1.50	1.00
12	0.12	1.00	1.00	1.13	1.00
13	0.11	1.00	1.00	1.00	1.00
14	0.16	1.00	1.00	1.14	0.98
15	0.16	1.00	1.00	1.00	1.00
16	0.15	1.00	1.00	0.75	1.00
17	0.20	1.00	1.00	1.24	1.16
18	0.08	1.00	1.00	0.95	1.00
19	0.10	1.00	1.00	1.00	1.00
20	0.14	1.00	1.00	0.78	1.00
21	0.12	1.01	1.00	0.74	1.00
22	0.09	1.00	1.00	1.00	1.00
23	0.17	1.00	1.00	1.11	1.00
24	0.13	1.00	1.00	1.25	1.00
25	0.20	1.00	1.00	2.08	1.00
26	0.04	1.00	1.00	1.00	1.00
Average	**0.13**	**1.00**	**1.00**	**1.18**	**1.02**
Max	**0.26**	**1.01**	**1.00**	**2.18**	**1.16**
Min	**0.03**	**1.00**	**1.00**	**0.74**	**0.98**
S	**0.06**	**0.00**	**0.00**	**0.38**	**0.03**

In the modified Nakayasu method, the average error of hydrograph's shape is reduced from 22% to 13%, and the average discrepanty ratio of peak discharge is reduced from 9% to 0%. The results show that modified Nakayasu method in this research can lead the method to be more accurate.

VIII. Conclusion

Modified Nakayasu synthetic unit hidrograph method can be used to analyze the ungauge watershed based on the watershed characteristics. By choosing the coefficients as same as the watershed characteristics in this study, the results is more accurate. The modified coefficient applies only to meso scale watershed, and not recommended to use for micro and macro watersheds.

REFERENCES

Journal Papers:

[1] Ariani Budi Safarina, Hang Tuah Salim, Iwan K Hadihardaja, and M.Syahril BK, Clusterization of Synthetic Unit Hydrograph Methods Based on Watershed Characteristics, , *International Journal of Civil and Environmental Engineering, 11(06), 2011*, 76-85.

[2] Uhlenbrook Stefan, Stefan Roser, Nils Tilch, Hydrological Process Representation at the Meso-scale:the Potential of a Distributed, Conceptual Catchment Model, , *Journal of Hydrology, Institut of Hydraulic Engineering, University of Stuttgart, Germany, 291, 2004*, 278-296.

[3] Hundecha Y. and Andras B, Modeling of the Effect of Land Use Changes on The Runoff Generation of a River Basin Trough Parameter Regionalization of a Watershed Model, , *Journal of Hydrology, Institut of Hydraulic Engineering, University of Stuttgart, Germany, 292, 2004*, 281-295.

[4] Ajward M Hubail; Muzik I, A Spatially Varied Unit Hydrograph Model *Journal of Environmental Hydrology,2000, vol 8 paper 7*

Books:

[5] Chow VT, Maidment, Mays L. W *AppliedHydrology* (McGraw-Hill International Edition, 1988).

[6] Gray, D.M *Interrelationships of Watershed Characteristics* (J. Geophys. Res.66, 1961).

[7] Sosrodarsono Suyono and Kensaku *Hidrologi Untuk Pengairan* (Pradnya Paramita, Jakarta, Indonesia, 2003).

[8] Sharma RK *Hydrologi & Water Resources* (Dhanpat Rai & Sons, New Delhi,1980)

[9] Das Ghanshyam *Hydrology and Soil Conservation Engineering (*Prentice-Hall of India, New Delhi,2002)

[10] Soemarto CD *Hidrologi Teknik* (Usaha Nasional, Surabaya, Indonesia,1987)

[11] Ang,Alfredo H-S and Tang Wilson H *Probablity Concepts in Engineering* (John Wiley &Sons, Inc, USA,2007)

Thesses:

[12] Safarina A B, *Model Analisa Metoda Hydrograf Satuan Sintetik Untuk Berbagai Karakteristik Daerah Aliran Sungai*, doctoral diss., Bandung Institute of Technology, Indonesia, 2010.

Proceedings Papers:

[13] Balci, O, Quality Assessment, Verification, and Validation of Modeling and Simulation Applications, *Winter Simulation Conference*, Louisiana, USA, 2004, 122-129.