Construction Of Mixed Sampling Plans Indexed Through Mapd And Aql With Conditional Double Sampling Plan As Attribute Plan Using Weighted Poisson Distribution

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ABSTRACT

This paper presents the procedure for the construction and selection of mixed sampling plan with MAPD as a quality standard and Conditional double sampling plan as attribute plan using Weighted Poisson distribution as a base line distribution. The plans are constructed indexed through MAPD and AQL and also compared. Tables are constructed for the easy selection of the plans.

Keywords and Phrases: Maximum Allowable Percent Defective, Acceptable quality level, Operating Characteristic, Tangent intercept.
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1. INTRODUCTION

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plans is a reduction in sample size for a similar amount of protection.

It is the usual practice that while selecting a sampling inspection plan, to fix the operating characteristic (OC) curve in accordance with the desired degree of discrimination. The sampling plan is in turn fixed through suitably chosen parameters.

The entry parameters used in the acceptance sampling literature are acceptable quality level (AQL), indifference quality level (IQL), limiting quality level (LQL) and maximum allowable percent defective (MAPD). Several authors have provided procedures to design the sampling plans indexed through these parameters for various acceptance sampling plans.

The concept of MAPD (p_*) was introduced by Mayer (1967) and further studied by Soundararajan (1975) is the quality level corresponding to the inflection point on the OC curve. The degree of sharpness of inspection about this quality level ' p_* ' is measured through ' p_t ', the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis.

One of the desired properties of an OC curve is that the decrease of $P_a(p)$ should be slower for lesser values of 'p' and faster for greater values of 'p'. If we set p_* as the quality standard, the above property of the OC curve is obtained since p_* corresponds to the inflection point of the OC curve and hence

$$d^2P_a(p)/dp^2 = 0 \qquad \text{for } p = p_*$$
$$d^2P_a(p)/dp^2 < 0 \qquad \text{for } p < p_*$$
$$d^2P_a(p)/dp^2 > 0 \qquad \text{for } p > p_*$$

The mixed sampling plan has been designed under two cases of significant interest. In the first case sample size n_1 is fixed and a point on the OC curve is given. In the second case plans are designed when two points on the OC curve are given. The procedure for designing the mixed sampling plans to satisfy the above mentioned conditions was provided by Schilling (1967). Using Schilling's procedure, Devaarul (2003) has constructed tables for mixed sampling plans (independent case) having various sampling plans as attribute plans. Radhakrishnan and Sampath Kumar (2006 a, 2006 b, 2007, 2009) and Radhakrishnan et.al (2010) have made contributions to mixed sampling plan for independent case.

Sampath Kumar (2007) has constructed mixed sampling plan with various sampling plans as attribute plans using Poisson distribution as a base line distribution. Radhakrishna Rao (1977) suggested that the Weighted Binomial distribution can be used in designing sampling plans. Sudeswari (2002) studied the construction of sampling plans using Weighted Poisson distribution as a base line distribution. Mohana Priya (2008), Radhakrishnan and Mohana Priya (2008 a, 2008 b) have constructed the sampling plans using Weighted Poisson distribution as a base line distribution.

The Weighted Poisson distribution plays an important role in the acceptance sampling, mainly in the construction of sampling plans. Each outcome (number of defectives) is specific but can be assigned with different weights based on its importance or usage.

In this paper, mixed sampling plan (independent case) with Conditional double sampling plan as attribute plan is constructed using Weighted Poisson distribution as a base line distribution. The plans indexed through MAPD and AQL are constructed separately by fixing the values of c_1, c_2, c_3 and β_j'. The mixed plans indexed through MAPD (p_*) and AQL (p_1) are also compared.

2. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

p : submitted quality of lot or process

$P_a(p)$: probability of acceptance for given quality p

p_* : maximum allowable percent defective

p_t : the point at which the inflection tangent of the OC curve cuts the 'p' axis

p_1 : the submitted quality level such that $P_a(p_1) = 0.95$ (also called AQL)

h_* : relative slope at p_*

n_1 : sample size for the variable sampling plan

n_2 : sample size for the attribute sampling plan

c : acceptance number

β_j , : probability of acceptance for the lot quality 'p_j

β_j' : probability of acceptance assigned to first stage for percent defective p_j

β_j'' : probability of acceptance assigned to second stage for percent defective p_j

k : variable factor such that a lot is accepted if $\bar{X} \le A = U - k\sigma$

3. OPERATING PROCEDURE OF MIXED SAMPLING PLAN HAVING CONDITIONAL DOUBLE SAMPLING PLAN AS ATTRIBUTE PLAN

Schilling (1967) has given the following procedure for the independent mixed sampling plan with upper specification limit (U) and standard deviation (σ).

1. Determine the parameters of the mixed sampling plan n_1, $n_{1,2}$, $n_{2,2}$, k, c_1, c_2 and c_3.
2. Take a random sample of size n_1 from the lot.
3. If a sample average $\bar{X} \le A = U - k\sigma$, accept the lot
4. If the sample average $\bar{X} > A = U - k\sigma$, take a second sample of size $n_{1,2}$ and count the number of defectives 'd_1' in the sample.
5. If the number of defectives $d_1 \le c_1$, accept the lot.
6. If $c_1+1 \le d_1 \le c_3$, take a second sample of size $n_{2,2}$ from the remaining lot and find the number of defectives 'd_2'
7. If $d_2 \le c_2$ or $d_1+d_2 \le c_3$ accept the lot, otherwise reject the lot.

4. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING CONDTIONAL DOUBLE SAMPLING PLAN AS ATTRIBUTE PLAN USING WEIGHTED POISSON DISTRIBUTION

The detailed procedure adopted in this paper for the construction of mixed sampling plan having Conditional double sampling as attribute plan using Weighted Poisson distribution indexed through MAPD is given below:

- Assume that the mixed plan is independent
- Decide the sample size n_1 (for variable sampling plan) to be used.
- Calculate the acceptance limit for the variable sampling plan as $A = U - [z(p_*) + \{z(\beta_*')/\sqrt{n_1}\}]\sigma$, where z is standard normal variate corresponding to 't' such that t = $\int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$
- Split the probability of acceptance β_* as β_*' and β_*'' such that $\beta_* = \beta_*' + (1-\beta_*')\beta_*''$. Fix the value of β_*'

- Determine β*'', the probability of acceptance assigned to the attribute plan associated with the second stage sample as β*''= (β* - β*')/ (1- β*').
- Determine the appropriate second stage sample of n_2 from the relation

$$\beta_*'' = \sum_{i=1}^{c_1} P_i + P_{c_i+1}\sum_{i=1}^{c_3-c_1-1} q_i + P_{c_1+2}\sum_{i=1}^{c_3-c_1-2} q_i + \dots\dots + P_{c_2}\sum_{i=1}^{c_3-c_2} q_i$$

Where $P_i = \frac{e^{-np}(np)^{i-1}}{(i-1)!}$, $q_i = \frac{e^{-mnp}(mnp)^{i-1}}{(i-1)!}$

Using the above procedure, tables have been constructed to facilitate easy selection of mixed sampling plan using Conditional Double Sampling plan as attribute plans indexed through MAPD.

4.1 Construction of tables

The OC function of Weighted Poisson distribution is given by,

$$P_a(p) = \frac{x^\alpha p(x,\alpha)}{\sum_{x=0}^{\infty} x^\alpha p(x;\alpha)} \text{ ; x=0, 1, 2…}$$

The probability of acceptance for Conditional double sampling plan under Weighted Poisson distribution when α = 1 is used in this paper for determining the second stage probabilities and is given by

$$P_a(p) = \sum_{i=1}^{c_1} P_i + P_{c_i+1}\sum_{i=1}^{c_3-c_1-1} q_i + P_{c_1+2}\sum_{i=1}^{c_3-c_1-2} q_i + \dots\dots + P_{c_2}\sum_{i=1}^{c_3-c_2} q_i$$

Where $P_i = \frac{e^{-np}(np)^{i-1}}{(i-1)!}$,

$q_i = \frac{e^{-mnp}(mnp)^{i-1}}{(i-1)!}$, m ≥ 1.

In this paper, the probability mass function of the conditional double sampling plan is used for m=1.

For $n_{1,2}= n_{2,2}= n$ (say) the inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2} = 0$ and $\frac{d^3 p_a(p)}{dp^3} \neq 0$.

The relative slope of the OC curve h_* is given by, $h_* = \left(\frac{-p}{P_a(p)}\right)\frac{dP_a(p)}{dp}$ at $p=p_*$.

The inflection tangent of the OC curve cuts the 'p' axis at $p_t = p_* + (p_*/h_*)$. The values of n_2p_*, h_*, n_2p_t and $R = p_t / p_*$ are calculated for the specified β*' = 0.35 using C program and presented in Table 1.

Table 1: Various characteristics of the mixed sampling plan when (p_*, β_*) and (p_1, β_1) are known for a specified β_*' = 0.35, β_1=0.95, β_1' =0.35

c_1	c_2	c_3	np_1	β_*	β_*''	np_*	h_*	np_t	R =p_t/p_*
1	2	2	0.0801	0.5702	0.3388	1.0825	1.0825	2.0825	1.9238
1	3	4	0.5157	04854	0.2083	2.2772	2.5329	3.1762	1.3948
1	4	5	0.8080	0.4601	0.1694	2.9967	3.2811	3.9100	1.3048
1	4	6	1.0972	0.4788	0.1982	3.3807	3.3906	4.3778	1.2949
2	4	5	0.8859	0.5215	0.2638	2.9583	2.4116	4.1850	1.4147
2	5	5	08859	0.5088	0.2443	3.0519	2.5151	4.2653	1.3976
3	5	7	1.5831	0.5345	0.2838	4.0955	2.7617	5.5785	1.3621

3	6	7	1.6038	0.5285	0.2746	4.1573	2.8324	5.6251	1.3531
3	7	7	1.6038	0.5177	0.2580	4.2482	2.9299	5.6981	1.3413
3	7	8	1.9133	0.4940	0.2215	4.8063	3.5344	6.1662	1.2829
3	7	9	2.2441	0.4835	0.2054	5.3250	4.0205	6.6495	1.2487
4	7	9	2.3638	0.5268	0.2720	5.3768	3.2410	7.0358	1.3085
4	7	10	2.6594	0.5161	0.2555	5.7914	3.6548	7.3760	1.2736
4	7	12	3.1905	0.5038	0.2366	6.6453	4.2482	8.1964	1.2334
4	8	9	2.3691	0.5227	0.2657	5.4195	3.2869	7.0683	1.3042
4	8	10	2.6851	0.5403	0.2928	5.5837	3.4051	7.2235	1.2937
4	8	12	3.3420	0.4891	0.214	6.8937	4.6133	8.3880	1.2168
6	9	13	3.9442	0.5335	0.2823	7.6361	3.8082	9.6418	1.2627
6	10	13	3.9738	0.5284	0.2745	7.7063	3.8855	9.6896	1.2574
6	10	16	4.9142	0.4991	0.2294	9.0633	5.0758	10.8489	1.1970
7	10	16	5.0179	0.5236	0.2671	9.1506	4.4374	11.2128	1.2254
8	11	19	6.0569	0.5177	0.258	10.6257	4.9363	12.7783	1.2026
12	16	27	9.5777	0.5171	0.2571	15.1403	5.9384	17.6899	1.1684

4.2 Selection of the Plan

Table 1 is used to construct the plans when MAPD (p_*) and tangent intercept (p_t) are given. For any given values of c_1,c_2, p_t and p_* one can find the ratio R = p_t / p_*. Corresponding to the value of c_1 and c_2 find the value in Table1 under the column R which is equal to or just greater than the specified ratio, the corresponding value of c_3 is noted from this c_1,c_2 and c_3 values one can determine the value of 'n' using n = n p_*/ p_*.

Example 1: Given $c_1 = 6$, $c_2 = 10$, $p_* = 0.09$, $p_t = 0.11$ and $\beta_*' = 0.35$. Find the ratio R= p_t /p_* = 1.2220. Using Table 1, corresponding to $c_1 = 6$, $c_2=10$ select the value of R equal to or just greater than this ratio is 1.2627 which is associated with $c_1 = 6$, $c_2=10$, $c_3 = 13$ and n = n p_*/ p_*= (7.6361/0.09) = 85 .The Mixed Sampling plan with Conditional double sampling plan as attribute plan is $n_{1,2}$= 85,$n_{2,2}$=85,c_1 = 6, c_2=10 and c_3=13 .

Practical problem:

Suppose the plan n_1=12, k=2.0 is to be applied to the lot by lot acceptance inspection of a laptop battery, the characteristic to be inspected is the "operating temperature and ambient temperature" of the battery for which there is a specified upper limit of 160^0.0 F with known S.D (σ) of 2.0^0 F.

In this example, U= 160^0.0 F, σ=2.0^0 F and k=2.0

A = U-kσ = 160 - (2.0) (2.0) = 160-4 =156^0F.

Now, by applying the variable inspection first, take random sample of size n_1=12 from the lot. Record the sample results and find $\bar{X}$. If $\bar{X} \leq A = U - k\sigma$ =156^0F, then accept the lot. If $\bar{X}$ >A, take a random sample size $n_{1,2}$ and apply the attribute inspection.

Under attributes inspection, by taking Conditional double sampling plan as attribute plan, if the manufacturer fixes the values p_* =0.09 (9 non conformities out of 100), p_t=0.11(11 non conformities out of 100) and β_*' =0.35, take the sample of size $n_{1,2}$=85 and observe the number of defectives (d_1). If

$d_1 \leq 6$ accept the lot and if $d_1>13$, reject the lot. If $7 \leq d_1 \leq 13$, take a second sample of size $n_{2,2}=85$ from the remaining lot and find the number of defectives (d_2). If $d_2 \leq 10$ or $d_1+d_2 \leq 13$ accept the lot, otherwise reject the lot and inform the Management for further action.

5. SELECTION OF MIXED SAMPLING PLAN HAVING CONDITIONAL DOUBLE SAMPLING PLAN AS ATTRIBUTE PLAN INDEXED THROGH AQL

The general procedure given in section 4 is used for designing the mixed sampling plan having Conditional double sampling plan as attribute plan indexed through AQL (p_1). For $n_{1,2}= n_{2,2}=n$(say),under the assumption $\beta_1=0.95$ and $\beta_1'=0.35$, the np_1 values are calculated for different values of c_1,c_2 and c_3 using C program and presented in Table1.

5.1 Selection of the Plan

Table 1 is used to construct the plans when AQL (p_1), c_1, c_2, c_3 values are given. For any specified values of p_1,c_1, c_2 and c_3 one can determine n value using $n= np_1 / p_1$.

Example 2: Given $p_1= 0.03$, $c_1 = 4$,$c_2 = 7$,$c_3 = 10$ and $\beta_1'=0.35$. Using Table 1, find $n=np_1/p_1 = 2.6594/0.03=89$. For a fixed $\beta_1'=0.35$, the Mixed Sampling plan with Conditional double sampling plan as attribute plan is $n_{1,2}= 89$, $n_{2,2}=89$, $c_1= 4$, $c_2=7$ and $c_3=10$.

6. COMPARISON OF PLANS INDEXED THROUGH MAPD AND AQL

In this section the mixed sampling plan with Conditional double sampling plan as attribute plan indexed through MAPD is compared with the mixed sampling plan with Conditional double sampling plan as attribute plan indexed through AQL by fixing the parameters c_1,c_2 and β_1'.

For the specified values of p_* and p_t, one can find the values of c_1,c_2 , c_3 and 'n' indexed through MAPD for $\beta_*'=0.35$ as given in section 4. By fixing the values of c_1,c_2,c_3 and n, find the value of p_1 by equating $P_a(p) = \beta_1= 0.95$. By fixing $\beta_1'=0.35$ and c_1,c_2,c_3 and for the value of p_1, one can find the value of n using $n = np_1/p_1$ from Table 1,for different combinations of c_1,c_2, p_* and p_t, the values of n, c_3 (indexed through MAPD) and n, c_3 (indexed through AQL) are calculated and presented in Table 2.

Table 2: Comparison of plans

Given Values				Through MAPD n,c_3	Through AQL n,c_3
c_1	c_2	p_*	p_t		
1	4	0.10	0.12	34 , 6	39 , 6
1	2	0.05	0.07	59 , 5	69 , 5
4	7	0.10	0.12	66 , 12	72 , 12
3	7	0.10	0.13	43 , 7	84 , 7
6	10	0.09	0.11	85 , 13*	92 , 13*

* OC Curves are drawn

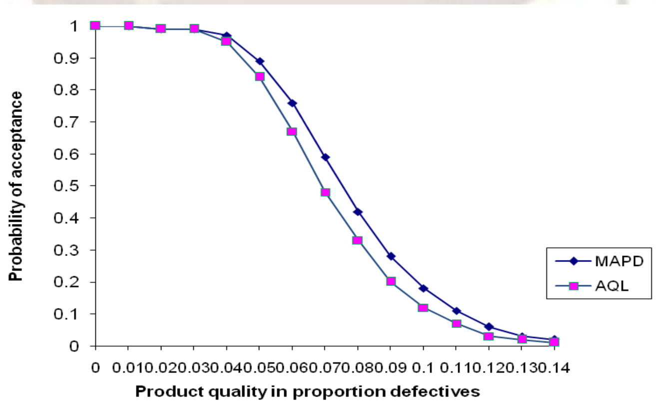

Figure 1 : OC Curves for CDSP with n=85 (MAPD), n=92 (AQL), $c_1=6$, $c_2=10$,$c_3=13$

7. CONCLUSION

In this paper the procedure for constructing mixed sampling plan's indexed through MAPD with Weighted Poisson distribution as the baseline distribution is presented. Suitable tables are also provided for the easy selection of the plans for the engineers who are working on the floor of the assembly. It is concluded from the study that the second sample size required for mixed sampling plan with Conditional double sampling plan as attribute plan indexed through MAPD is less than that of the second stage sample size of the Mixed sampling plan with Conditional double sampling plan as attribute plan indexed through AQL, justified by Sampath Kumar (2008) .These plans definitely help the producers, because of the lesser sample size which directly result in lesser sampling cost and indirectly reduces the total cost of the product. The different sampling plans can also be constructed by changing the first stage probabilities (β_*' and β_1') and can be compared for their efficiency.

REFERENCES

[1] Devaarul, S., 2003, *Certain studies relating to mixed sampling plans and reliability based sampling plans*, PhD Dissertation, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.

[2] Mayer, P.L., 1967, A note on sum of Poisson Probabilities and an Application, Industrial Quality Control, Vol 19, No.5, pp.12-15.

[3] Mohana Priya, L., 2008,*Construction and Selection of acceptance sampling plans based on Weighted Poisson distribution*, PhD Dissertation, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.

[4] Radhakrishna Rao, C., 1977, A natural example of a Weighted Binomial distribution, The American Statistician, Vol,31, No. 1.

[5] Radhakrishnan, R., and Mohana Priya, L., 2008 a, Selection of single sampling plan using Conditional Weighted Poisson distribution, *International Journal of Statistics and Systems*, Vol 3, No.1, pp. 91-98.

[6] Radhakrishnan, R., and Mohana Priya, L., 2008 b**,** Comparison of CRGS plans using Poisson and Weighted Poisson distribution, *Probstat Forum*, Vol 1, No.1, pp.50-61.

[7] Radhakrishnan, R., and Sampath Kumar, R., 2006 a, Construction of mixed sampling plan indexed through MAPD and IQL with single sampling plan as attribute plan, *National Journal of Technology*, Vol 2, No. 2, pp. 26-29.

[8] Radhakrishnan, R., and Sampath Kumar, R., 2006 b, Construction of mixed sampling plans indexed through MAPD and AQL with chain sampling plan as attribute plan, *STARS Int. Journal*, Vol 7, No 1, pp. 14-22.

[9] Radhakrishnan, R., and Sampath Kumar, R., 2007, Construction of mixed sampling plans indexed through MAPD and IQL with double sampling plan as attribute plan, *The Journal of the Kerala Statistical Association*, Vol 18, pp. 13-22.

[10] Radhakrishnan, R., and Sampath Kumar, R., 2009,Construction and comparison of mixed sampling plans having ChSP-(0,1) plan as attribute plan, *The International Journal of Statistics and Management System*,**Vol** 4, No.1-2, pp.134-149.

[11] Radhakrishnan, R., Sampath Kumar, R., and Malathi, M., 2010**,** Selection of mixed sampling plan with TNT- **(**n_1, n_2; 0) plan as attribute plan indexed through MAPD and MAAOQ, *International Journal of Statistics and System*, Vol 5, No 4, pp.477-484.

[12] Sampath Kumar, R., 2007**,** *Construction and selection of mixed variables-attributes sampling plan* - PhD Dissertation, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India.

[13] Schilling, E.G., 1967**,** *A general method for determining the operating characteristics of mixed variables-attributes sampling plans single side specifications*, S.D.known, PhD Dissertation- Rutgers- The State University, New Brunswick, New Jersy.

[14] Soundararajan, V., 1975, Maximum allowable percent defective (MAPD) single sampling inspection by attribute plan, *Journal of Quality Technology*, Vol 7, No. 4, pp.173-182.

[15] Sudeswari 2002, *Designing of sampling plan using Weighted Poisson distribution*, M.Phil. Dissertation, Department of Statistics, PSG College of Arts and Science, Coimbatore.