A New Method for Finding an Optimal Fuzzy Solution For Fully Fuzzy Linear Programming Problems

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Abstract

A new method namely, bound and decomposition method is proposed to find an optimal fuzzy solution for fully fuzzy linear programming (FFLP) problems. In the proposed method, the given FFLP problem is decomposed into three crisp linear programming (CLP) problems with bounded variables constraints, the three CLP problems are solved separately and by using its optimal solutions, the fuzzy optimal solution to the given FFLP problem is obtained. Fuzzy ranking functions and addition of nonnegative variables were not used and there is no restriction on the elements of coefficient matrix in the proposed method. The bound and decomposition method is illustrated by numerical examples.

Keywords: Linear programming, Fuzzy variables, Fuzzy linear programming, Bound and Decomposition method, Optimal fuzzy solution

1. Introduction

Linear programming (LP) is one of the most frequently applied operations research techniques. Although, it is investigated and expanded for more than six decades by many researchers from the various point of views, it is still useful to develop new approaches in order to better fit the real world problems within the framework of linear programming. In the real world situations, a linear programming model involves a lot of parameters whose values are assigned by experts. However, both experts and decision makers frequently do not precisely know the value of those parameters. Therefore, it is useful to consider the knowledge of experts about the parameters as fuzzy data [25]. Using the concept of decision making in fuzzy environment given by Bellman and Zadeh [5], Tanaka et al. [24] proposed a method for solving fuzzy mathematical programming problems. Zimmerman [26] developed a method for solving fuzzy LP problems using multiobjective LP technique. Campos and Verdegay [8] proposed a method to solve LP problems with fuzzy coefficients in both matrix and right hand of the constraint. Inuiguchi et al. [15] used the concept of continuous piecewise linear membership function for FLP problems. Cadenas and Verdegay [7] solved a LP problem in which all its elements are defined as fuzzy sets. Fang et al. [12] developed a method for solving LP problems with fuzzy coefficients in constraints. Buckley and Feuring [6] proposed a method to find the solution for a fully fuzzified linear programming problem by changing the objective function into a multiobjective LP problem. Maleki et al. [19] solved the LP problems by the comparison of fuzzy numbers in which all decision parameters are fuzzy numbers. Liu [17] introduced a method for solving FLP problems based on the satisfaction degree of the constraints.

Maleki [20] proposed a method for solving LP problems with vagueness in constraints by using ranking function. Zhang et al. [27] introduced a method for solving FLP problems in which coefficients of objective function are fuzzy numbers. Nehi et al. [22] developed the concept of optimality for LP problems with fuzzy parameters by transforming FLP problems into multiobjective LP problems. Ramik [23] proposed the FLP problems based on fuzzy relations. Ganesan and Veeramani [13] proposed an approach for solving FLP problem involving symmetric trapezoidal fuzzy numbers without converting it into crisp LP problems. Hashemi et al. [14] introduced a two phase approach for solving fully fuzzified linear programming. Jimenez et al. [16] developed a method using fuzzy ranking method for solving LP problems where all the coefficients are fuzzy numbers .

Allahviranloo et al. [1] solved fuzzy integer LP problem by reducing it into two crisp integer LP problems. Allahviranloo et al. [2] proposed a method based on ranking function for solving FFLP problems. Nasseri [21] solved FLP problems by using classical linear programming. Ebrahimnejad and Nasseri [10] solved FLP problem with fuzzy parameters by using the complementary slackness theorem. Ebrahimnejad et al. [11] proposed a new primal-dual algorithm for solving LP problems with fuzzy variables by using duality theorems. Dehghan et al. [9] proposed a FLP approach for finding the exact solution of fully fuzzy linear system of equations which is applicable only if all the elements of the coefficient matrix are non-negative fuzzy numbers. **Lotfi et al. [18]** proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints which can be applied only if the elements of the coefficient matrix are symmetric fuzzy numbers and the obtained solutions are approximate but not exact. Amit Kumar et al. [3, 4] proposed a method for solving the FFLP problems by using fuzzy ranking function in the fuzzy objective function.

In this paper, a new method namely, bound and decomposition method is proposed for finding an optimal fuzzy solution to FFLP problems. In this method, the FFLP problem is decomposed into three CLP problems and then using its solutions, we obtain an optimal fuzzy solution to the given FFLP problem. With the help of numerical examples, the bound and decomposition method is illustrated. The advantage of the bound and decomposition method is that there is no restriction on the elements of the coefficient matrix, fuzzy ranking functions and nonnegative variables are not used, the obtained results exactly satisfy all the constraints and the computation in the proposed method is more easy and also, simple because of the LP technique and the level wise computation. The bound and decomposition method can serve managers by providing an appropriate best solution to a variety of linear programming models having fuzzy numbers and variables in a simple and effective manner.

2. Preliminaries

We need the following definitions of the basic arithmetic operators and partial ordering relations on fuzzy triangular numbers based on the function principle which can be found in [2, 4, 25,26].

Definition 2.1 A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its member ship function $\mu_{\tilde{a}}(x)$ is given below:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x-a_1)/(a_2-a_1) & \text{for } a_1 \le x \le a_2 \\ (a_3-x)/(a_3-a_2) & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.2 Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then

(i) $(a_1,a_2,a_3) \oplus (b_1,b_2,b_3) = (a_1+b_1, a_2+b_2, a_3+b_3)$.

(ii) $(a_1,a_2,a_3)\ \Theta\ (b_1,b_2,b_3) = (a_1-b_3, a_2-b_2, a_3-b_1)$.

(iii) $k(a_1,a_2,a_3) = (ka_1, ka_2, ka_3)$, for $k \ge 0$.

(iv) $k(a_1,a_2,a_3) = (ka_3, ka_2, ka_1)$, for $k < 0$.

(v) $$(a_1,a_2,a_3) \otimes (b_1,b_2,b_3) = \begin{cases} (a_1b_1, a_2b_2, a_3b_3), & a_1 \ge 0, \\ (a_1b_3, a_2b_2, a_3b_3), & a_1 < 0, a_3 \ge 0, \\ (a_1b_3, a_2b_2, a_3b_1), & a_3 < 0. \end{cases}$$

Let $F(R)$ be the set of all real triangular fuzzy numbers.

Definition 2.3 Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be in $F(R)$, then

(i) $\tilde{A} \approx \tilde{B}$ iff $a_i = b_i, i = 1,2,3$; (ii) $\tilde{A} \preceq \tilde{B}$ iff $a_i \le b_i, i = 1,2,3$

(iii) $\tilde{A} \succeq \tilde{B}$ iff $a_i \ge b_i, i = 1,2,3$ and $\tilde{A} \succeq \tilde{0}$ iff $a_i \ge 0, i = 1,2,3$.

3. Fully Fuzzy Linear Programming Problem

Consider the following fully fuzzy linear programming problems with m fuzzy inequality/equality constraints and n fuzzy variables may be formulated as follows:

(P) $\quad \text{Maximize (or Minimize)} \ \tilde{z} \approx \tilde{c}^T \tilde{x}$

subject to $\tilde{A} \otimes \tilde{x} \; \{\preceq, \approx, \succeq\} \; \tilde{b}$, (1)

$\tilde{x} \succeq \tilde{0}$ (2)

where $\tilde{c}^T = (\tilde{c}_j)_{1\times n}$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{x} = (\tilde{x}_j)_{nx1}$ and $\tilde{b} = (\tilde{b}_i)_{mx1}$ and $\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in F(R)$, for all $1 \le j \le n$ and $1 \le i \le m$.

Let the parameters $\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j$ *and* $\tilde{b}_i$ be the triangular fuzzy number (p_j, q_j, r_j), (x_j, y_j, t_j), (a_{ij}, b_{ij}, c_{ij}) and (b_i, g_i, h_i) respectively. Then, the problem (P) can be written as follows:

(P) Maximize (or Minimize) $(z_1, z_2, z_3) \approx \sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, t_j)$

subject to

$$\sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, t_j) \{\preceq, \approx, \succeq\} (b_i, g_i, h_i), \text{ for all } i = 1,2,\ldots,m$$

$$(x_j, y_j, t_j) \succeq \tilde{0}.$$

Now, since (x_j, y_j, t_j) is a triangular fuzzy number, then

$$x_j \le y_j \le t_j, \quad j = 1,2,\ldots,m. \tag{3}$$

The relation (3) is called bounded constraints.

Now, using the arithmetic operations and partial ordering relations, we decompose the given FLPP as follows:

Maximize $z_1 = \sum_{j=1}^{n}$ lower value of $((p_j, q_j, r_j) \otimes (x_j, y_j, t_j))$

Maximize $z_2 = \sum_{j=1}^{n}$ middle value of $((p_j, q_j, r_j) \otimes (x_j, y_j, t_j))$

Maximize $z_3 = \sum_{j=1}^{n}$ upper value of $((p_j, q_j, r_j) \otimes (x_j, y_j, t_j))$

subject to

$\sum_{j=1}^{n}$ lower value of $((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, t_j))\{\le, =, \ge\} b_i$, for all $i = 1,2,\ldots,m$;

$\sum_{j=1}^{n}$ middle value of $((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, t_j))\{\le, =, \ge\} g_i$, for all $i = 1,2,\ldots,m$;

$\sum_{j=1}^{n}$ upper value of $((a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, t_j))\{\le, =, \ge\} h_i$, for all $i = 1,2,\ldots,m$

and all decision variables are non-negative.

From the above decomposition problem, we construct the following CLP problems namely, middle level problem (MLP), upper level problem (ULP) and lower level problem (LLP) as follows:

(MLP) Maximize $z_2 = \sum_{j=1}^{n}$ middle value of $\left((p_j, q_j, r_j) \otimes (x_j, y_j, t_j)\right)$

subject to

Constraints in the decomposition problem in which atleast one decision variable of the (MLP) occurs and all decision variables are non-negative.

(ULP) Maximize $z_3 = \sum_{j=1}^{n}$ upper value of $\left((p_j, q_j, r_j) \otimes (x_j, y_j, t_j)\right)$

subject to

$\sum_{j=1}^{n}$ upper value of $\left((p_j, q_j, r_j) \otimes (x_j, y_j, t_j)\right) \geq z_2^{\circ}$;

Constraints in the decomposition problem in which atleast one decision variable of the (ULP) occurs and are not used in (MLP) ;

all variables in the constraints and objective function in (ULP) must satisfy the bounded constraints ;

replacing all values of the decision variables which **are obtained in (MLP) and all decision variables are non-negative.**

and **(LLP)** Maximize $z_1 = \sum_{j=1}^{n}$ lower value of $\left((p_j, q_j, r_j) \otimes (x_j, y_j, t_j)\right)$

subject to

$\sum_{j=1}^{n}$ lower value of $\left((p_j, q_j, r_j) \otimes (x_j, y_j, t_j)\right) \leq z_2^{\circ}$;

Constraints in the decomposition constraints in which atleast one decision variable of the (LLP) occurs which are not used in (MLP) and (ULP);

all variables in the constraints and objective function in (LLP) must satisfy the bounded constraints;

replacing all values of the decision variables which are obtained in the (MLP) and (ULP) and all decision variables are non-negative,

where z_2° is the optimal objective value of (MLP).

Remark 3.1: In the case of LP problem involving trapezoidal fuzzy numbers and variables, we decompose it into four CLP problems and then, we solve the middle level problems (second and third problems) first. Then, we solve the upper level and lower level problems and then, we obtain the fuzzy optimal solution to the given FLP problem involving trapezoidal fuzzy numbers and variables.

4. The Bound and Decomposition Method

Now, we prove the following theorem which is used in the proposed method namely. Bound and decomposition method to solve the FFLP problem.

Theorem 4.1: Let $[x_M^{\circ}] = \{\ x_j^{\circ}, x_j^{\circ} \in M\ \}$ be an optimal solution of (MLP) , $[x_U^{\circ}] = \{\ x_j^{\circ}, x_j^{\circ} \in U\ \}$ be an optimal solution of (ULP) and $[x_L^{\circ}] = \{\ x_j^{\circ}, x_j^{\circ} \in L\ \}$ be an optimal solution of (LLP) where L, M and U are sets of decision variables in the $(LLP), (MLP)$ and (ULP) respectively. Then $\left\{\ \tilde{x}_j^{\circ} = (x_j^1, x_j^2, x_j^3),\ j = 1,2,...,n\ \right\}$ is an optimal fuzzy solution to the given problem (P) where each one of $x_j^{\circ 1}, x_j^{\circ 2}$ and $x_j^{\circ 3},\ j = 1,2,...,n$ is an element of L, M and U .

Proof: Let $[\tilde{y}_j] = \{\tilde{y}_j, j = 1,2,...,n\}$ be a feasible solution of (P). Clearly, $[y_M],[y_U]$ and $[y_L]$ are feasible solutions of $(MLP),(ULP)$ and (LLP) respectively.

Now, since $[x_M^\circ],[x_U^\circ]$ and $[x_L^\circ]$ are optimal solutions of $(MLP),(ULP)$ and (LLP) respectively, we have

$$Z_1([x_L^\circ]) \le Z_1([y_L]);\ \ Z_2([x_M^\circ]) \le Z_2([y_M]) \ \text{ and } \ Z_3([x_U^\circ]) \le Z_3([y_U])$$

This implies that $Z([\tilde{x}_j^\circ]) \le Z([\tilde{y}_j])$, for all feasible solution of the problem (P).

Therefore, $\{\tilde{x}_j^\circ \approx (x_j^{\circ 1}, x_j^{\circ 2}, x_j^{\circ 3}),\ j = 1,2,...,n\}$ is an optimal fuzzy solution to the given problem (P) where each one of $x_j^{\circ 1}, x_j^{\circ 2}$ and $x_j^{\circ 3},\ j = 1,2,...,n$ is an element of L, M and U.

Hence the theorem.

Now, we propose a new method namely, bound and decomposition method for solving a FFLP problem.

The bound and decomposition method proceeds as follows.

Step 1: Construct (MLP), (ULP) and (LLP) problems from the given the FFLP problems.

Step 2: Using existing linear programming technique, solve the (MLP) problem, then the (ULP) problem and then, the (LLP) problem in the order only and obtain the values of all real decision variables x_j, y_j *and* t_j and values of all objectives z_1, z_2 and z_3. Let the decision variables values be x_j°, y_j° *and* t_j°, j = 1,2,…,m and objective values be z_1°, z_2° and z_3°.

Step 3: An optimal fuzzy solution to the given FFLP problems is $\tilde{x}_j^\circ = (x_j^\circ, y_j^\circ, t_j^\circ)$, j=1,2,…,m and the maximum fuzzy objective is $(z_1^\circ, z_2^\circ, z_3^\circ)$ (by the Theorem 4.1.).

Now, we illustrate the proposed method using the following numerical examples

Example 4.1: Consider the following fully fuzzy linear programming problem (Objective function contains negative term)

Maximize $\tilde{z} \approx (-1, 2, 3) \otimes \tilde{x}_1 \oplus (2, 3, 4) \otimes \tilde{x}_2$

subject to

$$(0,1,2) \otimes \tilde{x}_1 \oplus (1,2,3) \otimes \tilde{x}_2 \approx (2,10,24);$$
$$(1,2,3) \otimes \tilde{x}_1 \oplus (0,1,2) \otimes \tilde{x}_2 \approx (1,8,21);$$
$$\tilde{x}_1 \text{ and } \tilde{x}_2 \succeq \tilde{0}.$$

Let $\tilde{x}_1 = (x_1, y_1, t_1)$, $\tilde{x}_2 = (x_2, y_2, t_2)$ and $\tilde{z} = (z_1, z_2, z_3)$.

Now, the decomposition problem of the given FLPP is given below.

Maximize $z_1 = -t_1 + 2x_2$

Maximize $z_2 = 2y_1 + 3y_2$

Maximize $z_3 = 3t_1 + 4t_2$

subject to

$$0x_1 + x_2 = 2;\ x_1 + 0x_2 = 1;$$

$y_1 + 2y_2 = 10$; $2y_1 + y_2 = 8$; .
$2t_1 + 3t_2 = 24$; $3t_1 + 2t_2 = 21$;
$x_1, x_2 \geq 0$, $y_1, y_2 \geq 0$, $t_1, t_2 \geq 0$.

Now, the Middle Level problem is given below:

(P_2) : Maximize $z_2 = 2y_1 + 3y_2$
subject to
$y_1 + 2y_2 = 10$; $2y_1 + y_2 = 8$; $y_1, y_2 \geq 0$.

Now, solving the problem (P_2) using simplex method, we obtain the optimal solution $y_1 = 2$; $y_2 = 4$ and $z_2 = 16$.

Now, the Upper Level problem is given below:

(P_3): Maximize $z_3 = 3t_1 + 4t_2$
subject to
$3t_1 + 4t_2 \geq 16$; $2t_1 + 3t_2 = 24$; $3t_1 + 2t_2 = 21$; $t_1 \geq y_1$; $t_2 \geq y_2$; $t_1, t_2 \geq 0$.

Now, solving the problem (P_3) with $y_1 = 2$; $y_2 = 4$ using simplex method , we obtain the optimal solution and $t_1 = 3$; $t_2 = 6$ and $z_3 = 33$.

Now, the Lower Level problem:

(P_1) : Maximize $z_1 = -t_1 + 2x_2$
subject to
$-t_1 + 2x_2 \leq 16$; $0x_1 + x_2 = 2$; $x_1 + 0x_2 = 1$; $x_1 \leq y_1$, $x_2 \leq y_2$; $x_1, x_2 \geq 0$.

Now, solving the problem (P_1) with $t_1 = 3; y_1 = 2$; $y_2 = 4$ by simplex method, we obtain the optimal solution $x_1 = 1; x_2 = 2$ and $z_1 = 1$.

Therefore, an optimal fuzzy solution to the given fully fuzzy linear programming problem is
$\tilde{x}_1 \approx (1, 2, 3)$, $\tilde{x}_2 \approx (2, 4, 6)$ and $\tilde{z} \approx (1, 16, 33)$.

Remark 4.1: The solution of the Example 4.1 is obtained at the 8th iteration with 10 non-negative variables by Amit Kumar et al.[4] method.

Example 4.2: Consider the following fully fuzzy linear programming problem:

Maximize $\tilde{z} \approx (1, 6, 9) \otimes \tilde{x}_1 \oplus (2, 3, 8) \otimes \tilde{x}_2$
subject to
$(2, 3, 4) \otimes \tilde{x}_1 \oplus (1, 2, 3) \otimes \tilde{x}_2 \approx (6, 16, 30)$;
$(-1, 1, 2) \otimes \tilde{x}_1 \oplus (1, 3, 4) \otimes \tilde{x}_2 \approx (1, 17, 30)$;
$\tilde{x}_1$ and $\tilde{x}_2 \succeq \tilde{0}$.

Now, by solving the bound and decomposition method, an optimal fuzzy solution to the given fully fuzzy linear programming problem is $\tilde{x}_1 \approx (1, 2, 3)$, $\tilde{x}_2 \approx (4, 5, 6)$ and $\tilde{z} \approx (9, 27, 75)$.

Remark 4.2: The solution of the Example 4.2, obtained by the bound and decomposition method is same as in Amit Kumar et al. [4].

Example 4.3: Consider the following fully fuzzy linear programming problem:

Maximize $\tilde{z} \approx (1, 6, 9) \otimes \tilde{x}_1 \oplus (2, 2, 8) \otimes \tilde{x}_2$
subject to

$$(0,1,1)\otimes\tilde{x}_1 \oplus (2,2,3)\otimes\tilde{x}_2 \succeq (4,7,14);$$
$$(2,2,3)\otimes\tilde{x}_1 \oplus (-1,4,4)\otimes\tilde{x}_2 \preceq (-4,14,22);$$
$$(2,3,4)\otimes\tilde{x}_1 \,\Theta\, (1,2,3)\otimes\tilde{x}_2 \approx (-12,-3,6);$$
$$\tilde{x}_1 \text{ and } \tilde{x}_2 \succeq \tilde{0}.$$

Now, by solving the bound and decomposition method, an optimal fuzzy solution to the given fully fuzzy linear programming problem is $\tilde{x}_1 \approx (0,1,2)$, $\tilde{x}_2 \approx (2,3,4)$ and $\tilde{z} \approx (4,12,50)$.

Remark 4.3: The solution of the Example 4.3, obtained by the Bound and Decomposition method is same as in Amit Kumar et al. [3].

Example 4.4: Consider the following fully fuzzy linear programming problem:

Maximize $\tilde{z} \approx (1,2,3)\otimes\tilde{x}_1 \oplus (2,3,4)\otimes\tilde{x}_2$

subject to

$$(0,1,2)\otimes\tilde{x}_1 \oplus (1,2,3)\otimes\tilde{x}_2 \preceq (1,10,27);$$
$$(1,2,3)\otimes\tilde{x}_1 \oplus (0,1,2)\otimes\tilde{x}_2 \preceq (2,11,28);$$
$$\tilde{x}_1 \text{ and } \tilde{x}_2 \succeq \tilde{0}.$$

Now, by solving the bound and decomposition method, an optimal fuzzy solution to the given fully fuzzy linear programming problem is $\tilde{x}_1 \approx (2,4,6)$, $\tilde{x}_2 \approx (1,3,5)$ and $\tilde{z} \approx (4,17,38)$.

Remark 4.4: The solution of the Example 4.4, obtained by the bound and decomposition method is same as in Amit Kumar et al. [3].

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