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Two-sample Statistical Hypotheses Test for Means with Imprecise Data

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Abstract
A new test procedure of two-sample statistical hypotheses for means in normal populations with interval data is proposed. The decision rules that are used to accept or reject the null and alternative hypotheses are given. With the help of the numerical example, the proposed test procedure is illustrated. The proposed test is extended to statistical hypotheses testing for fuzzy data.

Mathematics Subject Classification: 62F03

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1.Introduction

Testing statistical hypotheses is one of the most important areas of statistical analysis. In many situations, the researchers in the field of data analysis are interested in testing a hypothesis about the population parameter. In traditional testing [9], the observations of sample are crisp and a statistical test leads to the binary decision. However, in real life, the data sometimes cannot be recorded precisely. The statistical hypothesis testing under fuzzy environments has been studied by many authors. Arnold [3] discussed the fuzzy hypotheses testing with crisp data. The Neyman–Pearson type testing hypotheses was proposed by Casals and Gil [6] and Son et al. [17]. Saade [15, 16] considered the binary hypotheses testing and discussed the fuzzy likelihood functions in the decision making process. Casals and Gil [5, 7] considered the Bayesian sequential tests for fuzzy parametric hypotheses from fuzzy information. In the human sciences, Niskanen [14] discussed the applications of soft statistical hypotheses.The statistical hypotheses testing for fuzzy data by proposing the notions of degrees of optimism and pessimism was proposed by Wu [21]. Akbari and Rezaei [1] investigated a bootstrap method for inference about the variance based on fuzzy data. Viertl [18, 19] investigated some methods to construct confidence intervals and statistical tests for fuzzy data. Wu [22] proposed some approaches to construct fuzzy confidence intervals for the unknown fuzzy parameter. Arefi and Taheri [2] developed an approach to test fuzzy hypotheses upon fuzzy test statistic for vague data. The fuzzy tests for hypotheses testing with vague data were proposed by Grzegorzewski [11], Montenegro *et al.* [13], Baloui Jamkhaneh and Nadi Ghara [4] and Watanabe and Imaizumi [20]. A new approach to the problem of testing statistical hypotheses for fuzzy data using the relationship between confidence intervals and testing hypotheses is introduced by Chachi et al.[8].

In this paper, we propose a new statistical hypothesis testing procedure about population means when the data of the given two samples are real intervals. We provide the decision rules which are used to accept or reject the null and alternative hypotheses. In the proposed test, we split the given interval data into two different sets of crisp data namely, upper level data and lower level data ; then, we find the test statistic values for the two sets of crisp data and then we obtain a decision about the population means on the basis of the decision rules. In this testing procedure, we are not using degrees of optimism and pessimism and h-level set. To illustrate the proposed testing procedure, a numerical example is given. Further, we extend the proposed test to statistical hypotheses with fuzzy data.

2. Preliminaries

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [10,12].

Let $D = \{[a,b],\ a \le b$ and a and b are in R$\}$ the set of all closed bounded intervals on the real line R.

Definition 2.1: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then,

(i) $A \oplus B = [a+c, b+d]$; (ii) $A \Theta B = [a-d, b-c]$;

(iii) $kA = [ka, kb]$ if k is a positive real number ;

(iv) $kA = [kb, ka]$ if k is a negative real number and

(v) $A \otimes B = [p,q]$ where $p = \min\{ac, ad, bc, bd\}$ and $q = \max.\{ac, ad, bc, bd\}$.

Definition 2.2: Let $A=[a,b]$ and $B=[c,d]$ be in D. Then,

(i) $A \le B$ if $a \le c$ and $b \le d$; (ii) $A \ge B$ if $a \ge c$ and $b \ge d$ and

(iii) $A = B$ if $a = c$ and $b = d$.

3. Two-sample *t*-test

Let $x_1, x_2, ..., x_m$ be a random sample (X-sample) from a normal population with size m and $y_1, y_2, ..., y_n$ be another random sample (Y-sample) from an another normal population with size n such that $m + n \le 30$. Now, the mean values, denoted by $\bar{x}$ and $\bar{y}$ and the sample standard deviation, denoted by s_1 and s_2 of the above small samples are given by

$$\bar{x} = \frac{1}{m}\left(\sum_{i=1}^{m} x_i\right),\ \bar{y} = \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right),$$

$$s_1 = \sqrt{\frac{1}{m-1}\left(\sum_{i=1}^{m} (x_i - \bar{x})^2\right)} \text{ and}$$

$$s_2 = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} (y_i - \bar{y})^2\right)}.$$

Let μ_1 be the population mean of the X-sample and μ_2 be the population mean of the Y-sample.

In testing the null hypothesis $(H_\circ): \mu_1 = \mu_2$ with assumption of equal population standard deviations , one uses the statistic

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{m} + \frac{1}{n}}} \text{ where}$$

$$s = \sqrt{\frac{(m-1)s_1^2 + (n-1)s_2^2}{m+n-2}}.$$

In testing the null hypothesis $(H_\circ): \mu_1 = \mu_2$ with assumption of unequal population standard deviations , one uses the statistic

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}.$$

Now, the degrees of freedom used in this test is $\nu = n + m - 2$.

Let the level of significance be α. Let $t_{\alpha,\nu}$ denote the table value of t for ν degrees of freedom at α level.

Null hypothesis $H_\circ : \mu_1 = \mu_2$.

Now, the rejection region of the alternative hypothesis for level α is given below:

Alternative Hypothesis	Rejection Region for Level α test
$H_A : \mu_1 > \mu_2$	$t \geq t_{\alpha,m+n-2}$ (upper tailed test)
$H_A : \mu_1 < \mu_2$	$t \leq -t_{\alpha,m+n-2}$ (lower tailed test)
$H_A : \mu_1 \neq \mu_2$	$\lvert t \rvert \geq t_{\alpha/2,m+n-2}$ (two tailed test)

If $\lvert t \rvert < t_{\alpha,m+n-2}$ (one tailed test), the difference between μ_1 and μ_2 is not significant at α level . Then, the means of the populations are identical , that is, $\mu_1 = \mu_2$ at α level. Therefore, the null hypothesis is accepted. Otherwise, the alternative hypothesis is accepted.

If $\lvert t \rvert < t_{\alpha/2,m+n-2}$ (two tailed test), the difference between μ_1 and μ_2 is not significant at α level . Then, the means of the populations are identical , that is, $\mu_1 = \mu_2$ at α level. Therefore, the null hypothesis is accepted. Otherwise, the alternative hypothesis is accepted.

Now, the 100($1-\alpha$)% confidence limits for the difference of population means μ_1 and μ_2 corresponding to the given samples are given below:

$$(\bar{x}-\bar{y}) - t_{\alpha/2,m+n-2}\left(s\sqrt{\frac{1}{m}+\frac{1}{n}}\right) < \mu_1 - \mu_2 < (\bar{x}-\bar{y}) + t_{\alpha/2,m+n-2}\left(s\sqrt{\frac{1}{m}+\frac{1}{n}}\right).$$

Or

$$(\bar{x}-\bar{y}) - t_{\alpha/2,m+n-2}\left(\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}}\right) < \mu_1 - \mu_2 < (\bar{x}-\bar{y}) + t_{\alpha/2,m+n-2}\left(\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}}\right).$$

3.1. Testing Hypotheses for Interval Data

Let $\{[a_i, b_i], i = 1,2,\ldots,m\}$ be a random small sample (X- sample) with size m and $\{[c_j, d_j], j = 1,2,\ldots,n\}$ be a random small sample (Y-sample) with size n such that $\{[a_i, b_i], i = 1,2,\ldots,m\}$ is a random sample from a normal population with mean $[\eta_1, \mu_1]$ and $\{[c_j, d_j], j = 1,2,\ldots,n\}$ is another random sample from another normal population with mean $[\eta_2, \mu_2]$.

Now, we are going to test the null hypothesis that the means of the populations of the given samples are equal , that is, $[\eta_1, \mu_1]$ = $[\eta_2, \mu_2]$,this implies that, $\eta_1 = \eta_2$ and $\mu_1 = \mu_2$.

Testing Hypothesis:

Null Hypothesis (H_o): $[\eta_1, \mu_1] = [\eta_2, \mu_2]$, that is $\eta_1 = \eta_2$ and $\mu_1 = \mu_2$.

Alternative Hypothesis (H_A): (i) $[\eta_1, \mu_1] \neq [\eta_2, \mu_2]$, that is, $\eta_1 \neq \eta_2$ or $\mu_1 \neq \mu_2$.

(ii) $[\eta_1, \mu_1] > [\eta_2, \mu_2]$, that is, $\eta_1 > \eta_2$ and $\mu_1 > \mu_2$.

(iii) $[\eta_1, \mu_1] < [\eta_2, \mu_2]$, that is, $\eta_1 < \eta_2$ and $\mu_1 < \mu_2$.

Consider the following random sample consisting of the lower values of X- sample and Y- sample :

X^L (lower values of X-sample)	a_i , $i = 1,2,\ldots,m$
Y^L (lower values of Y-sample)	c_j , $j = 1,2,\ldots,n$

Now, the sample means of X^L and Y^L are $\bar{x}_L$ and $\bar{y}_L$ respectively and the sample S.Ds of X^L and Y^L are s_{x_L} and s_{y_L} respectively.

Consider the following random sample consisting of the upper values of X- sample and Y- sample :

X^U (upper values of X-sample)	b_i , $i = 1,2,...,m$
Y^U (upper values of Y-sample)	d_j , $j = 1,2,...,n$

Now, the sample means of X^U and Y^U are $\bar{x}_U$ and $\bar{y}_U$ respectively and the sample S.Ds of X^U and Y^U are s_{x_U} and s_{y_U} respectively.

Case (i): If the population standard deviations are assumed to be equal, we use the following test statistics .

$$t_L = \frac{\bar{x}_L - \bar{y}_L}{s_L \sqrt{\frac{1}{m} + \frac{1}{n}}} \quad \text{and}$$

$$t_U = \frac{\bar{x}_U - \bar{y}_U}{s_U \sqrt{\frac{1}{m} + \frac{1}{n}}}$$

where $s_L = \sqrt{\dfrac{(m-1)s_{x_L}^2 + (n-1)s_{y_L}^2}{m+n-2}}$

and $s_U = \sqrt{\dfrac{(m-1)s_{x_U}^2 + (n-1)s_{y_U}^2}{m+n-2}}$.

Case(ii): If the population standard deviations are assumed to be not equal, we use the following test statistics .

$$t_L = \frac{\bar{x}_L - \bar{y}_L}{\sqrt{\frac{s_{x_L}^2}{m} + \frac{s_{y_L}^2}{n}}} \quad \text{and}$$

$$t_U = \frac{\bar{x}_U - \bar{y}_U}{\sqrt{\frac{s_{x_U}^2}{m} + \frac{s_{y_U}^2}{n}}} .$$

Now, the rejection region of the alternative hypothesis for level α is given below:

Alternative Hypothesis	Rejection Region for level α test
$H_A : [\eta_1, \mu_1] > [\eta_2, \mu_2]$	$t_L \geq t_{\alpha, m+n-2}$ and $t_U \geq t_{\alpha, m=n-2}$ (upper tailed test)
$H_A : [\eta_1, \mu_1] <$	$t_L \leq -t_{\alpha, m+n-2}$ and $t_U \leq -t_{\alpha, m+n-2}$ (lower tailed test)
$H_A : [\eta_1, \mu_1] \neq [\eta_2, \mu_2] [\eta_2, \mu_2]$,	$\lvert t_L \rvert \geq t_{\alpha/2, m+n-2}$ or $\lvert t_U \rvert \geq t_{\alpha/2, m+n-2}$ (two tailed test)

Now, if $\lvert t_L \rvert < t_{\alpha, m+n-2}$ (one tailed test) and $\lvert t_U \rvert < t_{\alpha, m+n-2}$ (one tailed test), the difference between $[\eta_1, \mu_1]$ and $[\eta_2, \mu_2]$ is not significant at α level . Then, the means of the populations are equal, that is, $[\eta_1, \mu_1]$ = $[\eta_2, \mu_2]$. Therefore, the null hypothesis is accepted.

Now, if $\lvert t_L \rvert < t_{\alpha/2, m+n-2}$ (two tailed test) and $\lvert t_U \rvert < t_{\alpha/2, m+n-2}$ (two tailed test), the difference between $[\eta_1, \mu_1]$ and $[\eta_2, \mu_2]$ is not significant at α level . Then, the means of the populations are equal, that is, $[\eta_1, \mu_1] = [\eta_2, \mu_2]$. Therefore, the null hypothesis is accepted.

Now, the $100(1-\alpha)\%$ confidence limits for the difference of lower limit and upper limit of the population means $[\eta_1, \mu_1]$ and $[\eta_2, \mu_2]$ corresponding to the given samples are given below:

$$(\bar{x}_L - \bar{y}_L) - t_{\alpha/2, m+n-2}\left(s_L\sqrt{\frac{1}{m}+\frac{1}{n}}\right) < \eta_1 - \eta_2 < (\bar{x}_L - \bar{y}_L) + t_{\alpha/2, m+n-2}\left(s_L\sqrt{\frac{1}{m}+\frac{1}{n}}\right)$$

and

$$(\bar{x}_U - \bar{y}_U) - t_{\alpha/2, m+n-2}\left(s_U\sqrt{\frac{1}{m}+\frac{1}{n}}\right) < \mu_1 - \mu_2 < (\bar{x}_U - \bar{y}_U) + t_{\alpha/2, m+n-2}\left(s_U\sqrt{\frac{1}{m}+\frac{1}{n}}\right)$$

(or)

$$(\bar{x}_L - \bar{y}_L) - t_{\alpha/2, m+n-2}\sqrt{\frac{s_{x_L}^2}{m}+\frac{s_{y_L}^2}{n}} < \eta_1 - \eta_2 < (\bar{x}_L - \bar{y}_L) + t_{\alpha/2, m+n-2}\sqrt{\frac{s_{x_L}^2}{m}+\frac{s_{y_L}^2}{n}}$$

and

$$(\bar{x}_U - \bar{y}_U) - t_{\alpha/2, m+n-2}\sqrt{\frac{s_{x_U}^2}{m}+\frac{s_{y_U}^2}{n}} < \mu_1 - \mu_2 < (\bar{x}_U - \bar{y}_U) + t_{\alpha/2, m+n-2}\sqrt{\frac{s_{x_U}^2}{m}+\frac{s_{y_U}^2}{n}}$$

.The test procedure can be illustrated using the following numerical example.

Example 1: The deterioration of many municipal pipeline networks across the country is growing concerned. One technology proposed for pipeline rehabilitation uses a flexible liner threaded through existing pipe. The article " Effect of welding on a High-Density polyethylene Liner" reported the following data on tensile strength (psi) of liner specimens both when a certain fusion process was used and when this process was not used. It is assumed that the tensile strength distributions under the two conditions are both normal.

No fusion	[2728,2768] [2655,2745] [2600,2710] [2800,2844] [2496,2526] [3132,3166] [3200,3314] [3198,3228] [3206,3324] [2700,2806]
Fused	[3000,3054] [3340,3372] [3347,3371] [3290,3304] [3100,3150] [2900,2920] [2884,2894] [2900,2904]

We are going to analyze that the true average tensile strength $[\eta_1, \mu_1]$ for no-fusion treatment and the true average tensile strength $[\eta_2, \mu_2]$ for fusion treatment are equal.

That is, $H_0 : [\eta_1, \mu_1] = [\eta_2, \mu_2]$ and $H_A : [\eta_1, \mu_1] \neq [\eta_2, \mu_2]$.

We assume that the S.Ds of the populations are not equal and we use 5 % the level of significance.

Now, the table value of t for 16 degrees of freedom at 1% level, $T = 2.120$.

We have $m = 10$, $n = 8$, $\bar{x}_1 = 2871.5$, $\bar{y}_1 = 3095.125$, $s_{x_L} = 281.0628$, $s_{y_L} = 203.952$, $\bar{y}_1 = 2943.1$, $\bar{y}_2 = 3121.125$, $s_{x_U} = 286.8087$ and $s_{y_U} = 208.1685$

Test statistics:

$$t_L = \frac{\bar{x}_1 - \bar{y}_1}{\sqrt{\frac{s_{x_L}^2}{m}+\frac{s_{y_L}^2}{n}}}$$

$$= \frac{2871.5 - 3095.125}{\sqrt{\frac{(281.0628)^2}{10}+\frac{(203.952)^2}{8}}} = -1.9539$$

and

$$t_U = \frac{\bar{x}_1 - \bar{y}_1}{\sqrt{\frac{s_{x_U}^2}{m}+\frac{s_{y_U}^2}{n}}}$$

$$= \frac{2943.1 - 3121.125}{\sqrt{\frac{(286.8087)^2}{10} + \frac{(208.1685)^2}{8}}} = -1.52416 .$$

Now, since $|t_U| < T$ and $|t_L| < T$, we accept H_0. Therefore, the true average tensile strength for the no-fusion treatment and the true average tensile strength for the fusion treatment are equal at 1% level of significance.

3.2. Testing Hypotheses for fuzzy data

A triangular fuzzy number (a,b,c) can be represented as an interval number form as follows.

$$[(a,b,c)] = [a + (b-a)\lambda, c - (c-b)\lambda] ; \quad 0 \le \lambda \le 1. \qquad (1)$$

Suppose that the given sample is a fuzzy data that are triangular fuzzy numbers and we have to test the hypothesis about the population mean. Using the relation (1) and the proposed test procedure, we can test the hypothesis by transferring the fuzzy data into interval data. The solution procedure is illustrated with help of the following numerical example.

Example 2: We have two kinds of tire (A and B) for automobile and we set up each one on the some taxi, and then we request from taxi drivers to record consumption of the petrol. The data are recorded as triangular fuzzy number as given in the following table. Suppose that the random variables have normal distribution and their variance of both populations are known and equal with one. We investigate the effect of tires on consumption of the petrol at 5% level of significance.

$\tilde{A}$:	(4,5, 6)	(3.5,5 ,6.5)	(5,5.5 ,6)	(5.5,6 ,6.5)	(3,4 ,5)	
$\tilde{B}$:	(5,6. 5,8)	(4,5,6)	(5.5,7 ,8.5)	(5,6,7)	(6,7 ,8)	(6,7. 5,9)

Now, the interval representation of the above data is given below:

$[\tilde{A}]$	$[\tilde{B}]$
$[4+\alpha, 6-\alpha]$	$[5+1.5\alpha, 8-1.5$
$[3.5+1.5\alpha, 6.5-1$	$[4+\alpha, 6-\alpha]$
$[5+0.5\alpha, 6-0.5$	$[5.5+1.5\alpha, 8.5-$
$[5.5+0.5\alpha, 6.5-$	$[5+\alpha, 7-\alpha]$
$[3+\alpha, 5-\alpha]$	$[6+\alpha, 8-\alpha]$
-	$[6+1.5\alpha, 9-1.5$

Now, the lower level samples data and upper level samples data are given below:

Lower level samples		Upper level samples	
x_L	y_L	x_U	y_U
$4+\alpha$	$5+1.5\alpha$	$6-\alpha$	$8-1.5\alpha$
$3.5+1.5\alpha$	$4+\alpha$	$6.5-1.5\alpha$	$6-\alpha$
$5+0.5\alpha$	$5.5+1.5\alpha$	$6-0.5\alpha$	$8.5-1.5\alpha$
$5.5+0.5\alpha$	$5+\alpha$	$6.5-0.5\alpha$	$7-\alpha$
$3+\alpha$	$6+\alpha$	$5-\alpha$	$8-\alpha$
-	$6+1.5\alpha$	-	$9-1.5\alpha$

Now, we have : $m = 5$, $n = 6$ $\bar{x}_L = 4.2 + 0.9\lambda$, $\bar{y}_L = 5.25 + 1.25\lambda$, $\bar{x}_U = 6 - 0.9\lambda$, $\bar{y}_U = 7.75 - 1.25\lambda$,

$s^2_{x_L} = 0.175\lambda^2 - 0.68\lambda + 1.075$,

$s^2_{y_L} = 0.075\lambda^2 + 0.15\lambda + 0.575$,

$s_L = \sqrt{0.0672\lambda^2 - 0.1281\lambda + 0.4484}$ and $s_U = \sqrt{0.0672\lambda^2 - 0.1406\lambda + 0.4609}$.

Now, the null hypothesis, $\tilde{H}_0 : \tilde{\Lambda} \approx \tilde{\Omega}$ (two kinds of tire for automobile on consumption of the petrol are the same) and the alternative hypothesis, $\tilde{H}_A : \tilde{\Lambda} \not\approx \tilde{\Omega}$ (two kinds of tire for automobile on consumption of the petrol are not the same).

This implies that $[\tilde{H}_0]:[\tilde{\Lambda}]=[\tilde{\Omega}]$ and $[\tilde{H}_A]:[\tilde{\Lambda}]\neq[\tilde{\Omega}]$ where $[\tilde{\Lambda}]=[\eta_1,\mu_1]$ and $[\tilde{\Omega}]=[\eta_2,\mu_2]$. Therefore, $[\tilde{H}_0]:\eta_1=\eta_2$ and $\mu_1=\mu_2$; $[\tilde{H}_A]:\eta_1\neq\eta_2$ or $\mu_1\neq\mu_2$

Now, the table value of t at 5% level of significance with 9 degrees of freedom, T = 2.262 .

Test statistics:

$$t_L=\frac{\bar{x}_L-\bar{y}_L}{s_L\sqrt{\frac{1}{m}+\frac{1}{n}}}=\frac{-(1.05+0.35\lambda)}{\sqrt{0.0246\lambda^2-0.0469\lambda+0.1644}}$$ and

$$t_U=\frac{\bar{x}_U-\bar{y}_U}{s_U\sqrt{\frac{1}{m}+\frac{1}{n}}}=\frac{0.35\lambda-1.75}{\sqrt{0.0246\lambda^2-0.0516\lambda+0.1690}}.$$

Now, since $|t_L|>T$ and $|t_U|>T$ for all $\lambda, 0\leq\lambda\leq1$, the null hypothesis is rejected. Therefore, the two kinds of tire for automobile on consumption of the petrol are not the same at 5% level of significance .

Remark 1: The result obtained by the proposed test procedure for the Example 2. is same as in Baloui Jamkhaneh and Nadi Ghara [4].

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