Comparative Study of Peak-to-Average Power Ratio Reduction Techniques for OFDM System

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is one well-known scheme, which has recently become a preferred choice for parallel-data-transmission which reduces the influence of multipath fading and avoids the use of complex equalizers. However, an OFDM signal has a large peak-to-average power ratio (PAPR), which can result in significant distortion when transmitted through a nonlinear device, such as a transmitter power amplifier (PA). This paper describes and makes some remark on the selection criteria for selecting the PAPR reduction techniques that are based on signal distortion, probabilistic (scrambling) and coding. Comparison indicates that coding technique is best for PAPR reduction but suffers from bandwidth efficiency as the code rate is reduced. We have also developed Recursive Golay Complementary Codes (RGCC) with low PAPR, high code rate and high information rate.

Keywords **– BER, OFDM, PAPR, PA, RGCC.**

1. INTRODUCTION

The demand for data communication over mobile radio channels has increased steadily over the last few years and will likely continue to grow. To create broadband multimedia mobile communication systems, it is necessary to use high-data-rate transmission of megabits per second. Generally, the mobile radio channel is characterized by multipath reception that causes the inter-symbol interference (ISI) (reflected, delayed waves due to multipath, interfere with the direct wave), which in turn causes significant degradation of network performance [1]. An adaptive equalization technique at the receiver is one method to compensate ISI. There are practical difficulties in operating this equalization at several megabits per second with compact, low-cost hardware. However to avoid the use of such complex equalizers, parallel data transmission scheme is employed known as OFDM. OFDM has gained a lot of interest in digital communication application. This has been due to its properties like high spectral efficiency, robustness to channel fading and easy implementation.

In spite of OFDM is employed in number of applications like [2] Asymmetric digital Subscriber Line (ADSL), Digital Audio Broadcasting (DAB), Digital Video broadcasting (DVB) and Local Area Network standards IEEE802.11a, IEEE802. 16, European HIPERLAN/2 and other high-speed data application for both wireless and wired communications, yet one major difficulty with OFDM is its high peak-to average power ratio (PAPR), that distorts the signal if the transmitter contains nonlinear components such as power amplifiers [3].

When N signals are added with the same phase, they produce a peak power that is N times the average power. A high PAPR brings disadvantages, such as an increased complexity of the A/D and D/A converters, and a reduced efficiency of the RF power amplifier [4]. The objective of the PAPR reduction technique is to reduce the peaks of the OFDM signal to a satisfactory level to ensure better usage of the PA.

There have been several papers summarizing the PAPR reduction schemes [6]–[18]. In these papers, PAPR reduction schemes are compared according to various criteria, which include the PAPR reduction capability, average power increase, BER degradation, data rate loss, computational complexity, and out-of band radiation. Although numerous schemes have been proposed to solve the PAPR problem, no specific PAPR reduction scheme can be considered as the best solution. A new modified Golay technique has been proposed in this paper.

The paper is organized as follows. In Section 2, OFDM system is described. Section 3 discusses the problem of PAPR. Section 4 presents various methods to reduce PAPR. Selection Criteria is explained in section 5. A new technique based on coding is discussed under section 6 Simulation and results for the performance of new technique is given in section 7 and Section 8 concludes the paper.

2. OFDM SYSTEM

OFDM is a multi-carrier modulation system employing FDM of orthogonal sub carriers, each modulating a low bit rate digital stream. The block diagram of OFDM system is shown in Fig. 1.

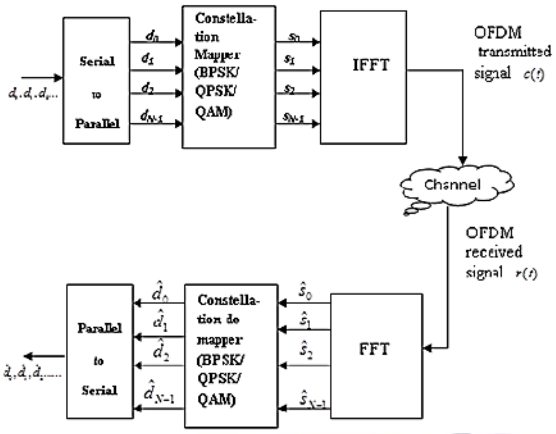

Figure 1. OFDM Block Diagram

Consider we want to transmit data bits using OFDM: $D=\{d_0, d_1, d_2.....\}$. The first thing that should be considered in designing the OFDM transmitter is number of sub-carriers required to send the given data [1]- [15]. Let us assume that there are N sub-carriers. Each sub-carrier is centered at frequencies that are orthogonal to each other (i.e. $f_1, 2f_1, 3f_1......$). The second design parameter could be the modulation format that can be BPSK, QPSK, QAM etc. The data has to be first converted from serial stream into N parallel stream depending on the number of sub-carriers (N). These N parallel streams are individually converted into the required digital modulation format denoted as $s=\{s_0, s_1, s_2.......s_N\}$, using constellation mapper which is essentially a Look up table (LUT). Modulated stream $s=\{s_0, s_1, s_2.......s_N\}$ is superimposed on the required orthogonal sub-carriers for transmission. The transmitted OFDM signal is given by,

$$c(t)=\sum_{n=0}^{N-1} s_n(t)\sin(2\pi nt) \qquad (1)$$

The IFFT ensures orthogonality among sub carriers and is decoded exactly with a FFT on the receiver side.

3. PROBLEM OF PAPR

The transmit signals in an OFDM system can have high peak values in the time domain since many subcarrier components are added via an IFFT operation. A low PAPR allows the transmit power amplifier to operate efficiently, whereas a high PAPR forces the transmit power amplifier to have a large back-off in order to ensure linear amplification of the signal [5]. Additionally, a high PAPR requires high resolution for the receiver A/D converter, since the dynamic range of the signal is much larger for high PAPR signals. High resolution A/D conversion places a complexity and power burden on the receiver front end.

The PAPR of the continuous transmitted signal (1) is measured by,

$$PAPR=\frac{Max\{|c(t)|^2\}}{E\{|c(t)|^2\}} \qquad (2)$$

where $E\{(\bullet)\}$ is expected value.

The discrete time baseband OFDM signals, which constitute the output of the IFFT block, are transformed to continuous time baseband OFDM signals by a low-pass filter called DAC, where the peak power can be increased while maintaining a costant average power. Usually, the PAPR of the continuous time baseband OFDM signals is larger than that of the discrete time baseband OFDM signals by 0.5~1.0 dB [7].

4. PAPR REDUCTION TECHNIQUES

PAPR reduction techniques are discussed in the following section.

4.1 Signal Distortion Techniques

These techniques reduce the amplitude of samples whose power exceeds a certain threshold by non-linearly distorting the OFDM signal at or around the peaks. There are number of ways to implement signal distortion.

- Clipping
- Windowing
- Cancellation

One simplest approach of reducing the PAPR is to *clip* the amplitude of the signal to a fixed level [6].

$$y=\begin{cases} -A & (if\ x<-A) \\ x & (if\ -A\le x\le A) \\ A & (if\ x>A) \end{cases}$$

where *x* denotes the signal before clipping and *y* denotes the signal after clipping. The pseudo-maximum amplitude in this approach is referred to as the clipping level and denoted by *A*. In other words, any signal whose amplitude exceeds clipping level *A* will saturate its amplitude to the clipping level *A*. However it is definitely the simplest solution, but two major problems arises, first is self-interference which degrades the BER and second is level of the out-of-band radiation increases.

In [7] *windowing*, large signal peaks are multiplied with a certain non-rectangular window. A Gaussian shaped window functions (cosine, Kaiser, and hamming windows) is proposed for this. To minimize the out-of-band interference, ideally the window should be as narrowband as possible and the window should not be too long in the time domain because that implies that many signal samples are affected, which increases the BER.

In [9] another approach linear *peak-cancellation*, in which a time-shifted and scaled-reference function is subtracted from the OFDM signal; such that each subtracted reference function reduces the peak power of at least one signal sample. One example of a suitable reference signal is a raised cosine window. A comparison of the three described signal distortion techniques is given in [8].

4.2 Scrambling Techniques

There are two techniques based on scrambling i.e. *Selected Mapping* (SLM) and *Partial Transmit Sequence* (PTS). In SLM method [10], input data block is subjected to scramble or (rotation in phase) by U different phase sequence. The IFFT of resulting data sequence generates a new sequence, among which the one with the lowest PAPR is selected for transmission. In another approach that is PTS [11], data block is partitioned into non-overlapping sub blocks and each sub block is rotated with a statistically independent rotation factor. The rotation factor, which generates the time domain data with the lowest peak amplitude, is also transmitted to the receiver as side information. The difference between the two is that the first applies independent scrambling rotations to all SCs, while the latter only applies scrambling rotations to groups of SCs.

The modernized SLM in which complex baseband signal is separated into real and imaginary part [12], by doing so the computational complexity can be reduced and concluded that MSLM technique reduces PAPR about 0.50db more as compared to traditional SLM technique.

4.3 Coding Techniques

The PAPR value can be reduced by using a code that only produces OFDM symbols for which the PAPR value is below some threshold. As given in the table [13], [14], it can be seen that by avoiding the transmission of codeword having higher PAPR, reduction in PAPR can be achieved. This can be done by block coding the data such that a 3 bit data word is mapped on to a 4 bit code word, the set of permissible words does not contain those that result in excessive PAPR. For example, avoid four words that result in the maximum PAPR of 16.00W and another four words result in PAPR of 9.45W. It is clear that we could reduce the PAPR of this multicarrier signal. However, a look-up table may be used if N is small. On the other hand, Golay complementary sets may be used to generate polyphase sequences with low PAPR. In [3], [15],[16] authors used the Golay complementary sequences to achieve the PAPR reduction, in which more than 3-dB PAPR reduction had been obtained. Codes with error correcting capabilities has been proposed in [17] to achieve more lower PAPR for OFDM signals by determining the relationship of the cosets of Reed-Muller codes to Golay complementary sequences. While these block codes reduce PAPR, they also reduce the transmission rate, significantly for OFDM systems with large number of subcarriers. An adaptive coding technique has been proposed by Zafar et al. [18] for reducing PAPR in COFDM to achieve reduction in PAPR as well as error correction capability. The adaptive approach has been adopted in order to reduce hardware for a slight increase in complexity.

5. CRITERIA FOR SELECTION OF PAPR REDUCTION TECHNIQUE

A number of factors must be considered while selecting a specific technique for PAPR reduction. These factors include PAPR reduction capability, distortion, rate hit, side information, complexity, power increase in the transmit signal, BER increase at the receiver, computation complexity increase and so on. It is shown in the table 1

Table1. Comparison of various PAPR reduction techniques

Method	Clipping	Selected Mapping	Partial Transmit Sequence	Coding
PAPR Reduction capability	High	Medium	Medium	High
Distortion	Yes	No	No	No
Rate hit	Low	Low-High	Low-High	High
Side Information	No	Yes	Yes	No
Complexity	Low	Low-High	Low-High	High
Power Increased	No	No	No	No
BER degradation	Medium	High	High	Low

6. RECURSIVE GOLAY COMPLEMENTARY CODES

V. Tarokh [19] has found a lower bound for achievable coding rate depending on the length of the code, minimum Euclidian distance, and maximum PAPR. The value of lower bound for the PAPR increases by increasing the coding rate. The need for low PAPR and at the same time overcoming the lower bound of coding rate has motivated us to develop recursive Golay complementary codes with low PAPR, high code rate and high information rate.

Definition. Two N-valued complex sequences x and y are called Recursive Golay Complementary Pairs (RGCP) if

- They are Golay complementary pairs.
- If Pav is the average power of the constellation,

$$P_x + P_y \leq 2NP_{av,} \quad (3)$$

where $x \approx y$

Theorem 1. The PAPR achieved by any RGCS is bounded up by 3dB.

Proof. By virtue of Theorem 1 and the fact that the instantaneous power of each code is always non-negative, we have

$$PAPR(x) = \frac{\max_t \{p_x(t)\}}{p_x} \le \frac{p_x(t) + p_y(t)}{NP_{av}} \le 2 = 3dB. \quad (4)$$

With the help of following theorems, we will find a way to develop recursive golay codes.

Theorem 2. The property of being Recursive Golay complementary pairs is invariant under the following transformations:

- Reflection w.r.t the origin
- Reflection w.r.t both axes.
- Multiplication of one or both sequences by a complex number with magnitude 1.
- Reflection w.r.t the bisectors of all regions.
- Rotation.

In all of these cases, the power of the sequences is preserved.

Theorem 3. If $x \approx y$ then,

$$a.\ \hat{x} \approx \hat{y}$$
$$b.\ x \approx \hat{y}^*$$
$$c.\ x \,|\, y \approx x \,|\, {-y}$$
$$d.\ x \downarrow y \approx x \downarrow -y$$

Specifically, if $x \approx y$ each with size N, then the following sequences are Recursive Golay pairs:

1) $\pm [j](x|y) \approx \pm [j](x|-y)$
2) $\pm [j](x \downarrow y) \approx \pm [j](x \downarrow -y)$
3) $\pm [j](x|y) \approx \pm [j](\hat{y}^*|-\hat{x}^*)$
4) $\pm [j](x \downarrow y) \approx \pm [j](\hat{y}^* \downarrow \hat{x}^*)$
5) $\pm [j](x \downarrow -y) \approx \pm [j](\hat{y}^* \downarrow \hat{x}^*)$

7. SIMULATION RESULTS

In this section, we present simulation results for PAPR reduction performance of recursive golay complementary codes. The CCDFs of PAPR (the probabilities that the PAPR of OFDM signal exceeds a given $PAPR_0$) are numerically obtained for the no coding, golay coding and modified golay coding. Comparison of PAPR, Code Rate and Information rate performance of the constructed codes with [17] show that the proposed codes are able to achieve twice the information rate. Fig. 2 shows CCDF plot for simple Golay complementary sequence and the proposed Recursive Golay complementary sequence. From Fig. 2 it has been concluded that the PAPR of recursive Golay code is bounded up to 3.6dB. Also for *N* = 256 the coding rate of 2 * 50/ (4 * 256) = 9.7% has been achieved while the PAPR remains bounded up to 3.6.

8. CONCLUSION

We reviewed the problem in multicarrier signal due to high PAPR. Various PAPR reduction techniques and criteria for the selection of these techniques have also been discussed. In this paper generation of Recursive Golay complementary sequence has been proposed. Results show that the PAPR of the constructed sequences is bounded up to 3.6 dB, and the information rate is twice as the rate for Golay QPSK codes. For *N* = 256 the coding rate of 9.7% has been achieved while the PAPR remains bounded up to 3.6 dB.

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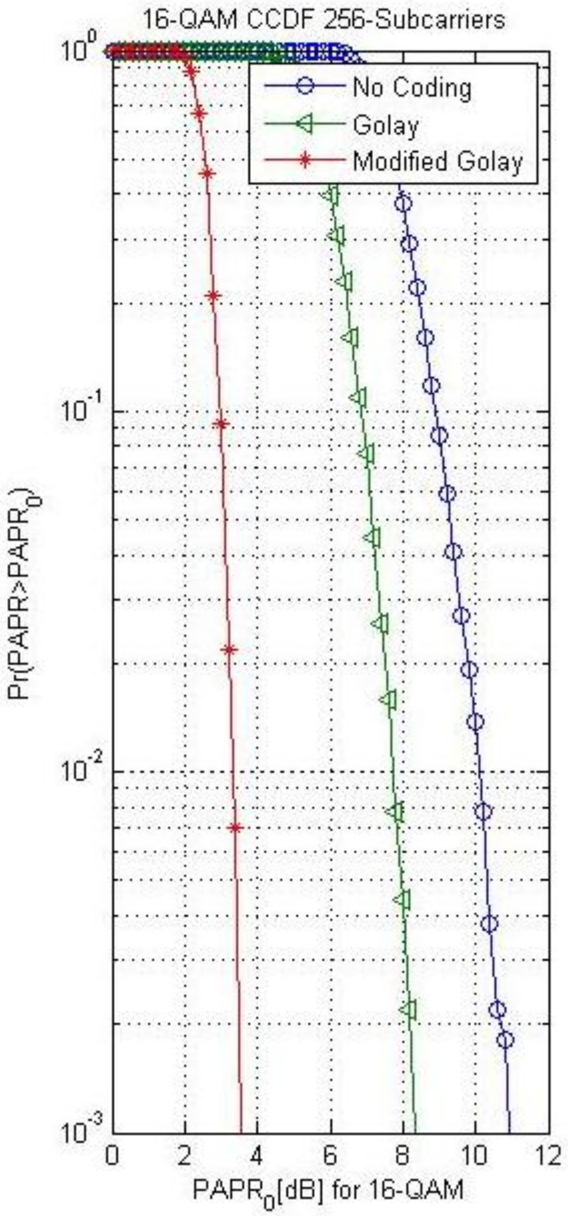

Figure 1. CCDF plot of 16-QAM in OFDM signal with 256 subcarriers