INTEGRAL AND FRACTIONAL OVER SAMPLING TECHNIQUES FOR REDUCTION OF JITTER NOISE POWER IN HIGH SPEED OFDM SYSTEM

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ABSTRACT

The OFDM system has multi sub carriers to send the high speed data. At high data rates there will be chance of timing jitter in OFDM system due to mismatch of the sampling clock at the receiver with the transmission speed. The effects caused by timing jitter is a significant limiting factor in the performance of very high data rate OFDM systems. Oversampling can reduce the noise caused by timing jitter. Both fractional over sampling achieved by leaving some band-edge OFDM sub carriers unused and integral over sampling are considered. Over sampling results in a 3 dB reduction in jitter noise power for every doubling of the sampling rate.

Keywords

Timing jitter, over sampling, Jitter noise power, OFDM

1. INTRODUCTION

Orthogonal frequency division multiplexing(OFDM) is becoming widely applicable in wireless communication systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay. It has been used in digital audio broadcasting (DAB) systems, digital video broadcasting (DVB) systems, digital subscriber line (DSL) standards, and wireless LAN. OFDM is used in many wireless broadband communication systems because it is a simple and scalable solution to inter symbol interference caused by a multi path channel. Data rates in optical fiber systems are typically much higher than in RF wireless systems. At these very high data rates, timing jitter is emerging as an important limitation to the performance of OFDM systems[2][4]. A major source of jitter is the sampling clock in the very high speed analog-to-digital converters (ADCs) which are required in these systems[6]. Timing jitter is also emerging as a problem in high frequency band pass sampling OFDM radios. In OFDM, fractional over sampling can be achieved by leaving some band-edge sub carriers unused. Very high speed ADCs typically use a parallel pipeline architecture not a PLL for this model.

2. OVERVIEW OF OFDM SYSTEM

In OFDM, sub carriers overlap. They are orthogonal because the peak of one sub carrier occurs when other subcarriers are at zero. This is achieved by realizing all the subcarriers together using Inverse Fast Fourier Transform (IFFT). The demodulator at the receiver parallel channels form an FFT block. Each sub carrier can still be modulated independently. An OFDM signal is a superposition of N sinusoidal carriers with frequency separation F_N, each sub-carrier is modulated by complex symbols with period T_N equal to the inverse of the frequency separation, i.e. $T_N=1/F_N$. The modulated carriers overlap spectrally but, since they are orthogonal within a symbol duration (the k^{th} carrier frequency is $f_k = f_o + k F_N$ where f_o is some reference frequency and $0< k < N$),the signal associated with each sinusoid can be recovered as long as the channel does not destroy the orthogonality. In practice, the samples of the OFDM signal are generated by taking the inverse discrete Fourier –transform (IDFT) of a discrete-time input sequence and passing the transform samples through a pulse shaping filter. At the receiver dual transformations are implemented.

Two periodic signals are orthogonal when the integral of their product, over one period, is equal to zero.

Definition of Orthogonality:

$$\sum_{k=0}^{N-1}\cos\left(\frac{2\pi kn}{N}\right)\times\cos\left(\frac{2\pi km}{N}\right)=0(n\neq m) \quad (1)$$

The carriers of an OFDM system are sinusoids that meet this requirement because each one is a multiple of a fundamental frequency.

3.SYSTEM MODEL

Consider the high-speed OFDM system .The OFDM symbol period, not including the cyclic prefix, is T. At the transmitter, in each symbol period, up to N complex values representing the constellation points are used to modulate up to N subcarriers. Timing jitter can be introduced at a number of points in a practical OFDM system but in this paper we consider only jitter introduced at the sampler block of the receiver ADC. Ideally the received OFDM signals sampled at uniform intervals of T/N. The effect of timing jitter is to cause deviation τ_n between the actual sampling times and the uniform sampling intervals. In OFDM systems while timing jitter degrades system performance, a constant time offset from the 'ideal' sampling instants is automatically corrected without penalty by the equalizer in the receiver.

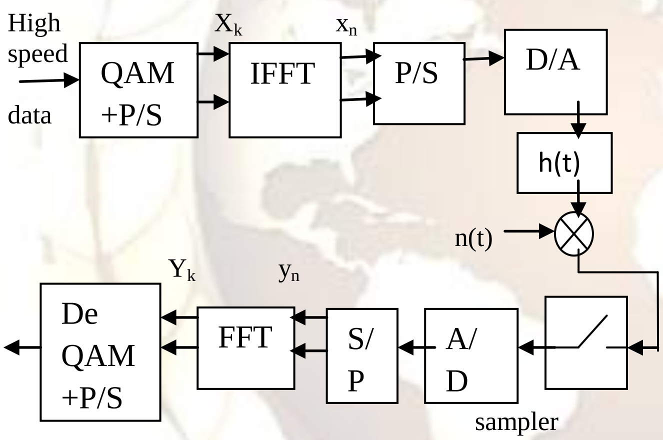

Fig 1: OFDM Transceiver

In OFDM system there is no chance for inter carrier interference(ICI) because the sub carriers are orthogonal to each other where as CDMA and GSM technologies are single carrier systems .At high data rates there will be chance of timing jitter in OFDM system due to mismatch of the sampling clock at the receiver with the transmission speed.

3.1 Quadrature Amplitude Modulation (QAM)

This modulation scheme is also called quadrature carrier multiplexing. This modulation scheme enables DSB-SC modulated signals to occupy the same transmission bandwidth at the receiver output. It is, therefore, known as a bandwidth-conservation scheme**.** The QAM Transmitter consists of two separate balanced modulators, which are supplied, with two carrier waves of the same frequency but differing in phase by 90°. The output of the two balanced modulators are added in the adder and transmitted.

3.2 FFT & IFFT:

An OFDM system treats the source symbols (e.g., the QPSK or QAM symbols that would be present in a single carrier system) at the Transmitter as though they are in the frequency domain. These symbols are used as the inputs to an IFFT block that brings the signal into the time domain. The IFFT takes in N symbols at a time where N is the number of sub carriers in the system. Each of these N input symbols has a symbol period of T seconds. The basis functions for an IFFT are N orthogonal sinusoids. These sinusoids each have a different frequency and the lowest frequency is DC. Each input symbol acts like a complex weight for the corresponding sinusoidal basis function. Since the input symbols are complex, the value of the symbol determines both the amplitude and phase of the sinusoid for that sub carrier.

The IFFT output is the summation of all N sinusoids. Thus, the IFFT block provides a simple way to modulate data onto N orthogonal sub carriers. The block of N output samples from the IFFT make up a single OFDM system. The length of the OFDM symbol is NT where T is the IFFT input symbol period mentioned above.

At the Receiver, an FFT block is used to process the received signal and bring it into the frequency domain. When plotted in the complex plane, the FFT output samples will form a constellation, such as 16-QAM.

4. TIMING JITTER IN OFDM:

Timing jitter is often modeled as a wide sense stationary (WSS) Gaussian process with zero-mean and variance $\sigma_n^{\ 2}$.

The effect of timing jitter can be described by a timing jitter matrix. The compact matrix form for OFDM systems with timing jitter is

$$Y = WHX^T + N \quad (2)$$

where **X**, **Y** and **N** are the transmitted, received and additive white Gaussian noise (AWGN) vectors respectively, **H** is the channel response matrix and **W** is the timing jitter matrix where

$$Y = [Y_{-N/2+}1 \ldots\ldots\ldots Y0 \ldots\ldots Y_{N/2}]^T \quad (3)$$

$$H = \mathrm{diag}(H_{-N/2} \ldots\ldots H_0 \ldots\ldots H_{N/2}) \quad (4)$$

$$X^T = [X_{-N/2+1} \ldots\ldots X_0 \ldots\ldots X_{N/2}] \quad (5)$$

Timing jitter causes an added noise like component in the received signal.

$$Y = HX^T + (W - I)HX^T + N \quad (6)$$

where **I** is the N×N identity matrix. The first term in (6) is the wanted component while the second term gives the jitter noise .

The elements of the timing jitter matrix **W** are given by

$$w_{l,k} = \frac{1}{N} \sum_{n=-N/2+1}^{N/2} e^{j2\pi k \frac{\tau_n}{T}} e^{j\frac{2\pi}{N}(k-l)n} \quad (7)$$

where is 'n' the time index ,'k' is the index of the transmitted subcarrier and 'l' is the index of the received sub carrier.

The timing jitter matrix is given by

$$W = \begin{bmatrix} w_{-\frac{N}{2}+1,-\frac{N}{2}+1} & \cdots & w_{-\frac{N}{2}+1,0} & \cdots & w_{-\frac{N}{2}+1,\frac{N}{2}} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ w_{0,-\frac{N}{2}+1} & \cdots & w_{0,0} & \cdots & w_{0,N/2} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ w_{\frac{N}{2},-\frac{N}{2}+1} & \cdots & w_{\frac{N}{2},0} & \cdots & w_{\frac{N}{2},\frac{N}{2}} \end{bmatrix} \quad (8)$$

5. OVERSAMPLING TYPES:

In signal processing, Oversampling is the process of sampling a signal with a sampling frequency significantly higher than twice the bandwidth or highest frequency of the signal being sampled. Oversampling helps avoid aliasing, improves resolution and reduces noise. An oversampled signal is said to be oversampled by a factor of β, defined as

$$\beta \stackrel{\text{def}}{=} \frac{f_s}{2B} \quad (9)$$

where f_s is the sampling frequency, B is the bandwidth or highest frequency of the signal; the Nyquist rate is 2B.

The effect of both fractional and integral over sampling in OFDM can be used to reduce the degradation caused by timing jitter. To achieve integral over sampling, the received signal is sampled at a rate of MN/T, where M is an integer. For fractional over sampling some band-edge sub carriers are unused in the transmitted signal. When all N sub carriers are modulated, the bandwidth of the baseband OFDM signal is N/2T, so sampling at intervals of T/N is Nyquist rate sampling. If instead, only the sub carriers with indices between $-N_L$ and $+N_U$ are non zero, the bandwidth of the signal is $(N_L+N_U)/2$. In this case sampling at intervals of T/N is above the Nyquist rate. The degree of over sampling is given by $(N_L+N_U)/N$.

6. EFFCT OF OVERSAMPLING ON JITTER NOISE POWER:

In the general case, where both integral and fractional over sampling are applied, the signal samples after the ADC in the receiver are given by

$$y_{n_M} = y\left(\frac{n_M T}{NM}\right) = \frac{1}{\sqrt{N}} \sum_{k=-N_L}^{N_u} H_k X_k e^{\left(\frac{j2\pi k}{T} \times \frac{n_M T}{NM}\right)} + \eta\left(\frac{n_M T}{NM}\right) \quad (10)$$

where η_M is the oversampled discrete time index and η is the AWGN. With integral oversampling, the N point FFT in the receiver is replaced by an 'oversized' NM-point FFT. The output of this FFT is a vector of length NM with elements.

$$Y_{l_M} = \frac{1}{\sqrt{M}} \frac{1}{\sqrt{NM}} \sum_{n_M=-NM/2+1}^{NM/2} y_{n_M} e^{\left(\frac{-j2\pi n_M l_M}{NM}\right)} \quad (11)$$

Where l_M is the index at the output of the NM point FFT. The modified weighting coefficients for the oversampling case,

$$w_{l_M,k} = \frac{1}{NM} \sum_{n_M=-NM/2+1}^{NM/2} e^{j2\pi k \frac{\tau_n M}{T}} e^{j\frac{2\pi}{NM}(k-l_M)n_M} \quad (12)$$

By using the approximation $e^{j\theta} = 1+j\theta$ for small θ

$$w_{l_M,k} = \frac{1}{NM} \sum_{n_M=-NM/2+1}^{NM/2} \left(1 + \frac{j2\pi k \tau_{n_M}}{T}\right) e^{j2\Pi(k-l_M)n_M/NM} \quad (13)$$

So for $k \neq l_M$ the variance of the weighting coefficients is given by

$$E\{|w_{l_M,k}|^2\} \approx \left(\frac{2\pi k}{MNT}\right)^2 E\{\tau_{n_M} \tau_{p_M}\} e^{j\frac{2\pi}{MN}(k-l)d_M} \quad (14)$$

Where $n_M - p_M = d_M$. When the timing jitter is white, then $\{\tau_{n_M} \tau_{n_M} - d_M\}$ =0 for $d_M \neq 0$ so

$$E\{|w_{l_M,k}|^2\} \approx \frac{1}{NM} \left(\frac{2\pi k}{T}\right)^2 E\{\tau^2{}_{nM}\} \qquad k \neq l_M \quad (15)$$

From (15) it can be seen that white timing jitter $E\{|w_{l_M,k}|^2\}$ is inversely proportional to M so increasing the integer over sampling factor reduces the inter carrier interference (ICI) due to timing jitter. Also that $E\{|w_{l_M,k}|^2\}$ depends on k^2 but not on l_M, so higher frequency sub carriers cause more ICI, but the ICI affects all sub carriers equally.

7. AVERAGE JITTER NOISE POWER FOR EACH SUBCARRIER

$$Y_{l_M} = H_{l_M} X_{l_M} + \sum_{k=-N_L}^{N_U} \left(w_{l,k} - I_{l_M,k}\right) H_k X_k + N(l) \qquad (16)$$

where the second term represents the jitter noise. We consider a flat channel with, $H_k = 1$ and assume that the transmitted signal power is distributed equally across the used subcarriers so that for each used subcarrier $E\{X_k^2\} = \sigma_s^2$. Then the average jitter noise power, $P_j(l)$to received signal power of l^{th} subcarrier is given by

$$\frac{Pj(l)}{\sigma_s^2} = \frac{E\{\left|\sum_{k=-N_l}^{N_v}\left(w_{l,k} - I_{l,k}\right)X_k\right|^2\}}{\sigma_s^2} \qquad (17)$$

$$= \sum_{k=-N_L}^{N_u} E\{\left|w_{l,k} - I_{l,k}\right|^2\}$$

Rearranging the terms in (17)equation

$$\frac{Pj(l)}{\sigma_s^2} = \frac{\pi^2}{3M}\left(\frac{N_v N}{T_N^2}\right) E\{\tau_n^2\} \qquad (18)$$

If there is no integral over sampling or fractional over sampling, M=1 and N_V=N,

$$\frac{Pj(l)}{\sigma_s^2} = \frac{\pi^2}{3}\left(\frac{N^2}{T_N^2}\right) E\{\tau_n^2\} \qquad (19)$$

Comparing (18) and (19) it can be seen that the combination of integral oversampling and fractional over sampling reduces the jitter noise power by a factor of Nv/NM.

8.SIMULATION PARAMETERS

The different Simulation Parameters are Oversampling factor, Subcarrier index and Average Jitter noise power in dB. Different subcarrier index values are considered for analyzing the Variation of power.

9. RESULTS & DISCUSSION

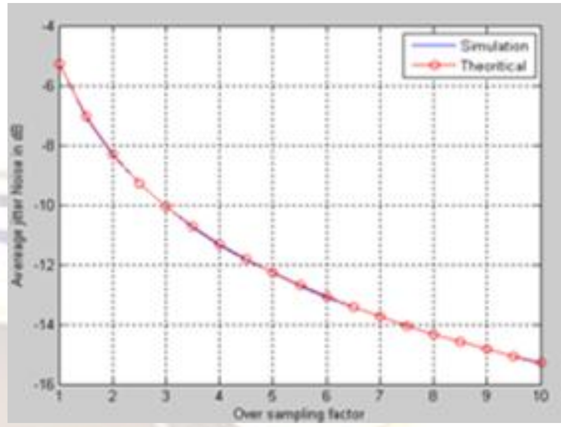

Fig 2: Reduction of Jitter Noise power

Fig (2) shows the graph of over sampling factor versus Average Jitter noise power in dB. It shows that due to over sampling whenever the sampling factor increases by two ,there was the decrease in jitter noise power by 3db .

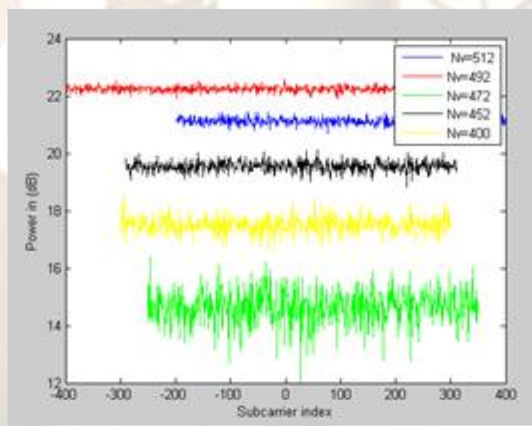

Fig 3 : Variation of power for different sub carrier indices

Fig (3) shows that the variance of the noise due to jitter as a function of received sub carrier index when band-edge sub carriers are unused. It shows that the power of the jitter noise is not a function of sub carrier index and that removing the band-edge sub carriers reduces the noise equally across all sub carriers. Average jitter noise power is a function of the over sampling factor. There is close agreement between theory and simulation.

10. CONCLUSION

It has been shown both theoretically and by simulation that oversampling can reduce the degradation caused by timing jitter in OFDM systems. Two methods of oversampling were used : fractional oversampling achieved by leaving some of the band-edge subcarriers unused, and integral oversampling implemented by increasing the sampling rate at the receiver.

The jitter variance is not changed when oversampling is applied, so the jitter represents a larger fraction of the sampling period for the oversampled systems For the case of white timing jitter both techniques result in a linear reduction in jitter noise power as a function of oversampling rate. Thus oversampling gives a 3 dB reduction in jitter noise power for every doubling of sampling rate.

11. REFERENCES

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