R. Sampath Kumar, R. Vijaya Kumar, R. Radhakrishnan / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue 3, May-Jun 2012, pp.2220-2226

SELECTION OF MIXED SAMPLING PLAN THROUGH MAPD AND LQL USING INTERVENED RANDOM EFFECT POISSON DISTRIBUTION

R. Sampath Kumar[1], R. Vijaya Kumar[2] and R. Radhakrishnan[3]

1. Assistant Professor, Department of Statistics, Government Arts College, Coimbatore-641 018.
2. Assistant Professor, Department of Statistics, SSM College of Arts & Science, Komarapalayam - 638 143.
3. Associate Professor, Department of Statistics, PSG College of Arts & Science, Coimbatore – 641 014.

ABSTRACT

This paper presents the procedure for the construction and selection of mixed sampling plan (MSP) using Intervened Random effect Poisson Distribution (IRPD) as a baseline distribution. The plans are constructed through limiting quality level (LQL) and maximum allowable percent defective (MAPD) having the single sampling plan as attribute plan,. Tables are constructed for easy selection of the plan.
Key words and phrases*: limiting quality level, intervention, mixed sampling plan, maximum allowable percent defective, operating characteristic, poisson, intervened random effect poisson distribution*.
AMS (2000) Subject Classification Number: Primary: 62P30 Secondary: 62D05

1. INTRODUCTION

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of mixed sampling plan over pure attribute sampling plan is a reduction in sample size for a similar amount of protection.

Schiling (1967) proposed a method for determining the operating characteristics of mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation. The mixed sampling plans have been designed under two cases of significant interest. In the first case, the sample size n_1 is fixed and a point on the OC curve is given. In the second case, plans are designed when two points on the OC curve are given. Devaarul (2003) has studied the mixed sampling plans and reliability based sampling plans.

Radhakrishnan and Sampath Kumar (2006, 2007a, b, c, and 2009) have constructed the mixed sampling plans using Poisson distribution as a baseline distribution. Sampath Kumar (2007) has constructed mixed variables – attributes sampling plans indexed through various parameters. Radhakrishnan et.al (2010) also made contributions to mixed sampling plans.

In the product control, the defective units are either rebuilt or replaced by new units during the sampling period. Quality engineers are always interested in improving the quality level of product to enhance the satisfaction of the customers and hence, they keep making changes in the production process. These actions trigger a change in the expected incidence of defective items in the remaining observational period. Any action for reducing the number of defectives during the sampling period is called an intervention and such intervention parameter ranges from 0 to 1.

In Intervened Random effect Poisson Distribution (IRPD), Poisson parameter is modified in two ways: one method is multiplying an intervention parameter ρ (a constant) and secondly, multiplying an unobserved random effect which follows Gamma probability distribution. The IRPD can be very useful to the quality and reliability engineers, who always make changes in the production system in the observational period of quality checking to ensure reliability of the system, because, the failure rate of the components may vary in different time intervals. The other areas of application of IRPD are queuing, demographic studies, and process control and so on.

Shanmugam (1985) has used Intervened Poisson Distribution (IPD) in the place of Zero Truncated Poisson Distribution (ZTPD) for the study on cholera cases. Radhakrishnan and Sekkizhar (2007a, b, and c) introduced intervened random effect Poisson distribution in the place of Poisson distribution for the construction of attribute sampling plans.

In this paper, using the operating procedure of mixed sampling plan (independent case) with single sampling plan as attribute plan, tables are constructed using IRPD as a baseline distribution. The tables are constructed for mixed sampling plan (MSP) indexed through i) LQL ii) MAPD. The plan indexed through MAPD is compared with the plan indexed through LQL.

2. CONDITIONS FOR APPLICATIONS OF IRPD - MIXED SAMPLING PLAN

- Production process is modified during the sampling inspection by an intervention
- Lots are submitted substantially in the order of their production
- Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective
- Lot quality variation exists

3. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

p : submitted quality of lot or process.

$P_a(p)$: probability of acceptance for given quality ' p '

p_2 : submitted quality such that $P_a(p_2) = 0.10$ (also called LQL)

p_* : maximum allowable percent defective (MAPD)

n : sample size for each lot.

c : sample acceptance number.

d : number of defectives in the sample.

n_1 : sample size for variable sampling plan.

n_2 : sample size for attribute sampling plan.

β_j : probability of acceptance for the lot quality ' p_j '

β_j' : probability of acceptance assigned to first stage for percent defective ' p_j '

β_j'' : probability of acceptance assigned to second stage for percent defective ' p_j '

z (j) : 'z' value for the j^{th} ordered observation.

k : variable factor such that a lot is accepted if $\bar{X} \leq A = U - k\sigma$

4. OPERATING PROCEDURE OF MIXED SAMPLING PLAN HAVING SINGLE SAMPLING PLAN AS ATTRIBUTE PLAN.

The general procedure given by Schilling (1967) for the independent mixed sampling plan with upper specification limit (U) and standard deviation (σ).

Select a random sample of size n_1 from the lot assumed to be large.

- If a sample average $\bar{X} \leq A$, accept the lot
- If a sample average $\bar{X} > A$, take a second sample of size n_2.
- Inspect and find the number of defectives in the second sample.
  (i) If the number of defectives $d \leq c$, accept the lot.
  (ii) If the number of defectives d>c, reject the lot.

5. CONSTRUCTION OF MIXED SAMPLING PLANS HAVING SINGLE SAMPLING PLAN AS ATTRIBUTE PLAN USING IRPD.

The operation of Mixed Sampling plans can be properly assessed by the OC curve for a given value of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit 'U'. By symmetry, a parallel discussion can be made for lower specification limits.

The procedure for the construction of mixed variables – attributes sampling plans is provided by Schilling (1967) for a given 'n_1' and a point ' p_j ' on the OC curve is given below.

- Assume that the mixed sampling plans are independent

- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as $A = U - k\sigma = U - [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma$, where U is the upper specification limit and z(t) is the standard normal variate corresponding to 't' such that $t = \int_{z(t)}^{\infty} \left(\frac{1}{\sqrt{2\pi}}\right) e^{-u^2/2}\, du$
- Determine the sample average $\bar{X}$. If a sample average $\bar{X} > A = U - k\sigma$, take a second stage sample size 'n_2' using attribute sampling plan.
- Now determine β_j'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_j'' = (\beta_j - \beta_j')/(1-\beta_j')$
- Determine the appropriate second stage sample size 'n_2' from $P_a(p) = \beta_j''$ for $p = p_j$

Using the above procedure, tables can be constructed to facilitate easy selection of mixed sampling plan with single sampling plan as attribute plans using IRPD as a baseline distribution indexed through LQL and MAPD.

Radhakrishnan and Sekkizhar (2007a, b, and c) suggested the probability mass function of the IRPD as

$$P_a(p) = \sum_{x=0}^{c}\left[\frac{e^{-\theta}\theta^x}{(1+\rho\theta)^{\alpha}}\sum_{l=0}^{x}\left(\frac{\rho}{1+\rho\theta}\right)^{l}\frac{(\alpha+l-1)!}{l!(x-l)!(\alpha-1)!}\right], \text{ where } \theta = \left(\frac{np}{1+\rho}\right)$$

when α=1, $$P_a(p) = \sum_{x=0}^{c}\left[\frac{e^{-\theta}\theta^x}{(1+\rho\theta)}\sum_{l=0}^{x}\left(\frac{\rho}{1+\rho\theta}\right)^{l}\frac{1}{(x-l)!}\right], \text{ where } \theta = \left(\frac{np}{1+\rho}\right) \quad (1)$$

Using the above procedure, tables can be constructed to facilitate easy selection of MSP using IRPD as a baseline distribution. The tables furnished in this paper are for the case when α=1.

6. CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH MAPD

MAPD, introduced by Mayer (1967) and studied by Soundararajan (1975) is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defectives. For designing, Soundararajan (1975) proposed a selection procedure for SSP indexed with MAPD and $R = \frac{p_t}{p_*}$

Using the probability mass function of the IRPD, given in expression (1), the inflection point (p_*) is obtained by using $\frac{d^2P_a(p)}{dp^2} = 0$ and $\frac{d^3P_a(p)}{dp^3} \neq 0$. The n_2MAPD values are calculated for different values of c and ρ for $\beta_*' = 0.04$, using c++ program and presented in Table 1.

Table 1: n_2MAPD values for different values of ρ and c when $\beta_*' = 0.04$

c	ρ					
	0.9	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**
1	0.9170	0.9877	1.0679	1.1609	1.2667	1.3872
2	1.8756	2.0186	2.1796	2.3608	2.5671	2.8033
3	2.8711	3.0871	3.3275	3.5977	3.9010	4.2483
4	3.8952	4.1863	4.5079	4.8644	5.2624	5.7164
5	4.9436	5.3116	5.7148	6.1577	6.6501	7.2855
6	6.0102	6.4603	6.9440	7.4740	8.0595	8.7192
7	7.0937	7.6235	7.1803	8.8091	9.4871	10.2475
8	8.1874	8.8056	9.4613	10.1633	10.9314	11.7939
9	9.2907	9.9969	10.7423	11.5349	12.3911	13.3528
10	10.4029	11.2021	12.0364	12.9173	13.8627	14.9302

The MAAOQ (Maximum Average Outgoing Quality) of a Sampling Plan is designed as the Average Outgoing Quality (AOQ) at the MAPD.

By definition AOQ = $p\ P_a(p)$ and MAAOQ = $p_*\ P_a(p_*)$

The values of MAPD and MAAOQ are calculated for different values of c and ρ for $\beta_*' = 0.04$ and the ratio $R = \frac{MAAOQ}{MAPD}$ is presented in Table 2.

Table 2: R values for different values of ρ and c when $\beta_*' = 0.04$

c	ρ					
	0.9	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**
1	0.7682	0.7442	0.7166	0.6842	0.6471	0.6408
2	0.7164	0.6830	0.6444	0.6000	0.5488	0.4903
3	0.6880	0.6486	0.6031	0.5504	0.4902	0.4217
4	0.6683	0.6243	0.5734	0.5148	0.4480	0.3727
5	0.6529	0.6052	0.5500	0.4866	0.4145	0.3343
6	0.6404	0.5892	0.5306	0.4631	0.3867	0.3027
7	0.6297	0.5759	0.5138	0.4430	0.3630	0.2761
8	0.6207	0.5640	0.4991	0.4252	0.3423	0.2530
9	0.6129	0.5538	0.4861	0.4092	0.3239	0.2329
10	0.6060	0.5445	0.4744	0.3950	0.3075	0.2148

6.1 Selection of the plan

Tables 1 and 2 are used to construct the plan when ρ, MAPD and MAAOQ are given. For any given values of ρ, MAPD and MAAOQ one can find the ratio R. From Table 2, for a given value of ρ the nearest value of 'R' is found out and c value is noted. Using the values of 'c' and ρ, one can find the value of 'n_2' from Table 1 as $n_2 = \frac{n_2 MAPD}{MAPD}$.

6.2 Example 1: Given ρ=0.5, MAPD=0.032 and MAAOQ=0.014. Find the ratio $R = \frac{MAAOQ}{MAPD}$ =0.4375. Select the nearest value of R from Table 2 as 0.4480 which is associated with c=4. For the values of c=4, ρ=0.5 and MAPD=0.032, from Table1, the second stage sample size $n_2 = \frac{n_2 MAPD}{MAPD} = \frac{5.2624}{0.032} = 164$. Thus n_2=164, c=4 and ρ=0.5 are the parameters selected for the mixed sampling plan having SSP as attribute plan for a specified ρ=0.5, MAPD=0.032 and MAAOQ =0.014 by taking IRPD as a baseline distribution.

Practical problem:

Suppose the plan n_1=17, k=1.0 is to the lot by lot acceptance inspection of a car engine, the characteristic to be inspected is the "operating temperature and ambient temperature" of the battery for which there is a specified upper limit of 230^0F with known S.D (σ) =2.0^0F. In this example, U=230^0F, σ =2.0^0F and k=1.0. $A = U - k\sigma$ = 230 – (1.0) (2.0) = 230 – 2.0 = 228.0^0F

Now, by applying the variable inspection first, take random sample of size n_1=17 from the lot. Record the sample results and find $\bar{X}$. If $\bar{X} \le A = U - k\sigma$ = 228.0^0F, then accept the lot. If $\bar{X} > A$, take a random sample size n_2 and apply the attribute inspection.

Under attributes inspection, by using single sampling plan as attribute plan using Intervened Random effect Poisson Distribution (IRPD) as a baseline distribution, if the manufacturer fixes the values MAPD=0.032(32 non conformities out of 1000), MAAOQ=0.014(14 non conformities out of 1000) and $\beta_*' = 0.04$, take a sample of size n_2=164 and observe the number of defectives $d \le 4$, accept the lot and if $d > 4$ reject the lot and inform the management for further action.

7. CONSTRUCTION OF MIXED SAMPLING PLANS INDEXED THROUGH LQL

The procedure given in section 5 is used for constructing the mixed sampling plan indexed through LQL (p_2). By assuming the probability of acceptance of the lot be β_2 =0.10 and β_2'=0.04, the $n_2 p_2$ values are calculated for different values of 'c' and 'ρ' using c++ program and is presented in Table 3.

Table 3: n_2 LQL values for different values of ρ and c when β_2=0.10 and β_2'=0.04

c	ρ								
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0	3.4331	3.3493	3.2648	3.1799	3.0953	3.0121	2.9323	2.8593	2.8002
1	5.7335	5.5740	5.4133	5.2521	5.0916	4.9336	4.7814	4.6415	4.5270
2	7.8456	7.6115	7.3758	7.1395	6.9041	6.6722	6.4484	6.2416	6.0709
3	9.8725	9.5640	9.2535	8.9421	8.6319	8.3260	8.0303	7.7559	7.5279
4	11.8495	11.4664	11.0809	10.6944	10.3091	9.9289	9.5607	9.2181	8.9316
5	13.7934	13.3355	12.8748	12.4127	11.9521	11.4972	11.056	10.6445	10.2986
6	15.7136	15.1806	14.6443	14.1065	13.5702	13.0402	12.5256	12.0444	11.6382
7	17.6158	17.0074	16.3953	15.7815	15.1692	14.5638	13.9754	13.4240	12.9564
8	19.5039	18.8199	18.1318	17.4417	16.7531	16.0719	15.4092	14.7870	14.2575
9	21.3807	20.6209	19.8565	19.0898	18.3246	17.5674	16.8300	16.1366	15.5444
10	23.2483	22.4123	21.5714	20.7279	19.8859	19.0523	18.2399	17.4748	16.8192

7.1 Selection of the plan for a given LQL, c and ρ

Table 3 is used to construct the plans when LQL (p_2) and c are given. For any given values of p_2, c and ρ one can determine n_2 value using $n_2 = \frac{n_2 p_2}{p_2}$.

7.2 **Example 2:** Let the probability of acceptance of the lot be β_2 =0.10 and β_2' =0.04. For the given values of p_2 = 0.0562 ρ=0.5 and c=4 from Table 3, the second stage sample size $n_2 = \frac{n_2 p_2}{p_2} = \frac{10.3091}{0.0562}$ = 183. Thus n_2 = 183, ρ=0.5 and c =4 are the parameters selected for the mixed sampling plan for a specified p_2 = 0.0562, ρ=0.5 and c =4.

8. COMPARISON OF MIXED SAMPLING PLAN INDEXED THROUGH MAPD AND LQL

In this section MSP indexed through MAPD is compared with MSP indexed through LQL by fixing the parameters c and β_j'.

For the specified values of ρ, MAPD and MAAOQ with the assumption $\beta_*' = 0.04$ one can find the values of c and n_2 indexed through MAPD. By fixing the values of c and n_2, find the value of p_2 by equating $P_a(p) = \beta_2$ =0.10. Using β_2' =0.04, c and n_2 one can find the values of n_2 using $n_2 = \frac{n_2 p_2}{p_2}$ from Table 3. For different combinations of ρ, MAPD and MAAOQ the values of c, n_2 (indexed through MAPD) and c, n_2 (indexed through LQL) are calculated and presented in Table 4.

Table 4: Comparison of plans

Given values			INDEXED THROUGH MAPD		INDEXED THROUGH LQL	
MAPD	**MAAOQ**	**ρ**	**n_2**	**c**	**n_2**	**c**
0.061	**0.032**	**0.7**	**114**	**6**	**126**	**6**
0.032*	**0.014**	**0.5**	**164**	**4**	**183**	**4**
0.088	**0.042**	**0.4**	**32**	**2**	**36**	**2**

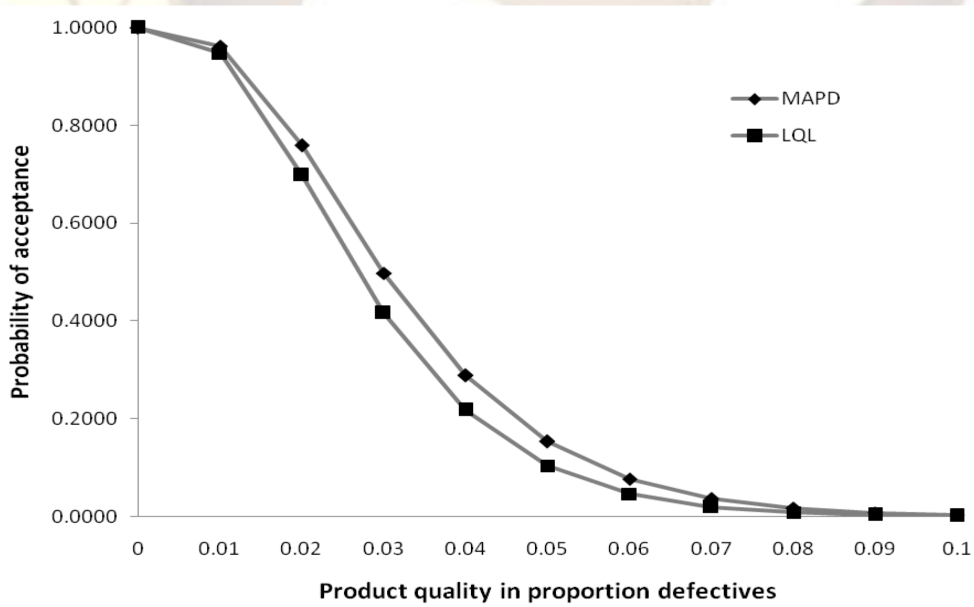

Figure 1: OC Curves for the plans (164, 4) and (183, 4)

9. CONCLUSION

In this paper the construction of mixed sampling plan with single sampling plan as attribute plan indexed through the parameters MAPD and LQL are presented by taking IRPD as a baseline distribution. Further the plan indexed through MAPD is compared with the plan indexed through LQL. It is concluded from the study that the second stage sample size required for single sampling plan indexed through MAPD is less than that of second stage sample size of the single sampling plan indexed through LQL. If the floor engineers know the levels of MAPD or LQL, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*' , β_2') and can also be compared for their efficiency.

REFERENCES

[1] Devaarul S (2003) Certain Studies Relating to Mixed Sampling Plans and Reliability Based Sampling Plans, Ph.D., Dissertation, Bharathiar University, Coimbatore, Tamil Nadu, India.
Mayer P.L (1967) A note on sum of Poisson probabilities and an application, *Annals of Institute of Statistical Mathematics*, Vol.19, pp.537-542.
[2] Radhakrishnan R and Sampath Kumar R (2006) Construction of mixed sampling plan Indexed through MAPD and AQL with chain sampling plan as attribute plan, STARS, Vol.7, No.1, June 2006, pp.14-22. .
[3] Radhakrishnan R and Sampath Kumar R (2007a) Construction and comparison of mixed Sampling plans having repetitive group sampling plan as attribute plan, *National Journal of Technology*, Vol. 4, No.3. pp. 1-6.
[4] Radhakrishnan R and Sampath Kumar R (2007b) Construction of mixed sampling plans Indexed through MAPD and IQL with double sampling plan as attribute plan, *The Journal of the Kerala Statistical Association*, Vol.18, pp. 13-22.
[5] Radhakrishnan R and Sampath Kumar R (2007c) Construction of mixed sampling plans Indexed through MAPD and AQL with double sampling plan as attribute plan, *The International Journal of Statistics and System*, Vol.2, No.2, pp. 33-39.
[6] Radhakrishnan R and Sampath Kumar R (2009) Construction and comparison of mixed Sampling plans having ChSP-(0, 1) plan as attribute plan, *The International Journal of Statistics And Management System*, Vol.4, No.1-2, pp. 134-149.
[7] Radhakrishnan R, Sampath Kumar R and Malathi M (2010) Selection of mixed sampling Plan with TNT-(n_1, n_2; 0) plan as attribute plan indexed through MAPD and MAAOQ, *International Journal of Statistics and System*, Vol.5, No.4, pp. 477-484.
[8] Radhakrishnan R and Sekkizhar J (2007a) Construction of sampling plans using Intervened random effect Poisson distribution, The *International Journal of Statistics and Management Systems*, Vol.2, 1-2, pp 88-97.
[9] Radhakrishnan R and Sekkizhar J (2007b) Construction of conditional double sampling Plans using intervened random effect Poisson distribution, Proceedings volume of *SJYSDNS-2005, Acharya Nagarjuna University, Guntur.* Pp.57-61.
[10] Radhakrishnan R and Sekkizhar J (2007c) Application of intervened random effect Poisson distribution in process control plans, *International Journal of Statistics and Systems.* Vol. 2. No.1. pp.29-39.
[11] Sampath Kumar R (2007) Construction and Selection of Mixed Variables – Attributes Sampling Plans, Ph.D., Dissertation, Bharathiar University, Coimbatore, Tamil Nadu, India.
[12] Schilling EG (1967) A General Method for Determining the Operating Charateristics of Mixed Variables – Attribute Sampling Plans Single Side Specifications, S.D. known, Ph.D Dissertation – Rutgers – The State University, New Brunswick, New Jersy.
[13] Shanmugam R (1985) An intervened Poisson distribution and its medical applications, *Biometrics*, 41, 1025-1029.
[14] Soundararajan V (1975) Maximum allowable percent defective (MAPD) single sampling Inspection by attributes plan, *Journal of Quality Technology*, Vol.7, No.4, pp.173-182.