Transmission loss allocation in a multiple transaction framework

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Abstract--This paper proposes fair schemes for the transmission loss allocation under a pool-based electricity market. The power generations or loads associated with the market are modeled as individual current injections based on a real-time solved AC power flow solution. Each load can be modeled as a current injection or equivalent constant impedance depending on whether it is required to be responsible for the system loss. Each current injection is then treated as an individual player of the transmission loss allocation game. The concept of Shapley value adopted from cooperative game theory is utilized to deal with the fairness of loss allocation. One alternative approach with a normalization procedure is presented to speedup the computation. Numerical results are presented and discussed to demonstrate the applicability of the proposed approaches to a pool-based market.

.I. INTRODUCTION

THE electric power industry is undergoing a series of challenging changes due to deregulation and competition. One of the most important issues is the allocation of transmission losses among market participants since system losses can typically represent from five to ten percents of the total generation and cost millions of dollars per year. However, it is not a trivial task to "fairly" allocate a component of system losses to an individual participant of the market. The main difficulty of loss allocation is caused by the highly nonlinear and non-separable properties of the loss function. To deal with the loss allocation problem, a number of allocation schemes have been proposed in the literature. These schemes fall into the following categories: *Prorata, proportional* haring, incremental transmission loss, loss formula, and circuit theory. Some approaches are based on DC power flow, while some use AC load flow for matching the calculation results and actual power flows. Some schemes are branch-power-flow based, while some focus on the branch-current based allocation techniques. For more detailed descriptions and discussions of their main features, please refer to some good related surveys in [1]-[6]. Different viewpoints and approaches may end up with different results and most of the existing allocation schemes face the problem due to a lack of economic foundations [7]. The motivation of this paper is then to offer an alternative scheme with economic features to handle the fairness issue.

Game theory provides well-behaved solution mechanisms with economic features for assessing the interaction of different participants in competitive markets and resolving the conflicts among players [8]. In particular, cooperative game theory is a most convenient tool to solve cost allocation problem [9]. Some game theory based solutions have been proposed for power engineering problems, such as transmission cost allocation [7] and wheeling transactions [10].

The application of Shapley value concept arisen from theco-operative game theory was investigated to allocate losses and the work is extended in this paper. Two basic models will be proposed in the paper: one basic model allocates losses to the power supply side (each generation) only, and the other attributes losses to both supply and demand sides (each generation and each load). The main difference is that the former treats the load demands as equivalent constant impedances based on a real-time solved AC power flow solution and accordingly the bus impedance matrix (*Z*bus) is then modified, while the later formulates the load demands as equivalent current injections.

In the proposed approaches, the power generations and/or loads associated the market transactions are modelled as individual current injections. Each current injection is then treated as an individual *player* of the *transmission loss allocation game*. The approaches are branch-current based, not branch-power-flow based. Without any approximations or assumptions like those made for a DC power flow or proportional sharing, the proposed approaches utilize the method of Shapley value [8] adopted from cooperative game theory to deal with the fairness issue of loss allocation. Some modified or alternative allocation approaches with or without a *normalization procedure* are also proposed to deal with the *aggregated player* of ancillary services and to speed up the computation when the number of players is large. The proposed approaches are consistent with the real-time AC power flow solution and recover the total system loss. The *Kirchhoff's laws* and *superposition principle* are satisfied and both the network configuration and the voltage-current relationships are reflected. The interactions of players are naturally and fully considered. Moreover, the effect of reducing transmission loss can be identified from the *negative* loss allocation and the *negative* allocation can provide economic signals for the players. The remainder of the paper is organized as follows: Section II introduces generation, load, and branch loss models. In addition, the *transmission loss* *allocation game* is established, and the proposed approaches are presented. Section IIIdemonstrates the application of the proposed schemes via several numerical tests. Discussions and one alternative version are also included. Conclusion in section IV will end up this paper.

II. METHODOLOGY

A. Generation and Load Models

Based on a solved AC power flow solution for a pool based electric power market, let the complex power injection in to a generator bus i be $S_i^G = P_i^G + jQ_i^G$ then the generation current injection is written as

$$I_i^G = [\frac{S_i^G}{V_i}]^* = [\frac{P_i^G + jQ_i^G}{V_i}]^* \quad (1)$$

Where Vi is its bus voltage.similarly, let the complex power injection in to a load bus j be $S_j^D = -(P_j^D + jQ_j^D)$, we can then have load current injection

$$I_j^D = [\frac{S_j^D}{V_j}]^* = -[\frac{P_j^D + jQ_j^D}{V_j}]^* \quad (2)$$

Or the equivalent load impedance

$$Z_j^D = [\frac{V_j}{-I_j^D}] = [\frac{|V_j|^2}{P_j^D - jQ_j^D}] \quad (3)$$

Accordingly two basic models are proposed:

1)basic model A (BMA)

BMB attributes losses to each generator and each load using the generation and load current injection models calculated by the (1)and (2), respectively.

2)basic model B (BMB)

BMB allocates losses only to the power supply side under each generation and load impedance models calculated by the (1)and (3)respectively.The bus impedance matrix is modified by including the equivalent load impedance and then denoted as Z'_{bus} .

B.individual voltage contribution

The voltage contribution to bus i by current injection Can be easily computed by $v_i^k = z_{ik} \cdot I_k$ for BMA or $v_i^k = z'_{ik} \cdot I_k$ (4)for BMB where z_{ik} (z'_{ik}) is the i-k element of Z_{bus} (Z'_{bus}) .by the super position principle ,the actual voltage contributions to that of bus by all current injections. Note that Kirchhoff's laws remain satisfied,

C.Transmission branch loss model

Consider a transmission line π-model between buses m and n as shown in fig, 1, where $z_{mn} = r_{mn} + jx_{mn}$ the serial impedance and jbc is is the shunt susceptance.After calculating the individual voltage contribution to each bus from every current injection, we can then calculate the individual current contribution to each line from every current injection.

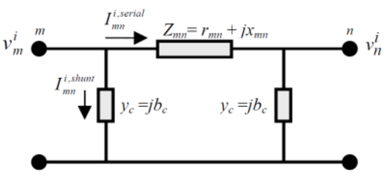


Fig.1.transmission line π-model between buses m and n

The current contribution to the transmission line m-n, measured at bus m,by current injection Ii can then be expressed as

$$I_{mn}^i = I_{mn}^{i,serial} + I_{mn}^{i,shunt} = \frac{v_m^i - v_n^i}{r_{mn} + jx_{mn}} + v_m^i \cdot jb_c \quad (5)$$

And by the superposition principle, the line current equals the sum of the individual current contributions to that line by all current injections i.e,

$$I_{mn} = \sum_{\forall i} I_{mn}^i = I_{mn}^{serial} + I_{mn}^{shunt} = \sum_{\forall i} I_{mn}^{i,serial} + \sum_{\forall i} I_{mn}^{i,shunt} \quad (6)$$

Consequently, the active power loss of line m-n can be calculated by the $|I_{mn}^{serial}|^2 r_{mn}$,and the individual transmission loss contribution by a current injection Ii,while all of the other current injections are represented by the open circuits, can be calculated by

$$|I_{mn}^{i,serial}|^2 r_{mn} = \left|\frac{v_m^i - v_n^i}{r_{mn} + jx_{mn}}\right|^2 r_{mn} \quad (7)$$

Inaddition,the reactive power loss of the line can also be calculated by the $|I_{mn}^{serial}|^2 x_{mn} - \left(|V_n|^2 + |V_m|^2\right) b_c$,if needed and the individual reactive loss contribution by a current injection Ii is equal to

$$|I_{mn}^{i,serial}|^2 x_{mn} - \left(|v_n^i|^2 + |v_m^i|^2\right) b_c \quad (8)$$

D.Transmission loss allocation game

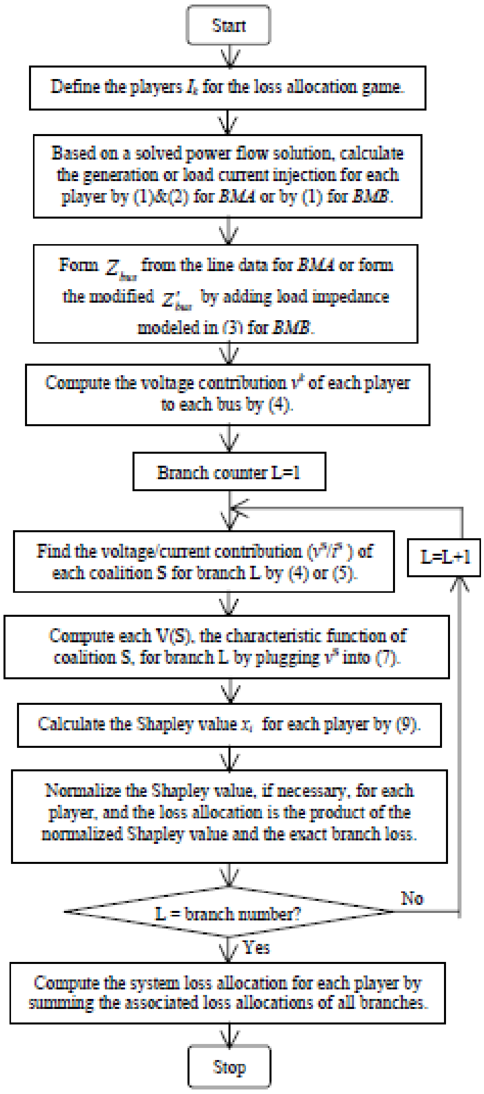
For an *n*-participant cooperative *transmission loss allocation game*, let **N** = {*I1*, *I2*, …, *In*} be the set of all *players*(current injections), and any nonempty subset **S** of **N** is called a *coalition*. The real-valued *characteristic function* of each possible coalition **S** for one transmission element is defined as:V(**S**)= the transmission loss contribution of the coalition current injection $\sum_{\forall I_i \in S} I_i$ to that transmission branch element while all the rest current injections are open-circuited. Note that by the superposition principle, the voltage (or current) contribution *v***S** (or *i***S**) of $\sum_{\forall I_i \in S} I_i$ to a bus (or branch) equals the sum of individual voltage (or current) contributions of all *Ii* in **S**. V(**S**) is obtained by plugging the associated *v***S** or *i***S** into (7). Under such a game setting, when the characteristic functions of all coalitions are computed, we can set up one fair and reasonable allocating mechanism for each *player*.

E. Proposed Loss Allocation Schemes

Let a fair and reasonable loss allocation for each *player* on the transmission element be denoted as ***X*** □ □ *x*(*1*, *x2*, …, *xn*), i.e. the loss allocation for player *Ii* is *xi*, then *xi* can be calculated by the Shapley value [8] as follows

$$x_i = \sum_{\forall S, I_i \notin S} \text{Prob}_n(S)[V(S \cup \{I_i\}) - V(S)] \quad (9)$$

where $[V(S \cup \{I_i\}) - V(S)]$ representsthe loss increment or decrement due to the player Ii joining the coalitionS;and $\text{Prob}_n(S) = \frac{s!(n-s-1)!}{n!}$ is the is the probability that player *Ii* is the (*s*+1)th participant joining the game, given that there have been *s* players in coalition **S** already, which can be seen as the sharing factor of the loss impact for one player joining the other coalition. Note that the self-induced loss of player *Ii* is V(*Ii*) and is taken into account in the term $[\mathrm{V}(\mathrm{S}\cup\{I_i\})-\mathrm{V}(\mathrm{S})]$ when S is empty. The loss allocation process according to (9) is then repeated for every transmission branch. Consequently, the system loss allocation for one player is the sum of the associated loss allocations of all transmission branches. The flow chart of the proposed loss allocation schemes is shown in Fig. 2.Note that for a pool-based market, the bid result of the generation dispatch and market clearing price may be determined initially through a merit-order approach that neglects network constraints, transmission losses, and reactive powers. In the beginning of the proposed allocation schemes, the players of the loss allocation game, i.e. who are going to pay for the exact transmission losses, are defined first according to the market rules. In (1), the generation current injection model may contain some power injections from ancillary services such as generation-demand balance, reactive powers. In the beginning of the proposed allocation schemes, the players of the loss allocation game, i.e. who are going to pay for the exact transmission losses, are defined first according to the market rules. In (1), the generation current injection model may contain some power injections from ancillary services such as generation-demand balance, reactive supply, or loss compensation. One possible modification may be made by aggregating those supplementary power injections as one *aggregated player* in the loss allocation game. However, the allocated loss of the *aggregated player* might be re-allocated to the original winning bidders of the energy market. Thus, the market may agree on allocating the system loss to the bid winners without re-allocation by initially e x c l u d i n g t h e *a g g r e g a t e d p l a y e r*



from the allocation process; but, a *normalization procedure* will be needed to guarantee that the exact amount of losses is allocated. That is, the branch loss allocation would then be the product of the *normalized Shapley value* and the *exact* amount of branch loss. For instance, assume that initially there are *n* players *Ii* (*i=1, 2,…, n*) and the actual branch active loss is $\left|\sum_{i=1}^{n} I_i\right|^2 R$ If the line loss is allocated to only m ($m < n$) players, say $m = n$-1, then their Shapley values are normalized as $x_i' = \left(x_i / \sum_{j=1}^{m} x_j\right)$ Accordingly, the loss allocation to *Ii* is

Computed by $\left(x_i' \left|\sum_{i=1}^{n} I_i\right|^2 \cdot R\right)$, and the exact amount of loss equals the sum of every player's allocation.

III. NUMERICAL RESULTS AND DISCUSSIONS

Several systems have been used to test the proposed method. In this paper, the test results of a six-bus system [12] and a 14-bus system [4], [13] are presented and discussed.

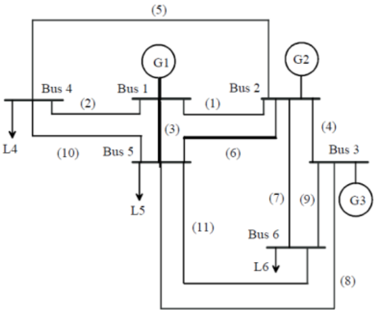


Fig .3.one line diagram of six bus system

TABLE I
A SOLVED POWER FLOW SOLUTION

Bus No.	P (pu)	Q (pu)	V (pu)	Angle (rad)
1	1.0788	0.1596	1.0500	0
2	0.5000	0.7436	1.0500	-0.0641
3	0.6000	0.8963	1.0700	-0.0746
4	-0.7000	-0.7000	0.9894	-0.0732
5	-0.7000	-0.7000	0.9854	-0.0921
6	-0.7000	-0.7000	1.0044	-0.1038

*System real power loss = 0.07876 pu

The one-line diagram of a six-bus system with three generation buses, three load buses, and eleven transmission lines (numbered (1), (2), …, (11)) is shown in Fig. 3. A solved power flow solution is shown in Table I. The players of the loss allocation game are defined as the bus injected complex powers according to the solution listed in Table I. The losses allocated to all generators and loads for each transmission line and the total system loss allocations using *BMA* (allocated to generator and load buses) and *BMB* (allocated to generator buses only) are listed in Tables II and III, respectively

TABLE II
TRANSMISSION LOSS ALLOCATION FOR *BMA* (pu)

Line No.	G1	G2	G3	L4	L5	L6
1	0.0088	-0.0005	-0.0002	0.0003	0.0000	0.0006
2	0.0068	-0.0006	-0.0003	0.0048	-0.0002	0.0003
3	0.0079	0.0009	-0.0011	-0.0019	0.0039	0.0010
4	0.0001	-0.0004	0.0008	0.0002	-0.0000	-0.0003
5	-0.0024	0.0050	0.0034	0.0127	-0.0003	-0.0034
6	0.0005	0.0026	0.0001	-0.0015	0.0036	-0.0003
7	0.0018	0.0026	-0.0026	-0.0024	0.0005	0.0059
8	-0.0009	0.0009	0.0089	0.0006	0.0056	-0.0041
9	-0.0000	-0.0001	0.0053	0.0001	-0.0001	0.0049
10	0.0004	0.0002	-0.0001	-0.0008	0.0006	0.0001
11	0.0000	0.0001	0.0005	0.0001	0.0006	-0.0008
Total	0.0230	0.0107	0.0148	0.0122	0.0140	0.0040

Table II shows that the losses allocated to generators are 0.0230, 0.0107, and 0.0148 pu, and those to loads are 0.0122, 0.0140, and 0.0040 pu, respectively. Table III shows that the losses allocated to generators are 0.03375, 0.01971, and 0.02531 pu, respectively. The total allocated loss is consistent with the power flow solution and can reasonably reflect the amounts of transactions injected complex powers according to the solution listed in Table I. The losses allocated to all generators and loads for each transmission line and the total system loss allocations using *BMA* (allocated to generator and load buses) and *BMB*(allocated to generator buses only) are listed in Tables II and III, respectively.

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2	0.0068	-0.0006	-0.0003	0.0048	-0.0002	0.0003
3	0.0079	0.0009	-0.0011	-0.0019	0.0039	0.0010
4	0.0001	-0.0004	0.0008	0.0002	-0.0000	-0.0003
5	-0.0024	0.0050	0.0034	0.0127	-0.0003	-0.0034
6	0.0005	0.0026	0.0001	-0.0015	0.0036	-0.0003
7	0.0018	0.0026	-0.0026	-0.0024	0.0005	0.0059
8	-0.0009	0.0009	0.0089	0.0006	0.0056	-0.0041
9	-0.0000	-0.0001	0.0053	0.0001	-0.0001	0.0049
10	0.0004	0.0002	-0.0001	-0.0008	0.0006	0.0001
11	0.0000	0.0001	0.0005	0.0001	0.0006	-0.0008
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TABLE III
TRANSMISSION LOSS ALLOCATION FOR *BMB* (pu)

Line No.	G1	G2	G3	Line Loss
1	0.00936	-0.00035	0.00004	0.00905
2	0.00902	0.00075	0.00111	0.01088
3	0.00925	0.00161	-0.00012	0.01074
4	0.00005	-0.00041	0.00076	0.00040
5	0.00026	0.00811	0.00668	0.01505
6	0.00104	0.00315	0.00078	0.00498
7	0.00326	0.00369	-0.00111	0.00583
8	-0.00028	0.00157	0.00965	0.01094
9	0.00145	0.00139	0.00720	0.01003
10	0.00032	0.00015	-0.00011	0.00036
11	0.00002	0.00005	0.00043	0.00050
Total	0.03375	0.01971	0.02531	0.07876

According to Tables II and III, the solved power flow solution, and its network configuration, most of the heavily loaded lines are directly connected to G1 (lines 1, 2, and 3)and G3 (lines 8 and 9), and thus the losses allocated to G1 and G3 are high. In addition, high loss shares indicate that the associated shared transmission branches are heavily loaded. It can also be seen that the loss allocated to a generator or load bus is mainly contributed by those lines which are directly connected with that bus and are heavily loaded. For example, from Tables II and III, the loss allocations of lines 1, 2, and 3for G1 are adding up to about 82% and 98% of its total
allocations, respectively. The results can reasonably reflect the transaction positions, the network configuration, and the operation status of transmission.

From Tables II and III, *negative loss* allocations for some branches are presented and can be explained using the phasor diagram of the individual current contributions. For example, the individual current contributions on line 4 for *BMB* show that the current contribution by G2 plays a role of reducing the net transmission loss and can be interpreted as contributing a *counter flow* to that branch against to the net flow direction. Table IV shows that the allocation percentage of each generator with respect to the sum of all generators' allocations is consistent in both models. Also, for *BMA*, the

TABLE IV
LOSS ALLOCATION RESULTS FOR TWO MODELS

GEN. / MODEL	G1	G2	G3	SUM
BMA	0.0230	0.0107	0.0148	0.0485
	47.4%	22.1%	30.5%	100%
BMB	0.0337	0.0197	0.0253	0.0787
	42.8%	25.0%	32.2%	100%

losses allocated to all three generators are 0.0485 pu which is 61.6% of the total system loss, and about 38.4% of the system loss is allocated to the loads. Since the network configuration and the location of each player are taken into account by the proposed schemes, the system loss is not evenly allocated to the supply side and the demand side. In addition, from Table II, for those lines directly connected between one generator bus and one load bus, the corresponding branch losses are mainly allocated to their supply (generation) sides, respectively. For example, the losses of lines 3, 6, and 9 are mostly allocated to G1, G2, and G3, respectively. Thus, there is no need to specify the sharing factors of losses to be allocated to the supply side and demand side.

For comparison, a 14-bus system used in [4] is tested using some approaches proposed earlier in [4] and [13]. The one line diagram [4] is duplicated in Fig. 4. Table V shows the loss shares for all generators by five approaches: incremental, proportional, quadratic, bus-oriented allocations [4], and *BMB*.

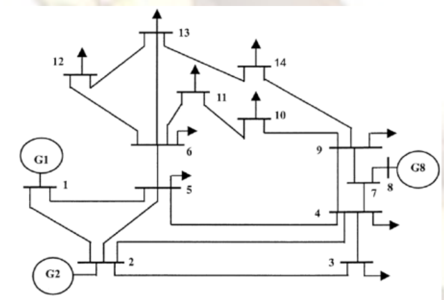

Fig. 4. One line diagram of a 14-bus power system.

As shown in Table V, the loss allocation results assigned to a generator may differ significantly by different approaches. The result of proposed scheme *BMB* is very close to that of incremental loss approach which needs a great number of repeated integrations. The purpose of this paper is to offer an alternative scheme to handle the fairness issue, instead of judging which one is the most accurate or the fairest.

TABLE V
LOSS ALLOCATION RESULTS FOR FIVE APPROACHES

Generator / Approach	G1	G2	G8
Incremental	0.0266	0.0182	0.0087
Proportional	0.0278	0.0201	0.0055
Quadratic	0.0294	0.0215	0.0025
Bus-oriented	00255	0.0097	0.0535
BMB	0.0265	0.0189	0.0088

However, it is worth noting that when the number of players of the game is getting bigger, the computation burden is getting heavier. Thus, *BMB* is recommended since its number of players is usually much smaller than that of *BMA*and will be much faster. Furthermore, some alternatives of the proposed schemes are provided in the following.

Alternative Version:

To speedup the allocation process, one slightly different but acceptable *alternative version* of (9) for *BMA* and *BMB* is then proposed as follows:

$$x_i = x_i^{(1)} + x_i^{(n)}$$

Where $x_i^{(1)} = \frac{1}{n}[V(\{I_i\}) - V(\{\})] = \frac{1}{n}[V(\{I_i\})]$ the loss is impact due to the player joining the game and can also represent the self induced term;

$x_i^{(n)} = \frac{1}{n}[V(\mathbf{N}) - V(\mathbf{N} - \{I_i\})]$ is the term when the player is the last one joining the game and also stands for the cross-induced term. That is, $x_i = \frac{1}{n}[V(\{I_i\}) + V(\mathbf{N}) - V(\mathbf{N} - \{I_i\})]$ Note that when (11) or (12) is utilized to speedup the allocation process, a *normalization procedure* is needed as shown in Fig. 2. Such a simplified version is still reasonable and acceptable since the self-induced term and the interaction with the rest of players of the game are both considered and followed by a *normalization procedure*.

IV. CONCLUSION

Based on the concept of Shapley value and the widely used current injection models in distribution load flow analysis, some fair and acceptable transmission loss allocation schemes have been proposed in the paper. The proposed schemes have the following properties:

1. It is consistent with a solved AC load flow and recovers the total system loss.
2. It is branch-current based, not branch-power-flow based, i.e., it emphasizes the interactions among complex currents rather than power flows.
3. It obeys the *Kirchhoff's laws* and *superposition Principle* and reflects both the network configuration and the voltage-current relationships.
4. The loss impacts between one player and any other coalitions of players are taken into account and the choice of cross term sharing factors is not uniform or arbitrary. Also, there is no need to specify the sharing factors of losses to be allocated to the supply side and demand side.
5. It can provide players with appropriate economic Signals, such as the negative loss indicates the Potential effect of reducing branch loss and good

transaction positions.

To speed up the allocation process, one alternative version has also been presented. The proposed schemes are also applicable to bilateral or hybrid pool-bilateral environments. The branch with negative loss allocation may provide one interesting application on congestion management, which is currently under investigation

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