Application of Cepstrum in Passive Sonar

*P.V.Ravi Kiran, **Amit Kumar Verma, ***G. Appala Naidu
*M.Tech Student, Department of ECE, GITAM University, Visakhapatnam
**Scientist-D, Signal Processing Division , NSTL, Visakhapatnam
***Assistant professor, Department of ECE, GITAM University , Visakhapatnam

ABSTRACT

In shallow water passive sonar, the received signal is characterized by multipath effects due to short distance between surface and bottom surfaces. Furthermore, interference noise makes the shallow water environment very difficult for target detection. To nullify the effects of multipath and noise, cepstrum based approach is used here. This approach takes the advantage of the fact that the underwater channel multipath effect occupies higher cepstral coefficients, while the signal remains concentrated at lower cepstral coefficients. The objective of this work is to recover the radiated sonar signal from the received signal, which is corrupted by multipath effects and interference noise.

Key words: **Cepstral liftering, SONAR, homomorphic, cepstrum**.

1. INTRODUCTION:

Recovering a signal that has been distorted both in frequency and in time caused by the multipath effects and noise is a common problem mainly encountered in shallow water environments. Multipath effects are due to one or more reflecting surfaces in between the target and the receiver. In shallow water channels the received signal is modeled as sum of the original signal and its multipath effects because of the nearness of the surface and the bottom surfaces. It is difficult to remove these multipath effects (delayed components), by using inverse filtering because of its inexactness. So in this work we have used cepstral processing which separates the signal components form the multipath effects. Further we recover the original radiated signal form the received signal with different SNR levels and notice how well Cepstral liftering (analogous to time domain filtering) nullifies multipath and noise effects.

2. HOMOMORPHIC SIGNAL PROCESSING:

We use linear filtering to split signals that have been additively combined. The principle advantage with linear filtering when applied to added signals, is that if the behavior of the filter for each of the signal components are separately know, then the behavior for the sum is the sum of the responses which satisfies the superposition principle.

In contrast, when determining a filtering procedure to separate signals that have been non- additively combined, such as through multiplication or convolution, it is usually more difficult to separate, and in many cases it is less meaningful to use a linear system. This leads to model a new class of system. By considering the system inputs as vectors in a vector space with the rule "o" corresponds to vector addition and the system transformation Φ as an algebraically linear transformation on that space.

$$s(n) = s_1(n) o s_2(n) \qquad (2.1)$$

$$\Phi[s_1(n) o s_2(n)] = \Phi[s_1(n)] + \Phi[s_2(n)] \qquad (2.2)$$

We confine the operation *o* so that it satisfies the algebraic assumption of vector addition and associate with the set of inputs a rule for combining inputs with scalars, which we will call scalar multiplication and denote by ":" To generalize the notion of linear filtering, then, we require that the class of systems, in addition to satisfying (2.2), also have the property that

$$\Phi[c{:}s_1(n)] = c{:}\Phi[s_1(n)] \qquad (2.3)$$

When the rule *"o"* equivalent to addition of the functions and the rule *":"* equivalent to the product of the input with the scalar, then equation (2.2) and (2.3) reduce to satisfy the principle of superposition as it applies to linear systems. The systems satisfying above equations (2.2) and (2.3) in general are called homomorphic systems[1][2].

3. SIGNAL RECOVERY WITH CEPSTRAL PROCESSING:

In shallow water channels we assume that received signal contains radiated signal and its delayed versions due to channel multipath effects. To explain how cepstral processing separates signal and multipath effects we take two functions which are convolved. One function represents the radiated signal and the other function represents time delayed multipath effects [3].

The received signal *y(n)* that is the convolution of a radiated signal *v(n)* and multipath function *h(n)* shown in Fig 3.1 and is given by

$$y(n) = h(n) * v(n) \quad (3.1)$$

Where '$*$' denotes convolution. The multipath function can be expressed as, here N=16

$$h(n) = 1 + \sum_{q=1}^{L} \alpha_q \delta(n - qN) \quad (3.2)$$

$$y(n) = v(n) + \sum_{q=1}^{L} \alpha_q v(n - qN) \quad (3.3)$$

Z transform of *h(n)* is given by

$$H(z) = 1 + \sum_{q=1}^{L} \alpha_q z^{-qN} \quad (3.4)$$
$$= \prod_{q=1}^{L}(1 + \alpha_q z^{-N}) \quad (3.5)$$

The radiated signal in the Z domain is defined as

$$V(z) = k\frac{\prod_{k=1}^{N_1}(1+a_k z^{-1})}{\prod_{j=1}^{N_2}(1+b_j z^{-1})} \quad (3.6)$$

Received signal *y(n)* in the Z domain is represented as

$$Y(z) = k\frac{\prod_{q=1}^{L}(1+\alpha_q z^{-N})\prod_{k=1}^{N_1}(1+a_k z^{-1})}{\prod_{j=1}^{N_2}(1+b_j z^{-1})} \quad (3.7)$$

To make the received signal minimum phase we have scaled it by a constant β.

Let $\beta = min\{|a_k|^{-1}, |b_j|^{-1}, |\alpha_q|^{-1/N}, 1+\varepsilon\} - \varepsilon$ (3.8)

where ε is a small positive number, if $x(n) = \beta^n y(n)$.

Now Z domain representation of the scaled received signal is

$$X(z) = k\frac{\prod_{q=1}^{L}(1+\alpha_q\beta^N z^{-N})\prod_{k=1}^{N_1}(1+a_k\beta z^{-1})}{\prod_{j=1}^{N_2}(1+b_j z^{-1})} \quad (3.9)$$

$$X(z) = k\frac{\prod_{q=1}^{L}(1+\hat{\alpha}_q z^{-N})\prod_{N_1}^{N_1}(1+d_k z^{-1})}{\prod_{j=1}^{N_2}(1+e_j z^{-1})} \quad (3.10)$$

where $|\alpha_q\beta^N| = |\hat{\alpha}_q| < 1$, $q = 1 \ldots L$; $|a_k\beta| = |d_k| < 1$, $k = 1 \ldots N_1$; $|b_k\beta| = |e_j| < 1, j = 1, 2 \ldots N_2$; Thus *X(z)* is a minimum phase signal shown in Fig 3.2.

Applying the logarithm transform to *X(Z)*, one obtains.

$$\hat{X}(z) = log\, k + \sum_{q=1}^{L} log[1 + \hat{\beta}_q z^{-N}] + \sum_{k=1}^{N_1} log[1 + d_k z^{-1}] - \sum_{j=1}^{N_2} log[1 + e_j z^{-1}] \quad (3.11)$$

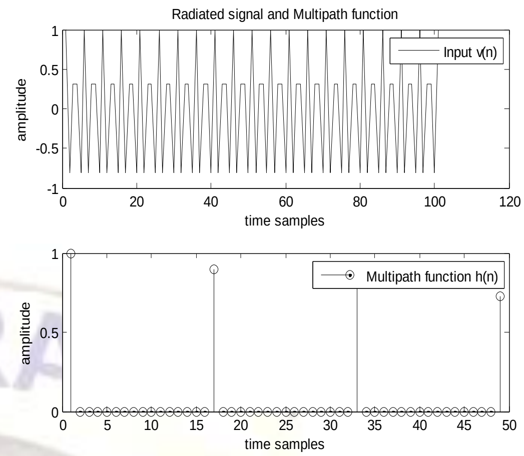

Figure 3.1

Where $\hat{X}(z) = log(X(z))$. Considering logarithmic expansion in the above equation i.e,

$$log[1 + z] = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} z^n}{n} \quad |z| < 1 \quad (3.12)$$

Now Equation 3.11 becomes

$$\hat{X}(z) = log\, k - \sum_{q=1}^{L}\sum_{m=1}^{\infty}\frac{(-\hat{\beta}_q)^{n+1} z^{-mn}}{m} - \sum_{k=1}^{N_1}\sum_{m=1}^{\infty}\frac{(-d_k)^m z^{-m}}{m} - \sum_{j=1}^{N_2}\sum_{m=1}^{\infty}\frac{(-e_j)^m z^{-m}}{m} \quad (3.13)$$

Taking the inverse Z transform of $\hat{X}(z)$ is denoted by $\hat{x}(n)$ and is given by

$$\hat{x}(n) = \begin{cases} 0, & m<0 \\ log(K), & m=0 \\ \sum_{j=1}^{N_2}\frac{(-e_j)^m}{m} - \sum_{k=1}^{N_1}\frac{(-d_k)^m}{m}, & m>0, m\neq iN \\ \sum_{j=1}^{N_2}\frac{(-e_j)^{iN}}{iN} - \sum_{k=1}^{L}\frac{(-\hat{\beta}_q)^i}{m} - \sum_{k=1}^{N_1}\frac{(-d_k)^{iN}}{iN}, & m>0, m=iN \end{cases} \quad (3.14)$$

Where 'i' is real positive integer

Equation (3.14) represents the signal *x(n)* in the cepstral domain. It is clear that cepstrum of the signal occupies lower indices $(m > 0, m \neq iN)$ and the multipath function occupies $(m > 0, m = iN)$. To filter out the multipath effects we take a comb filter.

$$\hat{C}(m) = [1 - \sum_{q=1}^{\infty}\delta(m - iN)] \quad (3.15)$$

Then the resulting output is, shown in Fig.3.2

$$\hat{y}_v(m) = \hat{C}(m)\,\hat{x}(m) \quad (3.16)$$

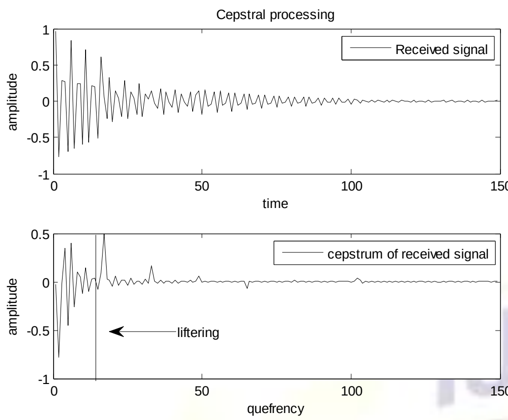

Figure 3.2

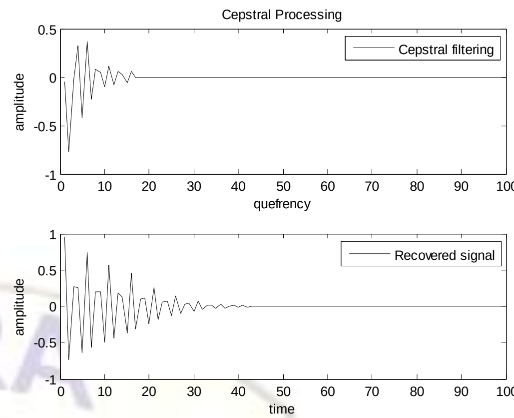

Figure 3.3

Now for $m= iN$.

$\hat{y}_v(iN)=0$ (3.17)

One way of obtaining sums for $\hat{y}_v(m)$ with known functional forms is to express $\hat{y}_v(iN)$ as

$$\hat{y}_v(m = iN) =-\sum_{k=1}^{N_1}\frac{(-d_k)^m}{m}+\sum_{k=1}^{N_1}\frac{(-d_k)^m}{m}+\sum_{j=1}^{N_2}\frac{(-e_j)^m}{m}-\sum_{j=1}^{N_2}\frac{(-e_j)^m}{m} \quad (3.18)$$

now to recover the original signal inverse cepstral processing is has to be performed where the z-transform of $\hat{y}_v(m)$ is,

$$\hat{y}_v(z) = \log k-\sum_{k=1}^{N_1}\sum_{m=1}^{\infty}\frac{(-d_k)^m z^{-m}}{m}+\sum_{k=1}^{N_1}\sum_{m=1}^{\infty}\frac{(-d_k)^{iN} z^{-iN}}{iN}+\sum_{j=1}^{N_2}\sum_{m=1}^{\infty}\frac{(-e_j)^m z^{-m}}{m}-\sum_{k=1}^{N_1}\sum_{m=1}^{\infty}\frac{(-e_j)^{iN} z^{-iN}}{iN} \quad (3.19)$$

and by taking inverse logarithm of $\hat{y}_v(z)$, one can obtains the new signal in terms of the original signal *v(n)*.

$$y_v(z) = \hat{V}(z)\prod_{k=1}^{N_1}[1-(-e_j z^{-1})^N]^{1/N} \times \prod_{k=1}^{N_1}[1-(-ejz-1)N]-1/N \quad (3.20)$$

where $\hat{V}(z) = V(\beta^{-1}z)$.A direct comparison of $Y(z)$ to $\hat{V}(z)$ can be made using (3.20). The binomial series in (3.20) can be expressed as

$$[1-(r)^N]^{-1/N}=1\pm\frac{(r)^N}{N}\pm\frac{(N\pm1)(r)^{2N}}{2!N^2}\pm\frac{(N\pm1)(2N\pm1)(r)^{2N}}{3!N^3} \quad (3.21)$$

Where $|x| < 1$ Hence (3.22) can be written as

$$Y_v(z)=\hat{V}(z)\prod_{j=1}^{N_2}[1 \pm\frac{(-e_j)^N}{N}z^{-N}\pm\frac{(N-1)(-e_j)^{2N}}{2!N^2}z^{-2N}\pm$$
$N\pm12N\pm1-ej2N3!N3z-3N-....]$ $k=1N1[1 \pm-dkNNz-N$ $\pm N-1-dk2N2!N2z-2N$ $\pm N\pm12N\pm1-dk2N3!N3z-3N-....]$ (3.22)

$v(n)$ and $Y_v(z)$ can be compared when the product of all binomial series in (3.22) be generated and then truncated at some index N_0, and above equation is expressed as

$Y_v(z)=\hat{V}(z)+\sum_{k=1}^{N_0} g_k z^{-kN}\hat{V}(z)$ (3.23)

This leads to the following representation for $y_v(n)$

$y_v(n)=\hat{v}(n)+\sum_{k=1}^{N_0} g_k\beta^{-n}\hat{v}(n-kN)$ (3.24)

where $\hat{v}(n) = \beta^n v(n)$. this is to make the signal minimum phase by moving the poles inside the unit circle, $y_v(n)$is scaled by $\beta^{-n}, \beta^{-n}y_v(n)$in order to obtain recovered waveform, $v_r(n)$(shown in Fig.3.3).

$v_r(n) = v(n)+\sum_{k=1}^{N_0} g_k\beta^{-KN} v(n-kN)$ (3.25)

By calculating this sum, we can obtain $v_r(n)$ to some accuracy. These derivations are taken from [3].

4. SIGNAL DETECTION IN PRESENCE OF NOISE:

Till now we have considered only the multipath noise and seen how cepstral processing successfully separated the radiated signal from the multipath noise. In this section we further consider the interference noise and its effect in the cepstral processing.

4.1 NOISE:

Interference noise is assumed to be white Gaussian noise with zero mean and variance equal to σ_w^2.

Each noise sample is assumed to be uncorrelated with the radiated signal and channel multipaths [4].

Now we will analyze the effect of added noise to signal in cepstral processing. Noise addition to the received signal in time domain does not map to a convenient function in the cepstral domain. So we add noise after convolution of *x(t)* and *h(t)*.

$$Y(t) = [x(t) * h(t)] + n(t) \quad (4.1)$$

When we take logarithm of the magnitude of the frequency response,

$$log\, Y(\omega) = log|X(\omega)H(\omega) + N(\omega)| \quad (4.2)$$

$$= log|X(\omega)H(\omega)(1 + K(\omega)| \quad (4.3)$$

where,

$$K(\omega) = \frac{N(\omega)}{X(\omega)H(\omega)} \quad (4.4)$$

Therefore,

$$log\,|Y(\omega)| = log|X(\omega)| + log|H(\omega)| + log|1 + K(\omega)| \quad (4.5)$$

$$log\,|Y(\omega)| = log|X(\omega)| + log\,|H(\omega)| + N^{"}(\omega) \quad (4.6)$$

When the inverse Fourier transform is performed on the above equation (4.6), the last term $N^{"}(\omega)$ can be concluded as noise effect in the cepstral domain $\hat{n}^{"}(\tau)$.

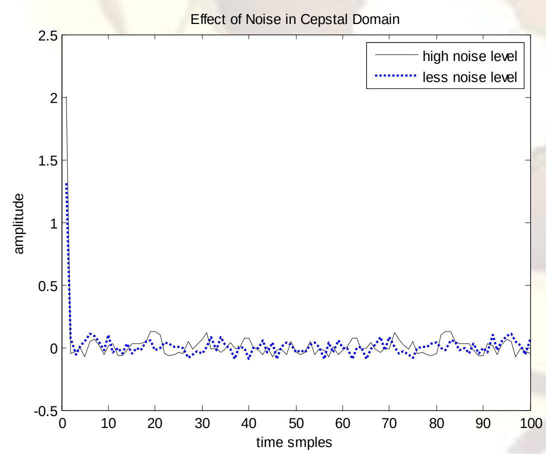

Figure 4.1

As we know the Fourier transform of Gaussian white noise $N(\omega)$ is a constant function in frequency domain. If the noise intensity increases, the magnitude of $N(\omega)$ increases and hence $N^{"}(\omega)$ also increases, and there will be no variations in shape of the signal but only a DC component is added to the signal. When we take inverse Fourier transform on $N^{"}(\omega)$ the effect of noise in cepstral domain $\hat{n}^{"}(\tau)$ can be seen. The dc component maps to the point $\tau = 0$ as shown in Fig. 4.1. The figure also shows that for stronger noise levels, the cepstrum of the signal increases at $\tau = 0$ and for $\tau \neq 0$, $\hat{n}^{"}(\tau)$ doesn't have any variations [5]. When we perform cepstral liftering the received signal which contains interference noise can be removed.

Now let us consider signals with different SNRs. In Fig. 4.2 the received signal with SNR of 9 dB is shown. On these received signals, we perform cepstral processing and later we do cepstral liftering to remove the effect of noise at τ=0, and also the multipath effects

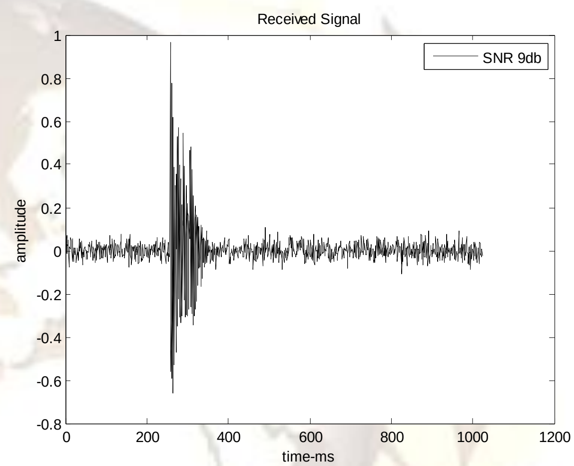

Figure 4.2

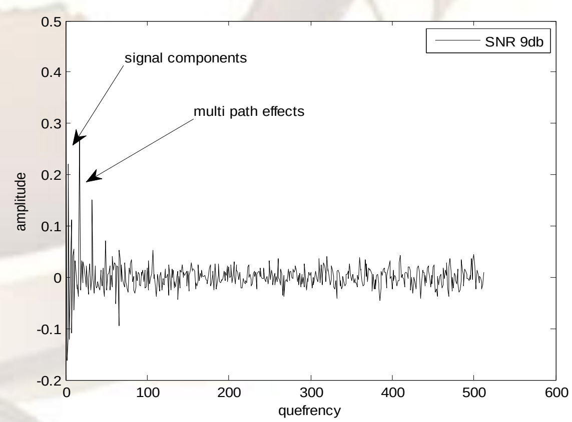

Figure 4.3

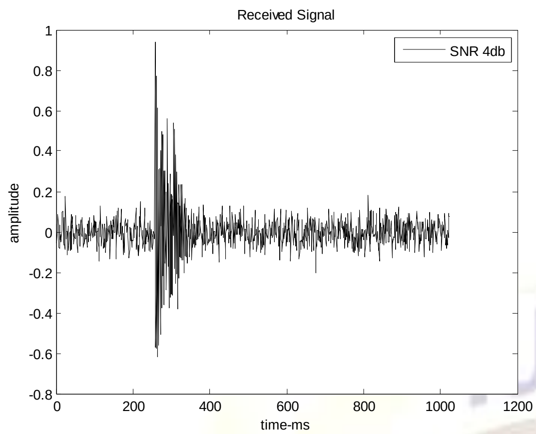

Figure 4.4

Fig.4.3 shows cepstrum output of the received signal with SNR 9 dB, where we can see the signal component and multipath components

Fig. 4.4 shows the received signal with SNR 4dB. Similarly Fig. 4.5 shows cepstrum output of the received signal with SNR 4 dB.

Through cepstrum processing, we can see that the original signal component and the multipaths impulses can be separated as seen in Fig. 4.3 and Fig.4.5. The signal component is captured by the lower cepstral coefficients and the multipath impulses are captured by the higher cepstral coefficients.

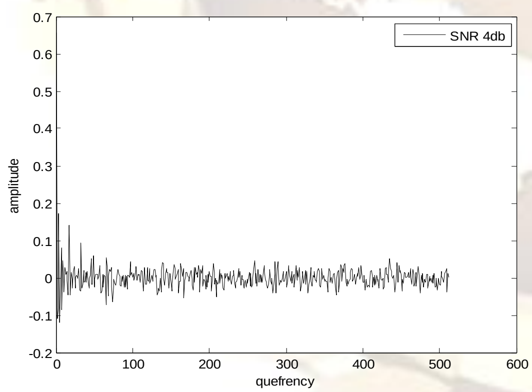

Figure 4.5

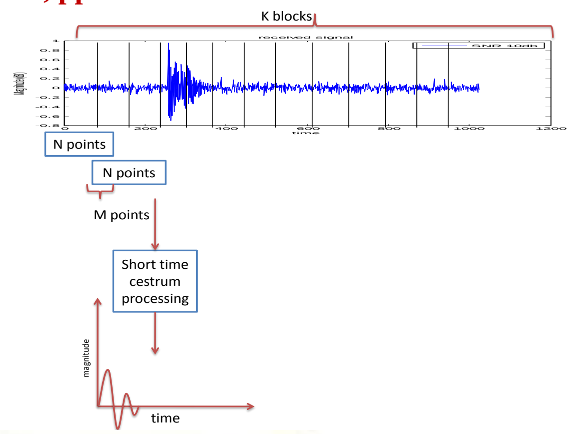

Figure 4.6

For our simulations, received time domain signal is blocked into K frames of N samples each, with adjacent frames spaced M samples apart. Typical values for N and M correspond to frames of duration 96ms, with frame shifts of 64 ms respectively hence adjacent frames overlap by 32ms shown in Fig 4.6

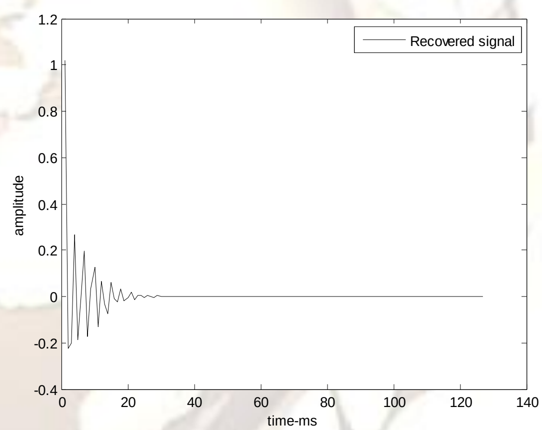

Figure 4.7

Processing the received signal block by block and performing cepstrum and inverse cepstrum we can reduce the effect of noise for recovering the signal component. We observe that signal component is recovered at 4^{th} and 5^{th} window as shown in the fig 4.7 and 4.8.

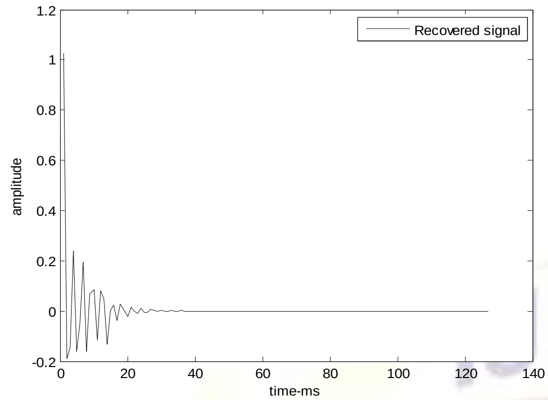

Figure 4.8

Thus the cepstral processing is useful in target detection in the presence of multipath effects and interference noise. It also recovers the signal frequency content. This may be used for target identification.

5. CONCLUSION

The study presented in this paper makes an observation that cepstral processing facilitates passive sonar target detection and identification. The effect of multipath and interference noise is minimized by using cepstral liftering. The received signal is the convolution of the sonar radiated signal and the impulse response of the shallow water channel in the time domain. However in cepstral domain it is superposition of the cepstrum of signal and cepstrum of the impulse response. Additionally signal cepstrum occupies low time cepstrum, impulses occupy high time cepstrum and noise occupies cepstrum at t=0. Therefore cepstrum of the signal can be extracted by using appropriate filter which nullifies the effect of noise and multipath effects.

REFERENCES

[1]. A. V. Oppenheim, R. W. Schafer and T. G. Stockham, "Nonlinear filtering of multiplied and convolved signals," *Proceedings of the IEEE, Vol. 56, No. 8*, pp. 1264-1291, Aug. 1968.

[2]. A. V. Oppenheim and R. S**.** Schafer, *Digital Signal Processing* (Englewood Cliffs, NJ: Prentice-Hall, 1975).

[3]. William Dale Blair, Jeff E. Conte, and Theodore Roger Rice, "An Instructive Example of Homomorphic Signal Processing*," IEEE Transaction on education*, *VOL. 38, NO. 3*, AUGUST 1995.

[4]. Arun Kumar and Rajendra Bahl. "Radiated signal characteristics of marine vessels in the cepstral domain for shallow underwater channel". *JASA express letters , Sep. 2010*.

[5]. Jean ok Nam, "*Signal recovery using cepstral smoothing in the shallow water environment*," Thesis submitted at Massachusettus Institute of Technology, Sep. 1994.