Denoising of Images corrupted by Random noise using Complex Double Density Dual Tree Discrete Wavelet Transform

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Abstract-This paper presents removal of random noisenoise by complex double density dual tree discrete wavelet Transform. In general in images noise suppression is a particularly delicate and difficult task. A tradeoff between noise reduction and the preservation of actual image features has to be made in a way that enhances the relevant image content. The main properties of a good image denoising model are that it will remove noise while preserving edges and contours. However, wavelet coefficients of natural images have significant dependencies. For many natural signals, the wavelet transform is a more effective tool than the Fourier transform. The wavelet transform provides a multiresolution representation using a set of analyzing functions that are dilations and translations of a few functions (wavelets). In this paper we have evaluated & compared performances of Standard Double Density DWT(SDDDWT), Real Double Density Dual Tree (RDDDTDWT) and Complex Double Density Dual Tree DWT(CDDDTDWT). Simulation and experimental results demonstrate that the complex double density dual tree discrete wavelet transform (CDDDTDWT) outperforms a number of other existing wavelet transform techniques and it is particularly effective for the very highly corrupted images.

KEYWORDS: **DWT, SDDDWT, RDDDTDWT& CDDDTDWT.**

I. INTRODUCTION

it is generally desirable for image brightness (or film density) to be uniform except where it changes to form an image. There are factors, however, that tend to produce variation in the brightness of a displayed image even when no image detail is present. This variation is usually random and has no particular pattern. In many cases, it reduces image quality and is especially significant when the objects being imaged are small and have relatively low contrast. This random variation in image brightness is designated ***random noise.***

Although noise gives an image a generally undesirable appearance, the most significant factor is that noise can cover and reduce the visibility of certain features within the image .The loss of visibility is especially significant for low-contrast objects. The visibility threshold, especially for low-contrast objects, is very noise dependent. In principle, when we reduce image noise, the "curtain" is raised somewhat, and more of the low-contrast objects within the body become visible. We can usually change imaging factors to reduce noise, we must always compromise.

The wavelet transform is a simple and elegant tool that can be used for many digital signal and image processing applications. It overcomes some of the limitations of the Fourier transform with its ability to represent a function simultaneously in the frequency and time domains using a single prototype function (or wavelet) and its scales and shifts. The wavelet transform comes in several forms. The critically-sampled form of the wavelet transforms provides the most compact representation; however, it has several limitations. For example, it lacks the shift-invariance property, and in multiple dimensions it does a poor job of distinguishing orientations, which is important in image processing. For these reasons, it turns out that for some applications, improvements can be obtained by using an expansive wavelet transform in place of a critically-sampled one.

An expansive transform is one that converts an N-point signal into M coefficients with M > N. There are several kinds of expansive DWTs such as dual tree DWT [1-3] and double density DWT [4]. The dual-tree complex wavelet transform overcomes these limitations, it is nearly shift-invariant and is oriented in 2-D. The 2- D dual-tree wavelet transform produces six sub bands at each scale, each of which is strongly oriented at distinct angles while the double-density DWT is an improvement upon the critically sampled DWT with important additional properties: (1) It employs one scaling function and two distinct wavelets, which are designed to be offset from one another by one half, (2) The double-density DWT is over complete by a factor of two, and (3) It is nearly shift-invariant.

The differences between the double-density DWT and the dual-tree DWT can be clarified with the following comparisons:

- In the dual-tree DWT, the two wavelets form an approximate Hilbert transform pair, whereas in the double-density DWT, the two wavelets are offset by one half.
- For the dual-tree DWT, there are fewer degrees of freedom for design (achieving the Hilbert pair property adds constraints), whereas for the double-density DWT, there are more degrees of freedom for design.

- Different filter bank structures are used to implement the dual-tree and double-density DWTs.
- The dual-tree DWT can be interpreted as a complex-valued wavelet transform, which is useful for signal modeling and denoising (the double-density DWT cannot be interpreted as such).
- The dual-tree DWT can be used to implement 2-D transforms with directional Gabor-like wavelets, which is highly desirable for image processing (the double-density DWT cannot be, although it can be used in conjunction with specialized post-filters to implement a complex wavelet transform with low-redundancy [5].

II. DISCRETE WAVELET TRANSFORM

***A.* Wavelet Transform**: The simplest wavelet transform for multi-dimensional digital data is the critically-sampled separable wavelet transform. This transform uses a 1-D wavelet transform in each dimension and is the one that is conventionally used. However, one way to improve the performance of wavelet-based signal and image processing algorithms is to use specialized wavelet transforms in place of the conventional wavelet transform. There are several advances in the design of specific wavelet transforms that lead to substantially improved performance. For example, the undecimated wavelet transform [6-7], the steerable pyramid [8], and curvelet transform [9] all give improved results in applications involving multidimensional data. Recently developed dual-tree transform, an oriented complex-valued wavelet transform shown to be highly beneficial for multi-dimensional signal and image processing. This transform has several advantages over the conventional multi-dimensional wavelet transform: (1) near shift invariance, (2) directional selectivity, and (3) improved energy compaction. The discrete wavelet transform are based on perfect reconstruction two-channel filter banks. It consists of recursively applying a 2-channel filter bank - the successive decomposition is performed only on the low pass output [2][5].

Mathematically the Discrete wavelet transform transform pair for one dimensional can be defined as

$$W_\phi(j_0,k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} f(x)\tilde{\phi}_{j_0,k}(x) \qquad (1)$$

$$W_\psi(j,k) = \frac{1}{\sqrt{M}} \sum_{x=0}^{M-1} f(x)\tilde{\psi}_{j,k}(x) \qquad (2)$$

for $j \geq j_0$ and

$$f(x) = \frac{1}{\sqrt{M}} \sum_k W_\phi(j_0,k)\phi_{j_0,k}(x) + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_k W_\psi(j,k)\psi_{j,k}(x) \qquad (3)$$

Where $f(x)$, $\phi_{j_0,k}(x)$, , and $\psi_{j,k}(x)$ are functions of discrete variable *x* = 0, 1, 2, ... ,

In two dimensions, a two-dimensional scaling function, $\phi(x,y)$, and three two-dimensional wavelet $\psi^H(x,y)$, $\psi^V(x,y)$ and $\psi^D(x,y)$ are required. Each is the product of a one-dimensional scaling function Φ and corresponding wavelet Ψ.

$$\phi(x,y) = \phi(x)\phi(y) \qquad (4)$$

$$\psi^H(x,y) = \psi(x)\phi(y) \qquad (5)$$

$$\psi^V(x,y) = \phi(y)\psi(x) \qquad (6)$$

$$\psi^D(x,y) = \psi(x)\psi(y) \qquad (7)$$

where ψ^H measures variations along columns (like horizontal edges), ψ^V responds to variations along rows (like vertical edges), ψ^D and corresponds to variations along diagonals.

a. 1-D filter Bank: The 1-D filter bank is constructed with analysis & synthesis filter bank. The analysis filter bank decomposes the input signal x(n) into two sub band signals, c(n) and d(n). The signal c(n) represents the low frequency part of x(n), while the signal d(n) represents the high frequency part of x(n). We have denoted the low pass filter by af1 (analysis filter 1) and the high pass filter by af2 (analysis filter 2).

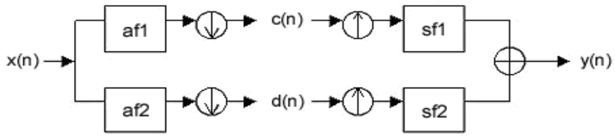

Figure 1.The Analysis filter bank Figure 2.The Synthesis filter bank

As shown in the figure 1, the output of each filter is then down sampled by 2 to obtain the two sub band signals c(n) & d(n) [3][10-12].The Synthesis filter bank combines the two sub band signals c(n) & d(n) to obtain a single signal y(n). The synthesis filters bank up-samples each of the two sub band signals. The signals are then filtered using a low pass and a high pass filter. We have denoted the low pass filter by sf1 (synthesis filter1) and the high pass filter by sf2 (synthesis filter 2) as shown in the figure 2. The signals are then added together to obtain the signal y(n). If the four filters are designed so as to guarantee that the output signal y(n) equals the input signal x(n), then the filters are said to satisfy the perfect reconstruction condition [3][10-12].

b. 2-D Filter Banks: To use the wavelet transform for image processing we must implement a 2-D version of the analysis and synthesis filter banks. In the 2-D case, the 1-D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has N1 rows and N2 columns, then after applying the 1-D analysis filter bank to each column we have two sub-band images, each having N1/2 rows and N2 columns; after applying the 1-D analysis filter bank to each row of both of the two sub-band images, we have four sub-band images, each having N1/2 rows and N2/2 columns. This is illustrated in figure 3. The 2-D synthesis filter bank combines the four sub-band images to obtain the original image of size N1 by N2 [3][10-12].

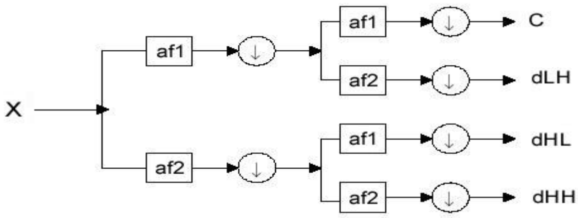

Figure3. One stage in multi-resolution wavelet decomposition of an image

The two-dimensional DWT can be implemented using digital filters and downsamplers and it is shown in the figure 4 & 5 respectively

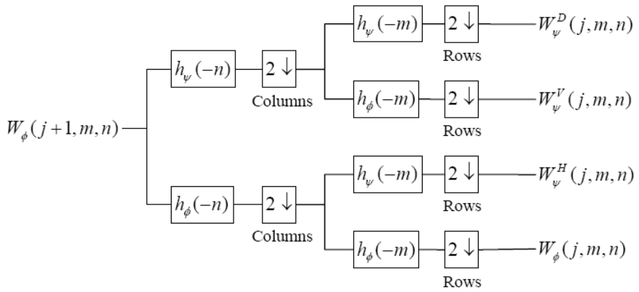

Figure 4. The two-dimensional DWT□ the analysis filter

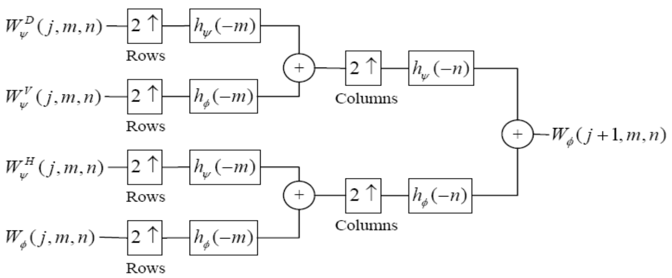

Figure 5. The two-dimensional DWT□ the synthesis filte

B. Dual Tree Discrete Wavelet Transform

The dual-tree complex DWT of a image x is implemented using two critically-sampled DWTs in parallel on the same data as shown in the figure 6.

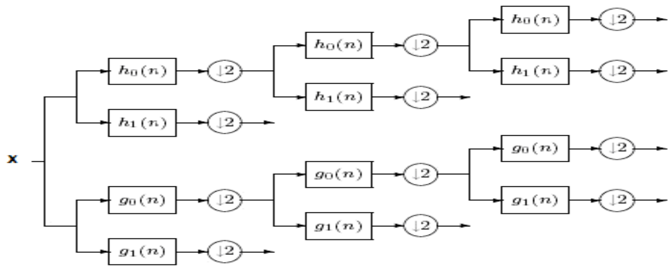

Figure 6. 2-D Dual Tree Discrete Wavelet Transform

This transform is 2-times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. However, if the filters are designed is a specific way, then the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and subband signals of the lower DWT can be interpreted as the imaginary part. Equivalently, for specially designed sets of filters, the wavelet associated with the upper DWT can be an approximate Hilbert transform of the wavelet associated with the lower DWT. When designed in this way, the dual-tree complex DWT is nearly shift-invariant, in contrast with the critically-sampled DWT. Moreover, the dual-tree complex DWT can be used to implement 2-D wavelet transforms where each wavelet is oriented, which is especially useful for image processing. For the separable 2-D DWT, recall that one of the three wavelets does not have a dominant orientation. The dual-tree complex DWT outperforms the critically-sampled DWT for applications like image enhancement. One of the advantages of the dual-tree complex wavelet transform is that it can be used to implement 2-D wavelet transforms that are more selective with respect to orientation than is the separable 2-D DWT[1-3][13-16].

There are two types of the 2-D dual-tree wavelet transform: the real 2-D dual-tree DWT is 2-times expansive, while the complex 2-D dual-tree DWT is 4-times expansive. Both types have wavelets oriented in six distinct directions. We describe the real version first.

1). Real 2-D Dual-Tree *Discrete* Wavelet Transform

The real 2-D dual-tree DWT of an image x is implemented using two critically-sampled separable 2-D DWTs in parallel. Then for each pair of subbands we take the sum and difference.

2).Complex 2-D Dual-Tree *Discrete* Wavelet Transform

The complex 2-D dual-tree DWT also gives rise to wavelets in six distinct directions, however, in this case there are two wavelets in each direction. In each direction, one of the two wavelets can be interpreted as the real part of a complex-valued 2-D wavelet, while the other wavelet can be interpreted as the imaginary part of a complex-valued 2-D wavelet. Because the complex version has twice as many wavelets as the real version of the transform, the complex version is 4-times expansive. The complex 2-D dual-tree is implemented as four critically-sampled separable 2-D DWTs operating in parallel. However, different filter sets are used along the rows and columns. As in the real case, the sum and difference of sub-band images is performed to obtain the oriented wavelets [1-3][13-16].

C. Bivariate Shrinkage Function

We have considered non-Gaussian bivariate probability distribution function to model the statistics of wavelet coefficients of natural images. The model captures the dependence between a wavelet coefficient and its parent. Using Bayesian estimation theory we derive from this model a simple non-linear shrinkage function for wavelet denoising, which generalizes the soft thresholding approach of Donoho and Johnstone. The shrinkage function, which depends on both the coefficient and its parent, yields improved results for wavelet-based image denoising [10-12].

Let w2 represent the parent of w1 (w2 is the wavelet coefficient at the same spatial position as w1, but at the next coarser scale). Then

$$y=w+n \qquad (8)$$

where w = (w1,w2), y = (y1,y2) and n = (n1,n2). The noise values n1, n2 are IID zero-mean Gaussian with variance \sigma_n^2. Now we define the following non-Gaussian bivariate pdf

$$p_w(w) = \frac{3}{2\pi\sigma^2}.\exp\left(-\frac{\sqrt{3}}{\sigma}\sqrt{w_1^2+w_2^2}\right) \qquad (9)$$

With this pdf, w1 and w2 are uncorrelated, but not independent. The MAP estimator of w1 yields the following bivariate shrinkage function

$$\hat{w} = \frac{\left(\sqrt{y_1^2+y_2^2}-\frac{\sqrt{3}\sigma_n^2}{\sigma}\right)}{\sqrt{y_1^2+y_2^2}}.y_1 \qquad (10)$$

For this bivariate shrinkage function, the smaller the parent value, the greater the shrinkage. This is consistent with other models, but here it is derived using a Bayesian estimation approach beginning with the new bivariate non- Gaussian mode [10-12].

D. MATLAB Implementation Procedure:

1. Set the window size. The image variance of a coefficient will be estimated using neighboring coefficients in a rectangular region with this window size.
2. Set how many stages will be used for the wavelet transform.
3. Extend the noisy image. The noisy image will be extended using symmetric extension in order to improve the boundary problem.
4. Calculate the forward dual-tree DWT.
5. Estimate the noise variance. The noise variance will be calculated using the robust median estimator.
6. Process each subband separately in a loop. First the real and imaginary parts of the coefficients and the corresponding parent matrices are prepared for each subband.
7. Estimate the image variance and the threshold value: The signal variance for each coefficient is estimated using the window size and the threshold value for each coefficient will be calculated and stored in a matrix with the same size as the coefficient matrix [10-12].
8. Estimate the magnitude of the complex coefficients The coefficients will be estimated using the magnitudes of the complex coefficient, its parent and the threshold value with the Bivariate Shrinkage Function.
9. Calculate the inverse wavelet transform.
10. Extract the image. The necessary part of the final image is extracted in order to reverse the symmetrical extension.

III. DOUBLE DENSITY DISCRETE WAVELET TRANSFORM

The double density Discrete Wavelet Transform is constructed with analysis & synthesis filter bank and it is shown in the figure 7.

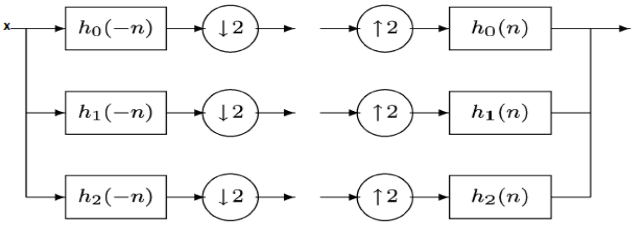

Figure 7. Oversampled analysis and synthesis filter bank.

In two dimensions, this transform outperforms the standard DWT in terms of enhancement; however, there is need of improvement because not all of the wavelets are directional. That is, although the double-density DWT utilizes more wavelets, some lack a dominant spatial orientation, which prevents them from being able to isolate those directions [4-5].

A solution to this problem is provided by the double-density complex DWT, which combines the characteristics of the double-density DWT and the dual-tree DWT. The double-density complex DWT is based on two scaling functions and four distinct wavelets, each of which is specifically designed such that the two wavelets of the first pair are offset from one other by one half, and the other pair of wavelets form an approximate Hilbert transform pair. By ensuring these two properties, the double-density complex DWT possesses improved directional selectivity and can be used to implement complex and directional wavelet transforms in multiple dimensions. We construct the filter bank structures for both the double-density DWT and the double-density complex DWT using finite impulse response (FIR) perfect reconstruction filter banks. These filter banks are then applied recursively to the low pass subband, using the analysis filters for the forward transform and the synthesis filters for the inverse transform. By doing this, it is then possible to evaluate each transforms performance in several applications including signal and image enhancement [4-5][17-20].

A. 1-D Double-Density DWT

The double-density DWT is implemented by recursively applying the 3-channel analysis filter bank to the low pass subband. This process is illustrated in figure 8. Conversely, the inverse double-density DWT is obtained by iteratively applying the synthesis filter bank.

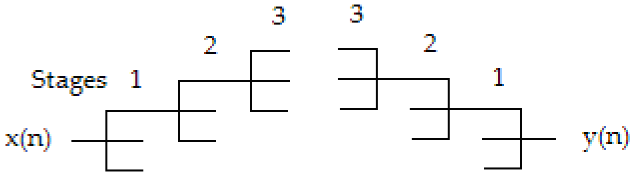

Figure 8. Three stage recursion of the 1-D double-density DWT

B. 2-D Double-Density DWT

To use the double-density discrete wavelet transform for 2-D signal and image processing, we must implement a two-dimensional analysis and synthesis filter bank structure. This can simply be done by alternatively applying the transform first to the rows, then to the columns of an image.

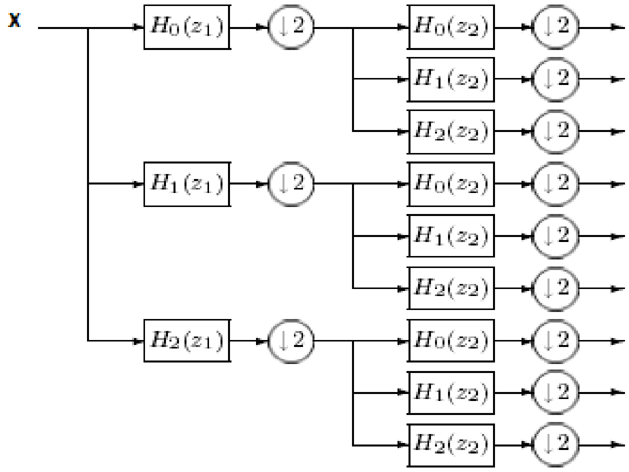

Figure 9. An Oversampled Filter Bank for 2-D Images

This gives rise to nine 2-D subbands, one of which is the 2-D low pass scaling filter, and the other eight of which make up the eight 2-D wavelet filters, as shown in figure 9.

C. 2-D Double-Density Dual-Tree DWT

The double-density dual-tree DWT, which is an over complete discrete wavelet transform (DWT) designed to simultaneously possess the properties of the double-density DWT and the dual-tree complex DWT. The double-density DWT and the dual-tree complex DWT are similar in several respects (they are both over complete by a factor of two, they are both nearly shift-invariant, and they are both based on FIR perfect reconstruction filter banks), but they are quite different from one another in other important respects. Both wavelet transforms can outperform the critically sampled DWT for several signal processing applications, but they do so for different reasons. It is therefore natural to investigate the possibility of a single wavelet transform that has the characteristics of both the double-density DWT and dual-tree complex DWT [4-5][17-20].

There are two types of the 2-D double-density dual-tree DWT: (1) The 2-D double-density dual-tree real-oriented DWT, which is 2-times expansive and (2) the 2-D double-density dual-tree complex-oriented DWT, which is 4-times expansive [4-5] [17-20].

1. Real 2-D Double-Density Dual-Tree DWT

The 2-D double-density dual-tree real DWT of an image *i* is implemented by using two oversampled 2-D double-density DWTs in parallel. Then, for each pair of sub bands, we take the sum and difference [4-5][17-20].

2. Complex 2-D Double-Density Dual-Tree DWT

The 2-D double-density dual-tree complex DWT is 4-times expansive, which means it gives rise to twice as many wavelets in the same dominating orientations as the 2-D double-density dual-tree real DWT. For each of the directions illustrated in Figure 7, one of the wavelets can be interpreted as the real part of a complex-valued 2-D wavelet function, while the other can be interpreted as the imaginary part. This transform is implemented by applying four 2-D double-density DWTs in parallel to the same input data with distinct filter sets for the rows and columns. As in the real DWT, we then take the sum and difference of the subband images. This operation yields the 32 oriented wavelets associated with the 2-D double-density dual-tree complex DWT [4-5][17-20].

D. MATLAB Implementation Procedure:

1. Set the window size. The image variance of a coefficient will be estimated using neighboring coefficients in a rectangular region with this window size.
2. Set how many stages will be used for the wavelet transform.
3. Extend the noisy image. The noisy image will be extended using symmetric extension in order to improve the boundary problem.
4. Calculate the Forward Double Density DWT.
5. Estimate the noise variance. The noise variance will be calculated using the robust median estimator.
6. Process each subband separately in a loop. First the real and imaginary parts of the coefficients and the corresponding parent matrices are prepared for each subband [1][5].
7. Estimate the image variance and the threshold value: The image variance for each coefficient is estimated using the window size and the threshold value for each coefficient will be calculated and stored in a matrix with the same size as the coefficient matrix.
8. Estimate the magnitude of the complex coefficients. The coefficients will be estimated using the magnitudes of the complex coefficient, its parent and the threshold value with the Bivariate Shrinkage Function.
9. Calculate the inverse Double Density
10. Extract the image. The necessary part of the final image is extracted in order to reverse the symmetrical extension

IV. SIMULATION & EXPERIMENT RESULTS SIMULATION

Five 8-bit images of dimensions MlxM2 (= 512x512) pixels is used for simulations. The pixels s(i, j) for 1 ≤i ≤ M1 and 1 ≤j

≤ M2, of the image is corrupted by adding random noise. The superiority of different wavelet transform is demonstrated. We have evaluated & compared performances of Standard Double Density DWT (SDDDWT), Real Double Density Dual Tree (RDDDTDWT) and Complex Double Density Dual Tree DWT (CDDDTDWT) by using peak signal to noise ratio (PSNR) value. Simulation and experimental results demonstrate that the Complex Double Density Dual Tree DWT (CDDDTDWT) noise removal transform outperform in comparison with others transform and it is particularly effective for highly corrupted image.

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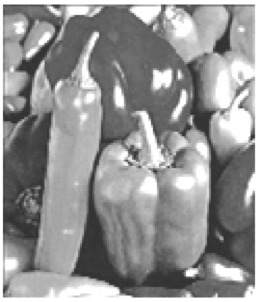
RESULTS

The results of all the 3 algorithms ,Standard Double Density DWT (SDDDWT), Real Double Density Dual Tree (RDDDTDWT) and Complex Double Density Dual Tree DWT (CDDDTDWT) when those are applied to denoise the 8-bit images having the same amount of random noise are given below.

Image1:

Original Image



noisy Image,PSNR=21.4904dB

Double-Density dual tree Method,PSNR=26.817dB

Double-Density Dual-Tree Real Method,PSNR=28.259dB

Double-Density Dual-Tree Complex Method,PSNR=29.3031dB

Image 2:

Original Image

noisy Image,PSNR=22.9494dB

Double-Density dual tree Method,PSNR=27.0931dB

Double-Density Dual-Tree Real Method,PSNR=27.6692dB

Double-Density Dual-Tree Complex Method,PSNR=27.7468dB

Image 3:

Original Image

noisy Image,PSNR=21.8258dB

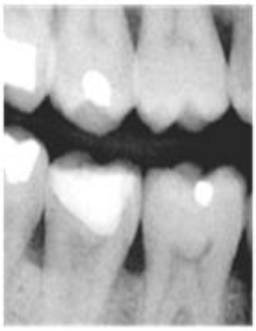
Double-Density dual tree Method,PSNR=27.3337dB

Double-Density Dual-Tree Real Method,PSNR=28.6759dB

Double-Density Dual-Tree Complex Method,PSNR=29.6901dB

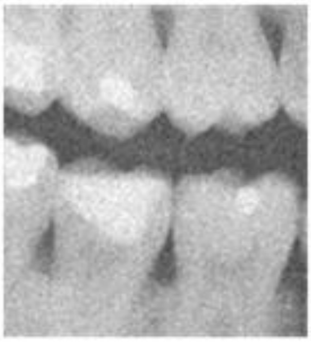
Image 4:

Original Image

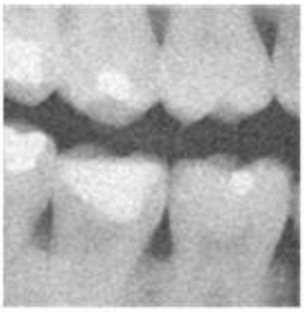


noisy Image,PSNR=19.2518dB

Double-Density dual tree Method,PSNR=23.6128dB



Double-Density Dual-Tree Real Method,PSNR=25.1515dE



Double-Density Dual-Tree Complex Method,PSNR=26.3608d

Image 5:

Original Image

noisy Image,PSNR=22.9694dB

Double-Density dual tree Method,PSNR=28.0677dB

Double-Density Dual-Tree Real Method,PSNR=28.6451dB

Double-Density Dual-Tree Complex Method,PSNR=28.9643dB

Performance tables

Image	Noisy image PSNR (dB)	Double Density dual tree method PSNR (dB)	Double Density dual tree real method PSNR (dB)	Double Density dual tree complex method PSNR (dB)
Image1	21.4904	26.817	28.259	29.3031
Image2	22.9494	27.0931	27.6692	27.7468
Image3	21.8258	27.3337	28.6759	29.6901
Image4	19.2518	23.6128	25.1515	26.3608
Image5	22.9694	28.0677	28.6451	28.9643

V. CONCLUSION

This paper highlighted wavelet based enhancement of gray scale digital images corrupted by random noise. In this study we have evaluated and compared the performances of wavelet transforms. The complex double density dual tree discrete wavelet transform (CDDDTDWT) outperforms in comparison with others wavelet transform in the highly corrupted images. In terms of image enhancement, the double-density complex wavelet transform performed much better at suppressing noise over the double-density wavelet transform. However, to improve the performance further it is necessary to use a different threshold for each subband because for this transform the wavelets associated with different subbands have different norms. The simulation results indicate that the complex double density dual tree discrete wavelet transform performances better than others wavelet transform.

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