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ENERGY BASED DISCRIMINATOR: SOME ISSUES

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Abstract: This paper introduces one of the most interesting energy operators, Teager Energy Operator in a new application in the context of signal processing. Modfied energy separation algorithm is proposed in this paper for the distortionless demodulation of the FM signals. This method is capable of estimating the distortion that results when an existing TEO method of FM signal demodulation is used. The simulation results are incorporated in support of the analytical analysis.

***Index Terms*—** TEO, DEO, ESA, Frequency Modulation, IQ signals, Harmonic Distortion.

I. Introduction

WITH the advancement of technology, the signal processing has become a very important operation in every field of engineering application. There are a lot of new techniques has been incorporated in the field of signal processing. One of them is Teager Energy Operator (TEO). The Teager Energy Operator is very attractive because of its simple definition, easy to implement and seems to be very powerful in certain cases of signal processing.

We start with a basic problem of communication engineering. In communication we transmit message signal(s) by the help of modulation. In modulation we first choose a high frequency signal called a carrier signal to modulate either amplitude or phase or frequency of the message signal and the message is transmitted by the carrier signal over a large distance. The purpose of the modulation is to transmit the message signal effectively over a communication channel by suitable modulation. At the receiving side the purpose of the receiver is to operate on the received signal to reconstruct a recognizable form of the original message signal. During the reconstruction of the message signal the demodulation is started by the proper tuning of the carrier signal in coherent reception. This starting step requires an precise information of the carrier signal which is tactically assumed that the transmitted frequency is known at the receiver side. Now the question arises if the receiver does not have the proper information of the carrier signal frequency is it still possible to carry out the demodulation process? This important question shows the importance of the so called TEO in the field of the communication where we shall see the TEO will be enable us to carry out the demodulation process without knowing the carrier signal. Moreover the TEO method of demodulation will lead us to several advantages which are not possible in the case of the conventional demodulation methods. Proper choice of the TEO based algorithm not only gives better result as compared to the conventional demodulation process. It is possible to design several new application of TEO in a signal processing. In this paper we shall incorporate several new applications of the TEO in communication systems as well as in the signal processing.

II. Definition of Teager Energy Operator

Basically the Teager Energy Operator is a differential energy operator (DEO) which can be defined for a harmonically vibrating signal x(t) as,

$$\psi(x(t)) = \left[\dot{x}(t)\right]^2 - \left[x(t)\ddot{x}(t)\right] \qquad (1)$$

And also in the discrete signal x[n] as,

$$\psi\left[x[n]\right] = x^2[n] - x[n-1]x[n+1] \qquad (2)$$

From the context of the differential energy operators we can define the discrete time TEO [4] as,

$$\left[x, \dot{x}\right] = \left(\dot{x}\right)^2 - x\ddot{x} = \psi(x) \qquad (3)$$

Before we proceed further we have to note the major advantages of the Teager Energy Operator. Some of the important advantages of the TEO are:

a. The TEO is very easy to implement in the circuit. Also we can use it as a core element which can be easily be fabricated in the integrated circuits (ICs).
b. The TEO has a very fast response to the input signal which makes it suitable for faster data reception in the communication systems. It is very important that the TEO is capable of the faster data response as the many conventional communication techniques.
c. The TEO is robust as it is free of division operation [4].
d. The TEO is basically nothing but a differential energy operator (DEO).

More properties of the TEO can be found in [2, 4, 6]. We now concentrate on the new proposed applications of the TEO.

III. Modified Energy Separation Algorithm (MESA)

A. Basics of FM signal

The frequency modulated signals or FM signals are those where information or message is transmitted with the help of a constant amplitude but varying frequency carrier signal. In this case, the spectral components in the modulated waveform depend on the amplitude as well as the frequency of the spectral components in the baseband signal. Consequently the modulation system is not linear and the superposition theorem is not applicable too.

An FM signal can be represented as

$$x_{FM}(t) = A\cos(\phi(t))$$

Where,

$$\phi(t) = \omega_c t + \omega_m \int_0^t m(\tau)\, d\tau \qquad (4)$$

The baseband signal and corresponding FM signal is shown in Fig 1.

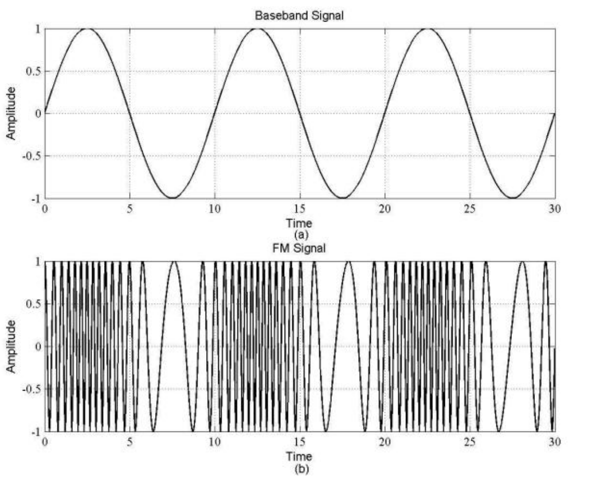


Figure 1: Frequency modulation. (a) Baseband or modulating signal (b) Frequency modulated signal with a sinusoidal carrier.

B. Theory of the demodulation of the FM signal

The conventional TEO based FM discriminator uses a single TEO to demodulate the incoming FM signal. But as per as the theory this demodulation process always have some distortions in the demodulated signal. We have proposed a new algorithm utilizing the two TEO to get rid of the distortions in the demodulated signal. We shall analyze it from the theory.

The basic demodulation scheme using TEO is well discussed in [5, 6, 8, and 9]. We have the TEO output for the FM signal given by,

$$\psi\left(x_{FM}(t)\right) = \psi\left(A\cos\left(\varphi(t)\right)\right)$$

$$= A^2 \dot{\varphi}^2(t) + A^2 \cos\left(\varphi(t)\right)\sin\left(\varphi(t)\right)\ddot{\varphi}(t) \qquad (5)$$

But in those cases distortions in the demodulated message signal appears. In order to remove the distortion at the output stage of the TEO demodulator, the incoming signal is converted to an in-phase (I) and quadrature (Q) components using a local IQ oscillator and a suitable filter. Although the received signal can be demodulated by only IQ outputs [See Appendix I] but the carrier information and other information of the received signal cannot be retrieved. Also the use of TEO ensures some noise suppression over the received signal. Now theoretically we can write for the In-phase FM signal,

$$\psi\left(x_I(t)\right) = \psi\left(A\cos\left(\phi(t)\right)\right)$$
$$= A^2 \dot{\varphi}^2(t) + A^2 \ddot{\varphi}(t)\frac{\sin\left(2\varphi(t)\right)}{2} \qquad (6)$$

And similarly for the Quadrature-phase FM signal we have

$$\psi\left[x_Q(t)\right] = \psi\left[A\sin\left(\varphi(t)\right)\right]$$
$$= A^2 \dot{\varphi}^2(t) - A^2 \ddot{\varphi}(t)\frac{\sin\left(2\varphi(t)\right)}{2} \qquad (7)$$

Adding (6) and (7) we have,

$$\frac{1}{2}\left[\psi\left(x_I(t)\right) + \psi\left(x_Q(t)\right)\right] = A^2 \dot{\phi}^2(t) \qquad (8)$$

Thus by using a square extractor we can easily determine the baseband signal. The theory of this demodulating scheme using dual TEO is a simple one and is different from [5, 6, 8, 9, 12].

C. Carrier Frequency Estimation

The carrier frequency of the FM signal can also be easily determined which leads us to another added advantage of using TEO demodulator over conventional demodulator.

We can approximately write that,

$$\dot{x}_{FM}(t) \cong -A\omega_c \sin(\phi(t)) \qquad (9)$$

Again we have,

$$\psi\left(\dot{x}_I\right) = A^2\omega_c^2\left[\dot{\phi}^2 - \ddot{\phi}\frac{\sin 2\phi}{2}\right] \qquad (10)$$

And, $$\psi\left(\dot{x}_Q\right) = A^2\omega_c^2\left[\dot{\phi}^2 + \ddot{\phi}\frac{\sin 2\phi}{2}\right] \qquad (11)$$

Hence we have from (11) and (11),

$$\psi\left(\dot{x}_I\right)+\psi\left(\dot{x}_Q\right)=\omega_c^2\left[\psi\left(x_I\right)+\psi\left(x_Q\right)\right]$$

$$\text{Or,}\ \omega_C=\sqrt{\frac{\psi\left(\dot{x}_I\right)+\psi\left(\dot{x}_Q\right)}{\psi\left(x_I\right)+\psi\left(x_Q\right)}} \qquad (12)$$

This is the important relationship to determine the carrier frequency of the incoming FM signal.

D. Approximation Error in Carrier Frequency Estimation

This carrier frequency estimation is not totally free from the error. The error is originated due to the approximation error present in (9). We now calculate this approximation error. We know that,

$$\dot{x}_{FM}(t)=-A\sin\left(\varphi(t)\right)\frac{d\varphi}{dt}$$
$$=-A\omega_c\sin\left(\varphi(t)\right)-A\omega_c\omega_m m(t)\sin\left(\varphi(t)\right) \qquad (13)$$

But the approximated relation used is

$$\dot{x}_{FM}(t)\cong-A\omega_c\sin\left(\phi(t)\right) \qquad (13a)$$

Thus from (13) and (13a) the approximation error is given by,

$$E(t)=-A\omega_c\omega_m m(t)\sin\left(\phi(t)\right) \qquad (14)$$

An approximate plot of the approximation error can easily be realized with SIMULINK and here we show the result of that simulation in Fig 2.

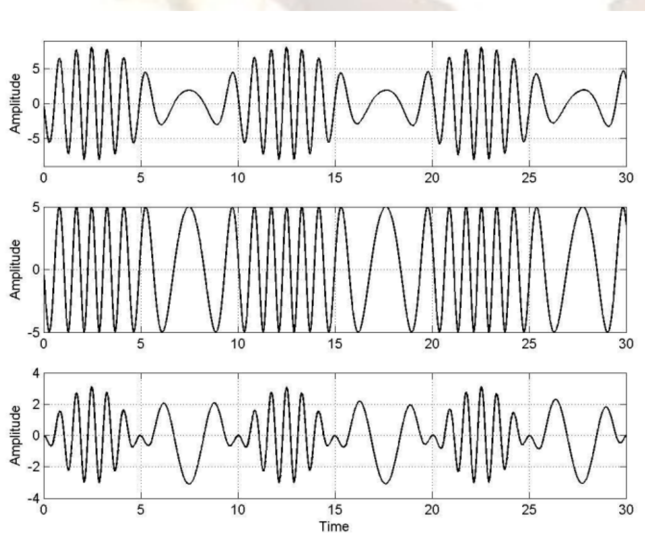


Figure 2: Plot showing the approximation errors. Up: Signal after derivation, Middle: Approximated signal, Bottom: Error due to approximation.

E. SIMULINK Realization of the Proposed Demodulator

The SIMULINK realization of the proposed demodulator using dual TEO is shown in Fig 3. Here the incoming signal is splitted into In-phase and Quadrature phase by a local IQ oscillator (See Appendix I for detailed study of IQ oscillator). The I-Q FM signals (Fig. 4) are then applied to two separate TEO as depicted in the theory and added and find out the square-root of the added signal to recover the distortion less demodulated baseband signal.

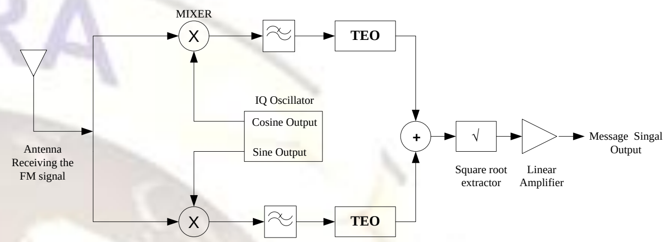


Figure 3: Block diagram of proposed dual TEO FM demodulator.

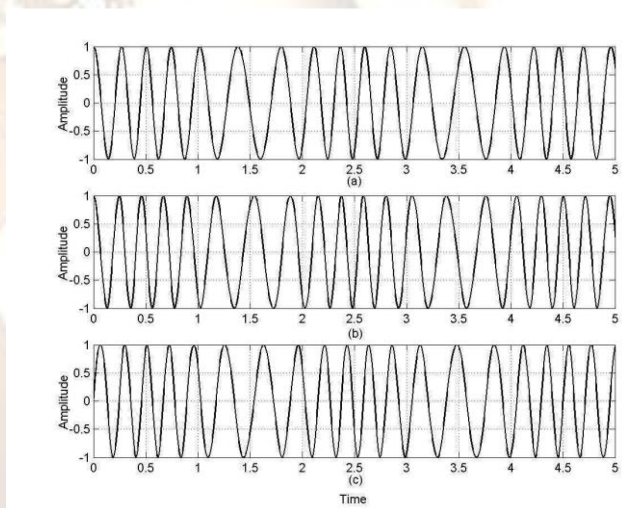


Figure 4: Generated FM signals. (a) Original signal after reception. (b) Generated Inphase signal. (c) Generated Quadrature signal.

IV. Estimation Of Harmonic Distortion Using The Proposed Dual TEO Demodulator

A. Theory

One of the most important advantages of using dual TEO demodulator is that it enables us to measure the distortion arises when we use single TEO based demodulator instead of the proposed demodulator. From (5) we have the output of TEO for FM signals as,

$$\psi\left(x_{FM}(t)\right)=A^2\dot{\phi}^2(t)+A^2\cos\left(\phi(t)\right)\sin\left(\phi(t)\right)\ddot{\phi}(t)$$

Here the desired part for recovering the baseband signal is the first part of the above expression while the last part of the above expression is the error term. Now it is interesting to note that the overall expression is a sum of constant term and an odd function. Hence instead of adding the two expressions (6) and (8) we simply subtract them and we get,

$$\frac{1}{2}\left(\psi[x_I(t)]-\psi[x_Q(t)]\right)=A^2\,\ddot{\phi}(t)\,\frac{\sin(2\phi(t))}{2} \qquad (15)$$

This simple modification leads us to measure the harmonic distortion. The measurement of harmonic distortion with changing the modulation index is quite important in the case of practical communication systems. This can also be studied with this proposed demodulator.

B. Variation of Harmonic Distortion vs. Modulation Index

As discussed earlier this novel technique leads us to measure the harmonic distortion, now we study the relationship between the harmonic distortion and modulation index.

Consider that the dual TEO demodulator is demodulating a single tone FM signal [3, 5]. Thus our message signal can be expressed as

$$m(t)=\sin(\omega_m t)$$

And thus we have,

$$\phi(t)=\omega_c t+k\omega_m\int_0^t m(\tau)d\tau$$
$$=\omega_c t+k\omega_m\int_0^t \sin(\omega_m t)dt \qquad (16)$$

So we have,

$$\dot{\phi}(t)=\omega_c+k\omega_m\sin\omega_m t$$

And,
$$\ddot{\phi}(t)=k\omega_m^2\cos(\omega_m t) \qquad (17)$$

Since the expression of harmonic distortion has the term $\ddot{\phi}(t)$ so we can say that the harmonic distortion has a linear relationship with the modulation index. A typical plot with data points is shown in Fig 5. Corresponding data are listed in Table-I. The data are obtained by the help of SIMULINK simulation in MATLAB R2009b.

C. Variation of Harmonic Distortion with Baseband Frequency

From (17) it is clear that the harmonic distortion varies directly with the square power of the message signal frequency. Thus there must be a practical trade off such that the value of harmonic distortion lies with in a desired or predefined limit with respect to the baseband signal frequency. A typical study with help of SIMULINK realization of the proposed system is shown Fig. 6 while data corresponding to the variation of harmonic distortion with baseband frequency is listed in Table II.

TABLE I

DATA POINTS FOR PLOTTING MODULATION INDEX VS. DISTORTION

Modulation Index	Measured Harmonic Distortion
0.0	0.00000
1.0	0.03350
2.0	0.06671
3.0	0.10010
4.0	0.13340
5.0	0.16680
6.0	0.20010
7.0	0.23350
8.0	0.26690
9.0	0.30020
10.0	0.33350
11.0	0.36690
12.0	0.40020
13.0	0.43360
14.0	0.46690
15.0	0.50020

The baseband used for this measurement of the harmonic distortion is a sinusoidal signal of 0.5 Hz frequency and amplitude of 2 V peak to peak

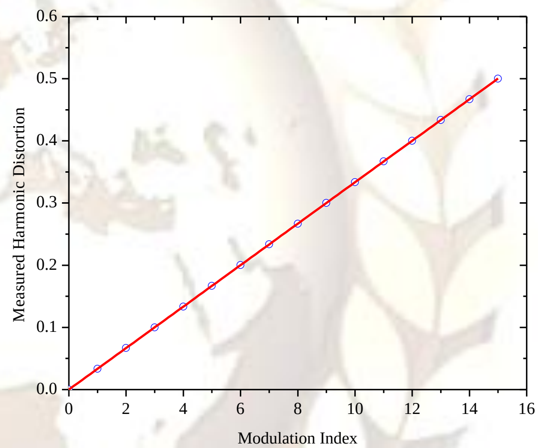


Figure 5: Plot of harmonic distortion with varying modulation index.

TABLE II

DATA POINTS FOR PLOTTING BASEBAND FREQUENCY VS. DISTORTION

Baseband Signal Frequency (rad sec^{-1})	Measured Harmonic Distortion
1.5	0.4812
2.3	0.4815
3.3	0.4835
4.3	0.4921
5.3	0.5023
6.3	0.5174
7.3	0.5394
8.3	0.5655

As suggested in the theory the harmonic distortion measurement can be easily done by the replacing the adder by a subtractor as shown in Fig 7.

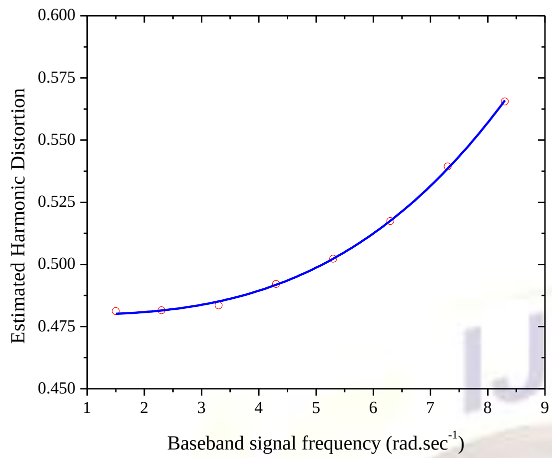


Figure 6: Plot showing the relationship between the baseband frequency and estimated harmonic distortion.

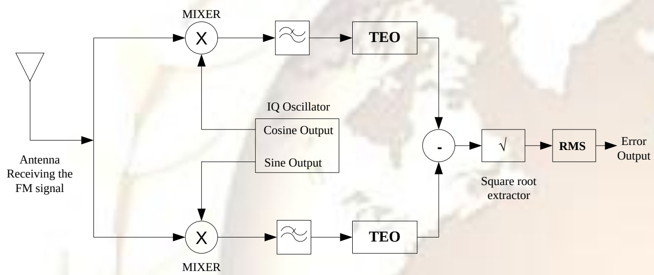


Figure 7: SIMULINK realization of harmonic distortion measurement scheme.

D. Effect of the offset error in the proposed algorithm

In the previous sections we donot include the error that may arise due to offset in the generated IQ signals at the receiver end. We now analyze the entire algorithm in the presence of the offset at the generated IQ FM signal.

Thus we can write that,

$$\left.\begin{aligned} y_I &= A\cos(\phi(t)+k) \\ y_Q &= A\sin(\phi(t)) \end{aligned}\right\} \qquad (17)$$

Thus we have,

$$\psi\left(y_I\left(t\right)\right) = A^2\dot{\phi}^2\left(t\right) + A^2\ddot{\phi}\left(t\right)\frac{\sin(2\phi(t)+k)}{2} \qquad (18)$$

And similarly,

$$\psi\left(y_Q\left(t\right)\right) = A^2\dot{\phi}^2\left(t\right) - A^2\ddot{\phi}\left(t\right)\frac{\sin(2\phi(t))}{2} \qquad (19)$$

Hence,

$$\psi\left[y_I\left(t\right)\right] + \psi\left[y_Q\left(t\right)\right] = 2A^2\dot{\phi}^2\left(t\right) - \frac{A^2}{2}\ddot{\phi}\left(t\right)[\sin 2\phi(t) - \sin(2\phi(t))] \qquad (20)$$

and,

$$\psi\left(y_I\left(t\right)\right) - \psi\left(y_Q\left(t\right)\right) = \frac{A^2}{2}\ddot{\phi}\left(t\right)[\sin 2\phi(t) + \sin(2\phi(t))] \qquad (21)$$

Thus clearly from the equation (20) and (21) the presence of the offset will create distortion in the demodulated signal. The simulation results are incorporated in the section E in support of the theory.

E. Simulation results showing the effects of the offset error

The simulation is again done with SIMULINK in support of MATLAB R2009b. A detailed comparison of the output of the FM detector in presence of the offset and with offset error of 0.5 radians in shown in the Fig 9. The corresponding data for distortion at various frequencies with the presence offset listed in the Table III and Table IV respectively. The corresponding graph is shown in the Fig 10.

TABLE III

DATA POINTS FOR PLOTTING SIGNAL FREQUENCY VS DISTORTION WITHOUT OFFSET ERROR

Signal Frequency (Hz)	Measured Harmonic Distortion
0.2	2.327
0.4	4.136
0.6	6.050
0.8	7.990
1.0	9.948

TABLE IV

DATA POINTS FOR PLOTTING SIGNAL FREQUENCY VS DISTORTION WITH OFFSET VALUE OF 0.5 RADIANS

Signal Frequency (Hz)	Measured Harmonic Distortion
0.2	67.0554
0.4	67.1225
0.6	67.2341
0.8	67.3901
1.0	67.0554

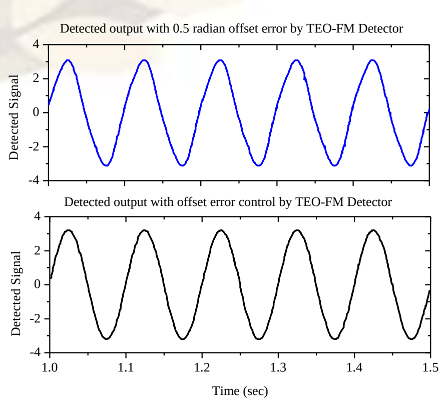
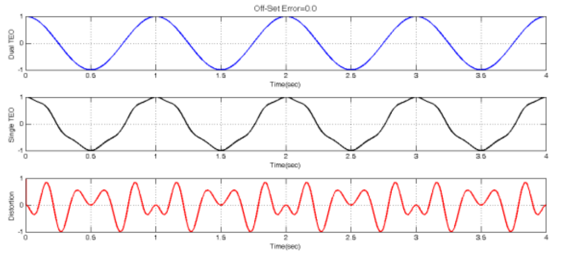
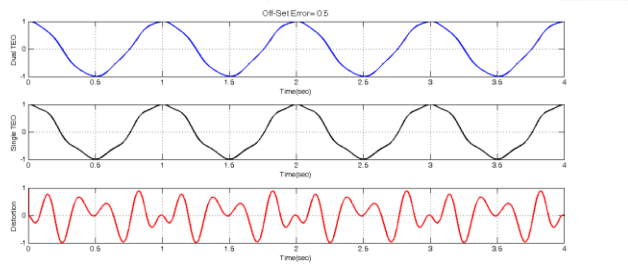


Figure 8: TEO FM detector output with offset error of 0.5 radians.



(a)



(b)

Figure 9: Comparison of TEO FM detector output. (a) without offset (b) without an offset value of 0.5 radians.

The effect of variation of harmonic distortion with the amplitude of the signal is also studied and it is shown in the Fig 11.

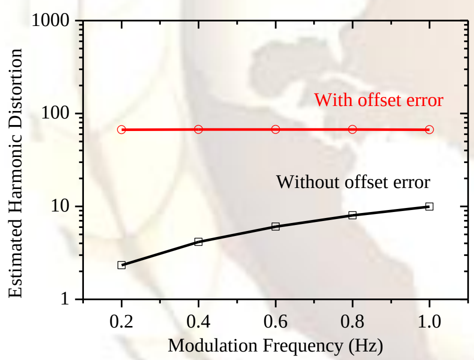


Figure 10: Study of the offset error in the algorithm. Data are listed in Table III and IV.

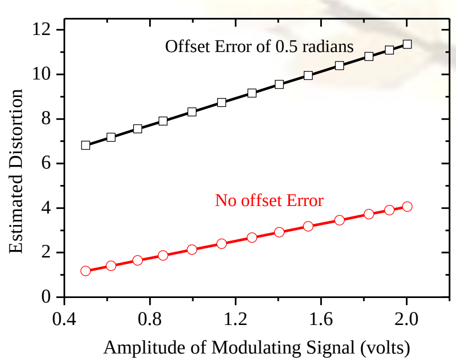


Figure 11: Study of the variation of the harmonic distortion with and without presence of the offset error.

Appendix

IQ OSCILLATORS

In the modern days the IQ oscillator have drawn a lot of attention as the field of communication based applications as there are a lot of advantages of using IQ oscillator instead of conventional oscillators. Modern transceivers requires an local quadrature oscillators as this quadrature signal is essentially required for the complex signal and down conversions and also used in image rejection. Also the modern communication circuits use IQ quadrature generators for efficient modulation and demodulations of the signals. There are a lot of techniques to generate this so called quadrature signal. Here we shall discuss only the SIMULINK realization of the IQ oscillator which is extensively used in the proposed TEO based demodulator.

This realization is done on very basic theory of differential equations. Consider two sinusoidal signals are expressed as,

$$\left.\begin{aligned} a_1(t) &= B\cos\omega t \\ a_2(t) &= B\sin\omega t \end{aligned}\right\} \tag{22}$$

Here B is the amplitude of oscillator output and ω is the oscillator output signal frequency. To generate these signals we have make a phase difference of 90°. But this is not practically possible to realize such an oscillator in the conventional phase shift methods. From (22) we have

$$\left.\begin{aligned} \dot{a}_1(t) &= -B\omega\sin\omega t = -\omega a_2(t) \\ \dot{a}_2(t) &= -B\omega\cos\omega t = -\omega a_1(t) \end{aligned}\right\} \tag{23}$$

Hence from (23) we can realize the IQ oscillator. The realization of IQ oscillator in SIMULINK and its outputs are also shown in Fig 12. Now we can generate phase parts of the incoming signal i.e. we can generate the in phase and quadrature parts. The output of the realized SIMULINK based IQ oscillator is shown in Fig 13.

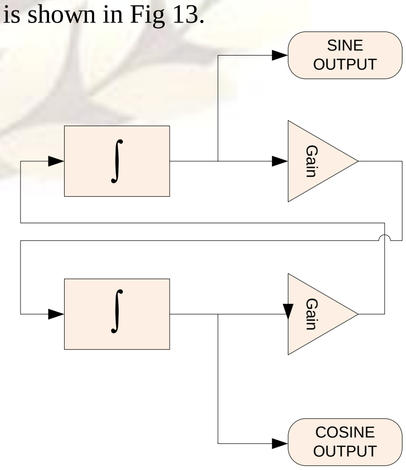


Figure 12: The SIMULINK realization of IQ oscillator.

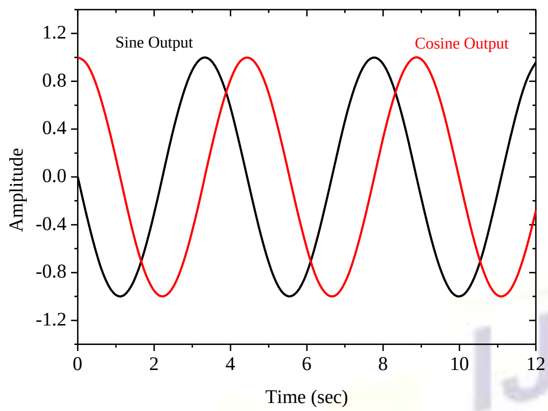


Figure 13: Output of the SIMULINK realized IQ oscillator.

Conclusion

In this paper we have proposed a novel energy separation algorithm for the distortionless demodulation of the FM signals. Simulation results verify the drastic improvement over the conventional algorithm like CESA, alternative CESA for the FM signals, however the effect of the offset still can be removed which is common in all practical oscillators. Thus we can utilize this MESA algorithm in the field of the communication engineering.

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