Rajpreet Singh, Amandeep Singh Sappal / International Journal of Engineering Research and Applications (IJERA) ISSN: 2248-9622 www.ijera.com
Vol. 2, Issue 3, May-Jun 2012, pp.1706-1710

Analytical Investigations on Cross-Coupled Passband Adaptive Equalizer for a 16-QAM Software Defined Radio Receiver

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Abstract- Quadrature Amplitude Modulation is widely used modulation format in software defined radios. The major problem in QAM scheme is the recovery of a quality signal at the receiving end. The purpose of the adaptive equalizer is to remove the intersymbol interference caused by the amplitude and phase distortions of the channel. In this paper Cross-Coupled Passband Adaptive Equalizer has been used for recovering the 16- QAM signal.

Key Words- Adaptive Equalizer, Cross-Coupled Passband Adaptive Equalizer, Equalizer Tap Values, Quadrature Amplitude Modulation. Software Defined Radio.

I. INTRODUCTION

Quadrature Amplitude Modulation (QAM) has become the most preferred modulation mechanism for high speed digital communication. QAM is widely used in the wireless 802.11 protocols to ADSL modems to personal communicators for the military. QAM has become a necessary part of our high speed digital systems. The major problem in QAM scheme is the recovery of a quality signal. An important milestone in high speed data transmission over narrow band channels was the invention and commercialization of the FIR adaptive equalizer by R.W. Lucky at AT&T Bell Laboratories in the early 1960's [1]. The purpose of the adaptive equalizer is to remove the intersymbol interference caused by the amplitude and phase distortions of the channel. Adaptive filters are used as the frequency response of the channel is not known accurately in many situations. Lucky's original equalizers used the zero forcing algorithm. It was soon replaced this algorithm by Widrow's [2], more powerful least-mean-square (LMS) algorithm. Most part of least-mean-square (LMS) algorithm used in this paper has been referred from [3]. The remarkable advances in VLSI technology has led to ever more powerful DSP's which allow complex algorithms to be implemented very inexpensively.

In this paper, we will analyze the complex cross-coupled passband equalizer and implement the LMS equalizer adjustment algorithm. This paper has been organised into three sections. Section-I is introduction, the design and investigations of complex cross-coupled passband adaptive equalizer has been disscused in section-II and section-III is conclusion.

II. THE COMPLEX CROSS-COUPLED PASSBAND ADAPTIVE EQUALIZER

The complex cross-coupled passband adaptive equalizer is shown in Figure-1. It operates on samples of the pre-envelope of the received signal. The sequence $r_+(nT/n_1)$ is given as the input to the equalizer. This sequence is obtained by evaluating the output of the Hilbert transform filter in the receiver front end at the desired times. Therefore, the equalizer operates on samples taken at the rate $f_1 = n_1 f_s$ where $f_s = 1/T$ is the symbol rate. The blocks containing $z^{-1/n}{}_1$ in the figure-1 represent complex signal delays of $T_1 = T/n_1$. The spectrum of the QAM pre-envelope is confined to the positive frequency interval $[f_s - 0.5(1+\alpha)f_s \le f \le f_c + 0.5(1+\alpha)f_s]$ where f_c is the carrier frequency and α is the excess bandwidth factor. To prevent the aliasing of the pre-envelope appropriate value of integer n_1 should be chosen. As $0 \le \alpha \le 1$, n_1 must be greater than or equal to 2 so that no aliasing occurs. In this paper, $n_1 = 2$ is used which generates a structure that is commonly known as *fractionally spaced equalizer*. The performance of fractionally spaced equalizers is better[3]. They can compensate for any fixed symbol clock timing phase offset and can act as interpolators.

The output of equalizer at time nT/n_1 is given by

$$\sigma_+(nT/n_1) = \sum_{k=0}^{N-1} h_k r + ((n-k)T/n_1) \tag{1}$$

The coefficients $h_0, \ldots, h_{N-1}$ of the equalizer are complex in nature and these coefficients are commonly called the equalizer tap values. It will be shown in this paper how to adaptively adjust the tap values which leads to minimization of ISI. For Adjusting the Equalizer Tap Values, LMS Method[3] is used. The symbol spaced sequence $\sigma_+(nT)$ generated by the Down Sampler is given by

$$\sigma_+(nT) = \sum_{k=0}^{N-1} h_k r + \left(nT - k\frac{T}{n_1} \right) \qquad (2)$$

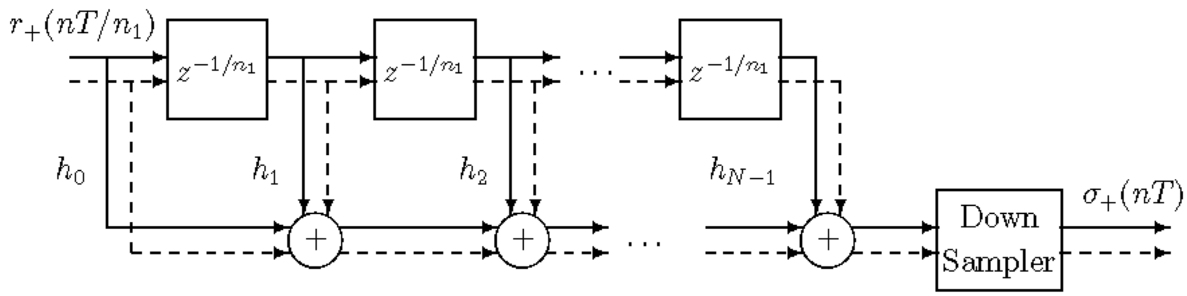

Figure-1: The Complex Cross-Coupled Passband Equalizer

The samples generated by equalizer are demodulated to baseband using the carrier angle ϕ_n generated by the carrier tracking system. The demodulated samples are given by

$$\tilde{\sigma}(nT) = \sigma_+(nT)e^{-j\varphi_n} \qquad (3)$$

The task of the equalizer is to make the baseband output samples as close as possible to a delayed version c_{n-nd} of the transmitted input symbol sequence. For a perfect channel, this can be achieved by setting h_{nd} to 1 and all other taps to 0. Time reference for the receiver can be effectively adjusted by the choice of n_d. The symbol at tap n_d is considered to be the current received symbol. The time reference n_d is usually selected to be near the center of the equalizer delay line. Thus, the equalizer can be thought of as a non-causal system where the taps before n_d operate on future samples and the taps after n_d operate on past samples. With real telephone lines, it has been found experimentally [3] that n_d should be chosen to be closer to $N - 1$ than 0. The optimum placement depends on the channel frequency response.

The mean-squared baseband or passband error can be minimized by choosing the optimum equalizer tap values and we have a mathematically tractable criterion for selecting the equalizer tap values.

The instantaneous baseband error present is given by

$$\tilde{\varepsilon}(nT) = c_{n-n_d} - \tilde{\sigma}(nT) \qquad (4)$$

and the instantaneous passband error is given by

$$\varepsilon_+(nT) = \tilde{\varepsilon}(nT)e^{j\varphi_n} = [c_{n-n_d} - \tilde{\sigma}(nT)]e^{j\varphi_n} = c_{n-n_d}e^{j\varphi_n} - \sigma_+(nT) \qquad (5)$$

The mean-squared error to be minimized is given by

$$\Lambda = E\left\{ \left| \tilde{\varepsilon}(nT) \right|^2 \right\} = E\left\{ \left| \varepsilon_+(nT) \right|^2 \right\} = E\left\{ \left| c_{n-n_d} - \tilde{\sigma}(nT) \right|^2 \right\} \qquad (6)$$

where E denotes statistical expectation. Similar results are obtained when E is thought of as a sum over n. Let the complex tap values have the representation

$$h_k = h_{R,k} + jh_{I,k} \quad where \quad h_{R,k} = \mathrm{Re}\{h_k\} \quad and \quad h_{I,k} = \mathrm{Im}\{h_k\} \qquad (7)$$

The optimum coefficients can be found by setting the derivatives of Λ with respect to the tap value components equal to zero. Since the mean-squared error is a quadratic function of the tap components, the error function is convex and a unique solution exists. The derivative with respect of $h_{R,m}$ is

$$\frac{\partial \Lambda}{\partial h_{R,m}} = -2E\left\{ \mathrm{Re}\left[\tilde{\varepsilon}(nT)e^{j\varphi_n}\overline{r_+(nT - mT/n_1)} \right] \right\} \qquad (8)$$

In terms of the passband instantaneous error, (8) can be written as

$$\frac{\partial \Lambda}{\partial h_{R,m}} = -2E\left\{ \mathrm{Re}\left[\varepsilon_+(nT)\overline{r_+(nT - mT/n_1)} \right] \right\} \qquad (9)$$

Similarly, the expression given below can be found

$$\frac{\partial \Lambda}{\partial h_{I,m}} = -2E\left\{ \mathrm{Im}\left[\varepsilon_+(nT)\overline{r_+(nT - mT/n_1)} \right] \right\} \tag{10}$$

Let the "derivative" with respect to the complex tap value h_m be defined as

$$\frac{\partial \Lambda}{\partial h_m} \overset{\Delta}{=} \frac{\partial \Lambda}{\partial h_{R,m}} + j\frac{\partial \Lambda}{\partial h_{I,m}} = -2E\left\{ \varepsilon_+(nT)\overline{r_+(nT - mT/n_1)} \right\} \tag{11}$$

This derivative with respect to tap h_m is proportional to the average of the product of the instantaneous passband error $\varepsilon_+(nT)$ and the complex conjugate of the passband data sample $r_+(nT - mT/n_1)$ sitting at tap m at time nT.
The optimum equalizer tap values must satisfy the equations

$$\frac{\partial \Lambda}{\partial h_m} = -2E\left\{ \varepsilon_+(nT)\overline{r_+(nT - mT/n_1)} \right\} = 0 \quad for \quad m = 0, \ldots., N-1 \tag{12}$$

Substituting (5) for the passband error and rearranging yields the set of equations

$$\sum_{k=0}^{N-1} h_k E\left\{ r_+(nT - kT/n_1)\overline{r_+(nT - kT/n_1)} \right\} = E\left\{ c_{n-n_d} e^{j\varphi_n} \overline{r_+(nT - mT/n_1)} \right\} \quad \text{for } m=0,\ldots,N-1 \tag{13}$$

These are called the normal equations in estimation theory and the fact that the error sequence must be uncorrelated with the data samples is called the orthogonality principle. Assuming that the channel and baseband symbol sequence information required to compute the expectations is available, this is a set of N linear equations in the N unknown equalizer coefficients. Let the transpose of the N-dimensional coefficient column vector be

$$h^t = \left[h_0, h_1, \ldots\ldots., h_{N-1} \right] \tag{14}$$

The elements of the N×N correlation matrix R for the received samples in the equalizer delay line are given by

$$R_{m,k} = E\left\{ r_+(nT - kT/n_1)\overline{r_+(nT - mT/n_1)} \right\} \quad \text{for } m=0,\ldots,N-1 \tag{15}$$

Also, let the N×1 column vector p of cross-correlations between the desired equalizer output and delay line samples have elements

$$p_{m,1} = E\left\{ c_{n-n_d} e^{j\varphi_n} \overline{r_+(nT - mT/n_1)} \right\} \quad \text{for } m=0,\ldots,N-1 \tag{16}$$

Then, the linear set of equations can be written as the matrix equation

$$Rh = p \tag{17}$$

When R is nonsingular, the solution for the optimum tap values is

$$h = R^{-1}p \tag{18}$$

and it can be derived that the resulting minimum mean-squared error is equal to

$$\Lambda_{min} = E\left\{ \left| c_{n-n_d} \right|^2 \right\} - (\overline{p})^t R^{-1} p \tag{19}$$

In many real world applications, the channel frequency response and noise statistics are known only roughly at the transmitter and receiver. Therefore, the correlation matrices cannot be computed and the optimum tap values cannot be calculated by (18). An adaptive tap adjustment algorithm can solve this problem. Least-mean square (LMS) algorithm is one of the most popular algorithm. The basic philosophy is to iteratively minimize Λ by incrementing the tap values by small amounts in the directions opposite the derivatives given by (11). This is a form of gradient search algorithm known as the method of steepest descent. When the channel is unknown, the expected value required to compute a derivative cannot be evaluated. However, a known training sequence is usually sent at the beginning of transmission, so $\varepsilon_+(nT)$ and $r_+(nT - mT/n_1)$ are known to the receiver and a time average of the products of these quantities can be used as an unbiased estimate of the true expected value. These ideas suggest using the following tap adjustment formula

$$h_m(n+1) = h_m(n) + \mu\varepsilon_+(nT)\overline{r_+(nT - mT/n_1)} \quad \text{for } m=0,\ldots,N-1 \tag{20}$$

where $h_m(n)$ is the value of the m-th tap at time n and μ is a small positive constant. This is the LMS tap adjustment algorithm. The parameter μ controls the speed and smoothness of the convergence of the taps to their optimum values. A large value of μ gives rapid initial convergence but large variations about the theoretically optimum final value because of the small averaging effect. A small value results in slow convergence but small tap variations around the optimum values.

Very large values of μ cause the algorithm to become unstable, while very small values can result in arithmetic underflow which causes the adjustments to stop. Practically, the adaptation is often started with a moderately large value of μ to get rapid initial convergence for a period of time and then "gear shifted" to a small value for precise final adjustment. The block diagram of a section of an adaptive passband equalizer illustrating the LMS algorithm for adjusting one tap is shown in Figure-2.

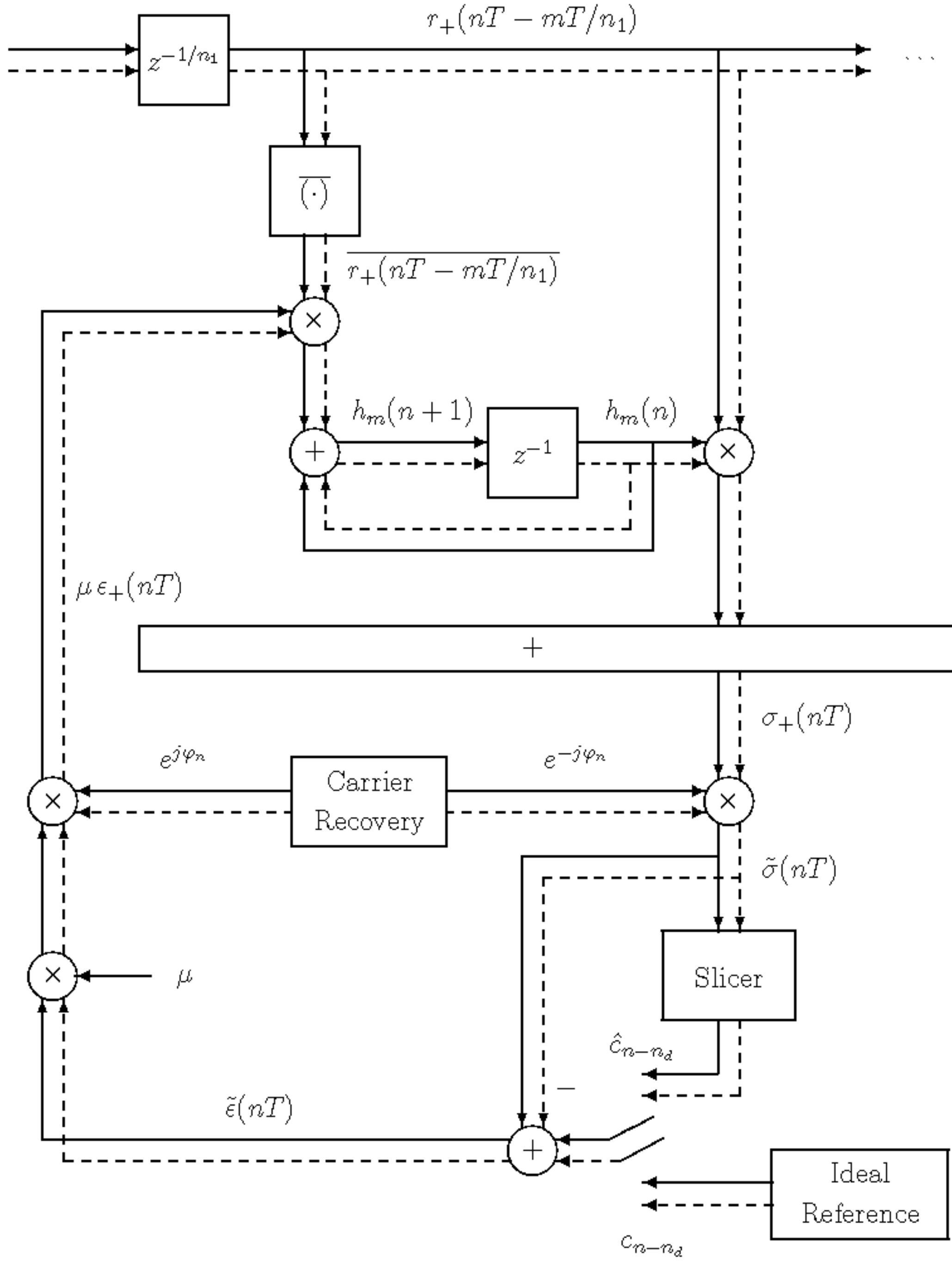

Figure-2: Block Diagram Illustrating the LMS Algorithm in a Passband Equalizer

The outputs of all the tap multipliers are summed in the box labeled "+" to form the passband output signal $\sigma_+(nT)$. The passband output is demodulated to the baseband signal $\tilde{\sigma} = (nT)$ using the angle ϕ_n generated by the carrier tracking system. The Slicer quantizes its input to the closest ideal constellation point. During initial training, a known sequence c_n is transmitted and a delayed version c_{n-nd} is generated in the receiver by the Ideal Reference block. The exact baseband error signal $\tilde{\varepsilon} = (nT)$ can be formed during the initial training period. This error signal is modulated to passband and correlated against the data sample at the tap being adjusted and scaled by μ to form the tap update increment. After the equalizer converges to the point where the baseband output symbols $\tilde{\sigma} = (nT)$ are close to the ideal constellation points, the switch can be moved to the slicer output and $\hat{c}_{n-nd}$ can be used as an accurate estimate of the delayed transmitted symbol sequence. This mode is called decision directed equalization. Decision directed equalization is required in practice because the receiver does not know the random symbol sequence transmitted during normal data transmission. If the majority of decisions are correct, the equalizer will converge because of the averaging effect of a small μ. Infrequent errors cannot move the equalizer taps very far from their optimum values.

III. CONCLUSION

Design of an adaptive Equalizer and optimum adjustment of its tap values is one of the important task in the design of a software defined radio receiver. From the design and investigations of the adaptive Equalizer, it has been concluded that LMS tap adjustment algorithm results in optimum solution and helps in proper recovery of the transmitted signal.

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