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EFFECT OF FLEXURAL-TORSIONAL-DISTORTIONAL INTERACTIONS ON THE BEHAVIOUR OF THIN- WALLED MONO SYMMETRIC BOX GIRDER STRUCTURES

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ABSTRACT

In this paper the differential equations of equilibrium for flexural-torsional-distortional analysis of thin-walled mono symmetric box girder structures were derived using Vlasov's theory. To ensure that the derived equations depict the real life behaviour of mono symmetric box girder structures the interaction between all the strain modes of flexure, torsion and distortion were taken into consideration.

By carrying out analysis using similar equations derived by the authors for flexural-torsional analysis, flexural-distortional analysis and torsional-distortional analysis of mono symmetric box girder frames, the effects of flexural-torsional-distortional strain modes interactions on the behaviour of such structures were established. This involved flexural-torsional analysis, flexural-distortional analysis, torsional-distortional analysis and flexural-torsional-distortional analysis of a mono symmetric box girder frame.

The derived equations are fourth order ordinary differential equations of equilibrium which were integrated by method of trigonometric series with accelerated convergence. The results show that flexural-torsional analysis, flexural-distortional analysis and torsional-distortional analysis have inherent errors due to the negligence of full interactions between the strain modes of flexure, torsion and distortion.

***Keywords*:** Box girder, deformation, distortion, flexure, interaction, mono symmetric, thin-walled, torsion.

1. Introduction

Basically, a curved structural element has two interacting forces: bending and torsion. The study of curved elrments offers only one instance where bending and torsional forces simultaneously ocure. Torsional loads consisting of opposing vertical forces result from gravity loads that are eccentric to the centre line of the girder and they give rise to bending and torsion. Torsional load can be modeled as a uniform (Saint Venant) torsional component and a distortional component. Therefore an eccentric load on a bridge structure introduces interaction between bending, pure torsion and distortion. The flexural-torsional-distortional interactions are of great interest particularly in a thin walled box structure where the geometry of the cross section comes into play.

Research [1] has shown that doubly symmetric sections have only one interaction of strain modes, i.e. torsional strain mode and distortional strain mode interaction. A mono-symmetric section on the other hand, has three strain modes interactions; torsion interacts with distortion and each of these strain modes interacts with flexure about the non axis of symmetry. Thus we have torsional-distortional interaction, flexural-torsional interaction, and flexural-distortional interaction. A non symmetric section has multiple strain modes interaction; each of torsion and distortion interacts with flexure about both axes of non-symmetry in addition to the interaction between themselves. Thus, we have torsional-distortional interaction, torsional-flexural interaction in major and minor axes and flexural-distortional interaction in major and minor axes. These strain modes interactions are quite inseparable that it is not possible to examine only one interaction in isolation of others.

The objective of this study is to derive a set of differential equations of equilibrium governing the flexural-torsoinal-distortional behaviour of thin-walled mono symmetric box girder structures on the basis of Vlasov's theory and to apply the obtained equatons in the analysis of single cell mono symmetric box girder structure to obtain the flexural, torsional and distortional deformations of the girder.

2. Review of Past Work

Recent literatures, Hsu et al [2], Fan and Helwig [3], Sennah and Kennedy [4], on straight and curved box girder bridges deal with analytical formulations to better understand the behaviour of these complex structural systems. Few authors; Okil and El-tawil [5], Sennah and Kennedy [4] have undertaken experimental studies to investigate the accuracy of existing methods. Before the advent of Vlasov's 'theory of thin-walled beams', Vlasov [6], the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure

Several investigators; Bazant and El-Nimeiri [7], Zhang and Lyons [8], Boswell and Zhang [9], Usuki [10], Waldron [11], Paavola [12], Razaqpur and Lui [13], Fu and Hsu [14], Tesar [15], have combined thin-walled beam theory of Vlasov and the finite element technique to develop a thin walled box element for elastic analysis of straight and curved cellular bridges. Osadebe and Chidolue [16], [17], obtained fourth order differential equations of torsional-distortional equilibrium, flexural-torsional equilibrium and flexural distortional equilibrium for the analysis of mono symmetric box girder structures using Vlasov's theory with modifications by Varbanov [18].

Various theories were therefore postulated by different authors examining methods of analysis, both classical and numerical. A few others however carried out tests on prototype models to verify the authenticity of the theories. The authors are of the view that Vlasov's theory captures all peculiarities of cross sectional deformations such as warping, torsion, distortion etc. The theory is therefore adopted in this work .

3. Potential Energy of a Thin-Walled Box Structure

The potential energy of a box structure in terms of the strain energy and the work done by external loads is as follows, Osadebe and Chidolue [16]:

$$\Pi = \frac{E}{2}\sum a_{ij}U_i{}'(x)U_j{}'(x)dx +$$
$$\frac{G}{2}\left[\sum b_{ij}U_i(x)U_j(x) + \sum c_{kj}U_k(x)V_j{}'(x)\right]dx +$$
$$+\frac{G}{2}\left[\sum c_{ih}U_i(x)V_h{}'(x) + \sum r_{kh}V_k{}'(x)V_h{}'(x)\right]dx +$$
$$+\frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx - \sum q_h V_h dx \qquad (1)$$

where Π = the total potential energy of the box structure,

$U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the box girder frame.

q_h = Line load per unit area applied in the plane of the box girder plates

E = Modulus of elasticity

G = Shear modulus

a_{ij}, b_{ij}, c_{kj}, r_{kh}, s_{hk} are Vlasov's coefficients given by the following expressions.

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA \qquad (a)$$
$$b_{ij} = b_{ji} = \int \varphi_i'(s)\varphi_j'(s)dA \qquad (b)$$
$$c_{kj} = c_{jk} = \int \varphi_k'(s)\psi_j(s)dA \qquad (c)$$
$$c_{ih} = c_{hi} = \int \varphi_i'(s)\psi_k(s)dA \qquad (d)$$
$$r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; \qquad (e) \qquad (2)$$
$$s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad (f)$$
$$q_h = \int q\psi_h ds \qquad (g)$$

These coefficients depend on a combination of elementary displacements or strain fields; three in the longitudinal direction and four in the transverse direction. The strain fields are:

φ_1 = out of plane displacement due to vertical load

φ_2 = out of plane displacement due to horizontal load normal to bridge longitudinal/vertical plane,

φ_3 =out of plane displacement due to warping of the cross section,

ψ_1 = in-plane displacement due to vertical load,

ψ_2 in-plane displacement due to horizontal load normal to bridge longitudinal/vertical plane,

ψ_3 in-plane displacement due to distortion of the cross section,

ψ_4 in-plane displacement due to pure rotation,

Some or all of these strain modes may be present in a given frame depending on the geometry of the cross section and the nature of loading.

4. Governing Equations of Equilibrium

The governing equations of flexural-torsional-distortional equilibrium are obtained by minimizing the energy functional eqn .(1), with respect to its functional variables u(x) and v(x) using Euler Lagrange technique, Elgolts[19]

Minimizing with respect to u(x) we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i{}''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k{}'(x) = 0 \qquad (3)$$

Minimizing with respect to v(x) we have;

$$\sum c_{ih} U_i'(x) - \kappa \sum s_{hk} V_k(x) + \sum r_{kh} V_k''(x) + \frac{1}{G}\sum q_h = 0 \quad (4)$$

where, $\kappa = \frac{E}{G} = 2(1+\nu)$ (5)

Equations (3) and (4) are Vlasov's generalized differential equations of flexural-torsional-distortional equilibrium for a box girder. They are presented in matrix form as follows:

$$\kappa \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} U_1'' \\ U_2'' \\ U_3'' \end{Bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} - \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix} \begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{Bmatrix} = 0 \quad (6)$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix} \begin{Bmatrix} U_1' \\ U_2' \\ U_3' \end{Bmatrix} - \kappa \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{41} & s_{43} & s_{44} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} + \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix} \begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4'' \end{Bmatrix} + \frac{1}{G} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0 \quad (7)$$

5. Strain Mode Diagrams and Evaluation of Vlasov's Coefficients

Fig.1a shows the cross section of a single cell mono symmetric box girder structure (regarded as a frame) for which the strain modes diagrams are to be obtained and Vlasov's coefficients computed for the analysis of the frame.

If we assume the normal beam theory, i.e., neutral axis remaining neutral before and after bending, then the displacement φ_1 (strain mode 1) at any distance R, from the centroid of the cross section is given by $\varphi_1 = R\theta$ where θ is the distortion angle. If we assume a unit rotation of the vertical z axis then $\varphi_1 = R$, at any point on the cross section. Thus, φ_1 is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian.

Similarly, if the load is acting in horizontal (y- y) direction, normal to the x-z plane then the bending is in x-z plane and y axis is rotated through angle θ_2 giving rise to φ_2 (strain mode 2) displacement out of plane. The values of φ_2 are obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian. The warping function φ_3 (strain mode 3), of the beam cross section is the out of plane displacement of the cross section when the beam is twisted about its axis through the pole, one radian per unit length without bending in either z or y direction and without longitudinal extension. ψ_1 and ψ_2 are in-plane displacements of the cross section in x-z and x-y planes respectively while ψ_3 is the distortion of the cross section.

In an un published work the authors have shown that these in-plane displacement quantities ψ_1, ψ_2 and ψ_3 are the same as the derivatives of their corresponding out of plane displacements. Consequently, ψ_1, ψ_2 and ψ_3 are obtained by numerical differentiation of φ_1, φ_2 and φ_3 diagrams respectively. Strain mode 4, ψ_4, is the displacement diagram of the beam cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section. ψ_4 is assumed to be positive if the member moves in the positive directions of the coordinate axes and negative otherwise.

Figs.1(b) to 1(h) show the strain modes diagrams for the single cell mono symmetric box girder frame in Fig.1(a). The coefficients $a_{ij}, b_{ij}, c_{kj}, c_{ih}$ and r_{kh}, of the governing equations of equilibrium are computed with the aid of Morh's integral chart using the strain modes diagrams.

6. Flexural-Torsional-Distortional Equilibrium Equations

The relevant coefficients for flexural-torsional-distortional equilibrium are those involving strain modes 2, 3 and 4. These are; a_{32}, a_{33}, b_{32}, b_{33}, c_{32}, c_{33}, c_{34}, c_{42}, r_{22}, r_{32}, r_{33}, r_{34}, r_{42}, s_{33}, and r_{44}. Substituting these into the matrix eqns .(6) and (7) and multiplying out, noting that $a_{ij} = a_{ji}$, $b_{ij} = b_{ji}$, etc, we obtain;

$$ka_{22}U_2'' + ka_{23}U_3'' - b_{22}U_2 - b_{23}U_3 - c_{22}V_2' - c_{23}V_3' - c_{24}V_4' = 0 \quad (8)$$

$$ka_{32}U_2'' + ka_{33}U_3'' - b_{32}U_2 - b_{33}U_3 - c_{32}V_2' - c_{33}V_3' - c_{34}V_4' = 0 \quad (9)$$

$$c_{22}U_2' + c_{23}U_3' + r_{22}V_2'' + r_{23}V_3'' + r_{24}V_4'' = -\frac{q_2}{G} \quad (10)$$

$$c_{32}U_2' + c_{33}U_3' - ks_{33}V_3 + r_{32}V_2'' + r_{33}V_3'' + r_{34}V_4'' = -\frac{q_3}{G} \quad (11)$$

$$c_{42}U_2' + c_{43}U_3' + r_{42}V_2'' + r_{43}V_3'' + r_{44}V_4'' = -\frac{q_4}{G} \quad (12)$$

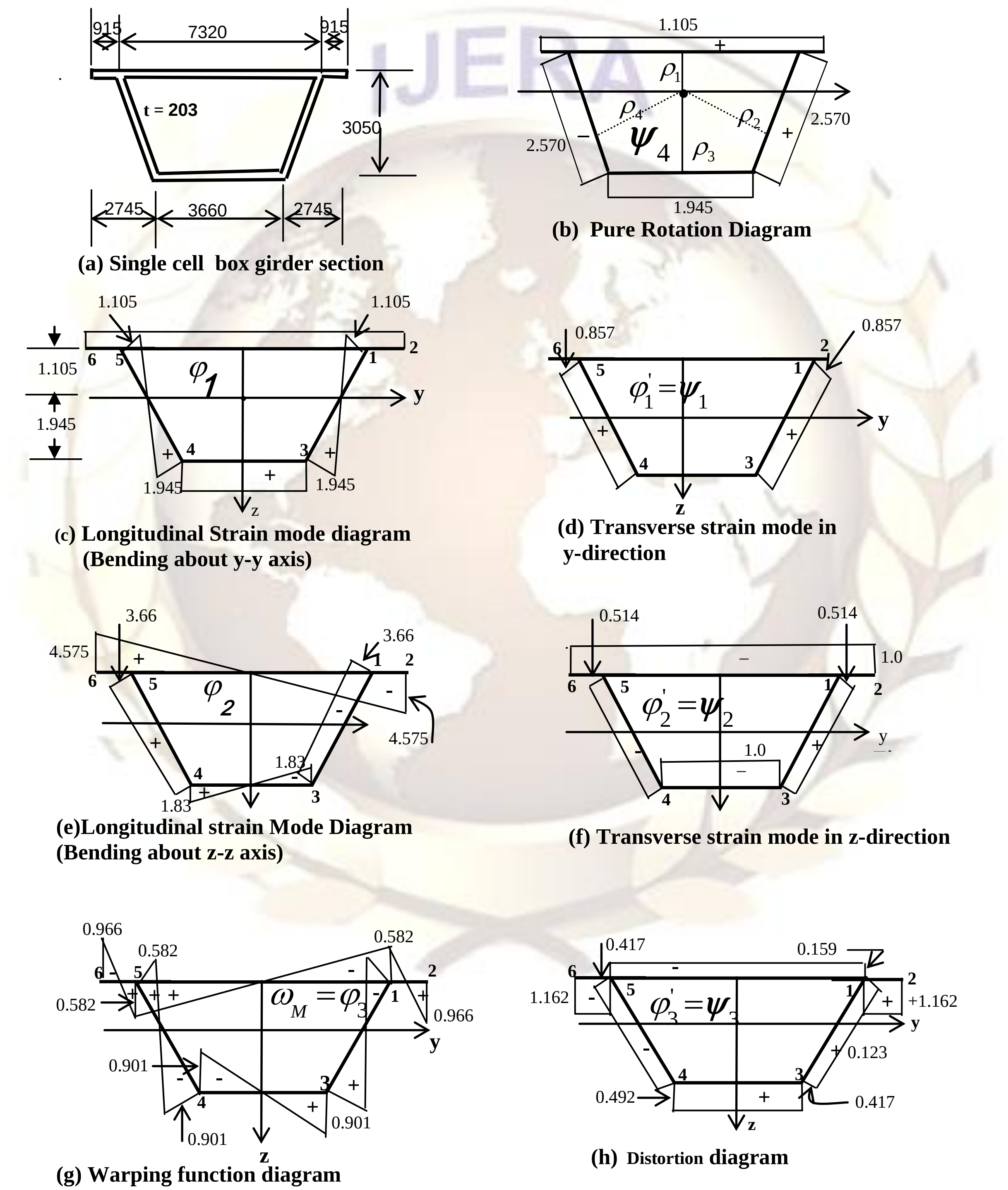


Fig.1 Generalized strain modes for single cell mono-symmetric box girder frame

Simplifying further we obtain the coupled differential equations of flexural-torsional-distortional equilibrium for mono symmetric sections as follows.

$$\beta_4 V_2'' + \beta_5 V_3'' + \beta_6 V_4'' - \gamma_1 V_3 = -K_3 \quad (a)$$

$$\alpha_4 V_2^{iv} + \alpha_5 V_3^{iv} + \alpha_6 V_4^{iv} - \beta_7 V_2'' - \beta_8 V_3'' - \beta_9 V_4'' + \gamma_2 V_3 = K_4 \quad (b) \qquad (13)$$

$$\alpha_7 V_2^{iv} + \alpha_8 V_3^{iv} + \alpha_9 V_4^{iv} - \beta_{10} V_2'' - \beta_{11} V_3'' - \beta_{12} V_4'' + \gamma_3 V_3 = K_5 \quad (c)$$

where

$$\alpha_1 = \left(\frac{r_{32}}{c_{33}} - \frac{r_{22}}{c_{23}}\right) / \left(\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right)$$

$$\alpha_2 = \left(\frac{r_{34}}{c_{33}} - \frac{r_{24}}{c_{23}}\right) / \left(\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right)$$

$$\alpha_3 = Ks_{33} / c_{33}\left(\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right)$$

$$\alpha_4 = ka_{22}\alpha_1;\ \alpha_5 = ka_{23}\beta_1;\ \alpha_6 = k(a_{22}\alpha_2 + a_{23}\beta_3)$$

$$\alpha_7 = ka_{32}\alpha_1;\ \alpha_8 = ka_{33}\beta_1;\ \alpha_9 = k(a_{32}\alpha_2 + a_{33}\beta_3)$$

$$\beta_1 = \left(\frac{r_{33}}{c_{32}} - \frac{r_{23}}{c_{22}}\right) / \left(\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right)$$

$$\beta_2 = \left(\frac{r_{34}}{c_{32}} - \frac{r_{24}}{c_{22}}\right) / \left(\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right)$$

$$\beta_3 = ks_{33} / c_{32}\left(\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right)$$

$$\beta_4 = (c_{42}\alpha_1 + r_{42});\ \beta_5 = (c_{43}\beta_1 + r_{43});$$

$$\beta_6 = (c_{42}\alpha_2 + c_{43}\beta_2 + r_{44});\qquad \beta_7 = (b_{22}\alpha_1 + c_{22})$$

$$\beta_8 = (ka_{23}\beta_3 + b_{23}\beta_1 + k\alpha_{22}\alpha_3 + c_{23});$$

$$\beta_9 = -(b_{22}\alpha_2 + c_{24} + b_{23}\beta_2);\ \beta_{10} = (b_{32}\alpha_1 + c_{32})$$

$$\beta_{11} = (ka_{32}\alpha_3 + b_{33}\beta_1 + k\alpha_{33}\beta_3 + c_{33});$$

$$\beta_{12} = (b_{32}\alpha_2 + c_{34} + b_{33}\beta_2)$$

$$\gamma_1 = (c_{42}\alpha_3 + c_{43}\beta_3);\ \gamma_2 = (b_{22}\alpha_3 + b_{23}\beta_3);$$

$$\gamma_3 = (b_{32}\alpha_3 + b_{33}\beta_3);$$

$$K_1 = \bar{q}_3 / c_{32}G\left(\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right) - \bar{q}_2 / c_{22}G\left(\frac{c_{23}}{c_{22}} - \frac{c_{33}}{c_{32}}\right)$$

$$K_2 = \bar{q}_3 / c_{33}G\left(\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right) - \bar{q}_2 / c_{23}G\left(\frac{c_{22}}{c_{23}} - \frac{c_{32}}{c_{33}}\right)$$

$$K_3 = \bar{q}_4 / G + c_{42}K_2 + C_{43}K_1;\qquad K_4 = b_{22}K_2 + b_{23}K_1;$$

$$K_5 = b_{32}K_2 + b_{33}K_1$$

7. Analysis of Single Cell Mono Symmetric Box Girder Structure

In order to determine the effect of the interaction between flexure, torsion and distortion on the mono symmetric box girder structure, the following analyses were carried out. (a) Flexural- torsional analysis, (b) flexural-distortional analysis, (c) torsional-distortional analysis and (d) flexural-torsional-distortional analysis.

Live loads were considered according to AASHTO-LRFD [19] , following the HL-93 loading: uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A 50m span simply supported bridge deck structure was considered. The obtained torsional loads are $\bar{q}_3 = 157.16KN,\ \ \bar{q}_4 = 1446.505KN$.

The full text on flexural-torsional analysis, flexural-distortional analysis and torsional-distortional analysis of the single cell mono symmetric box girder structure are given by Oadebe and Chidolue in references [21], [22] and [16] respectively. The obtained results are as follows.

7.1 Flexural-Torsional Analysis

The differential equations of flexural-torsional equilibrium for mono symmetric box girder structures are as follows:

$$157.502V_2^{iv} - 915.327V_4^{iv} + 49.910V_4'' = -4.4932*10^{-4} \quad (a) \qquad (14)$$

$$V_4'' = -1.206x10^{-5} \quad (b)$$

Integrating by method of trigonometric series with accelerated convergence we have,

$$V_2(x) = 79.73*10^{-3} Sin\frac{\pi x}{50};$$

$$V_4(x) = 3.055*10^{-3} Sin\frac{\pi x}{50} \qquad (15)$$

7.2 Flexural-Distortional Analysis

The coupled differential equations of flexural-distortional equilibrium for mono symmetric box girder structures are as follows :

$$62.625V_2^{iv} - 0.675V_3^{iv} - 8.2517*10^{-4}V_3'' = 4.47545*10^{-6} (a)$$

$$-0.675V_2^{iv} + 1.893V_3^{iv} - 6.04*10^{-4}V_3'' + 4.5487*10^{-4} = 2.0463*10^{-5} (b) \qquad (16)$$

Integrating by method of trigonometric series with accelerated convergence we have:

$$V_2(x) = 4.272*10^{-3} Sin\frac{\pi x}{50}$$
$$V_3(x) = 4.213*10^{-2} Sin\frac{\pi x}{50} \qquad (17)$$

7.3 Torsional-Distortional Analysis

The coupled differential equations of torsional-distortional equilibrium for mono symmetric box girder structures are as follows:

$$2.371V_3^{iv} + 27.405V_4^{iv} - 18.963V_4'' = 2.120*10^{-6} \quad (a)$$
$$-18.964V_4^{iv} - 5.503*10^{-4}V_3 = 1.9163*10^{-4} \quad (b) \qquad (18)$$

Integrating by method of trigonometric series with accelerated convergence we have

$$V_3(x) = 3.268*10^{-2} Sin\frac{\pi x}{50};$$
$$V_4(x) = 2.80*10^{-3} Sin\frac{\pi x}{50} \qquad (19)$$

7.4 Flexural-Torsional-Distortional Analysis

The governing equations of equilibrium are given by eqns (13) The relevant coefficients are as follows.

$a_{22} = 25.05, a_{23} = -0.270, a_{33} = 0.757,$

$b_{22} = c_{22} = r_{22} = 2.982, \; b_{23} = c_{23} = r_{23} = -0.153$

$b_{33} = c_{33} = r_{33} = 1.407, \; r_{44} = 14.616$

$c_{24} = c_{42} = r_{24} = r_{42} = -2.515$

$c_{34} = c_{43} = r_{34} = r_{43} = 1.265$

$s_{33} = 0.261*6.9712*10^{-4} = 1.8195*10^{-4}$

$\bar{q}_2 = 0.0KN, \bar{q}_3 = 154.58KN, \bar{q}_4 = 1446.505KN$ The coefficients of the governing equations are as follows.

$\alpha_1 = -1; \; \alpha_2 = 0.802; \; \alpha_3 = -1.6679*10^{-5};$

$\alpha_4 = -62.625; \; \alpha_5 = 0.675$

$\alpha_6 = 50.7734; \alpha_7 = 0.675 \; \alpha_8 = -1.893;$

$\alpha_9 = -2.078$

$\beta_1 = -1; \; \beta_2 = -0.812;$

$\beta_3 = -3.251*10^{-4}; \beta_4 = 0; \quad \beta_5 = 0; \; \beta_6 = 11572;$

$\beta_7 = 0; \; \beta_8 = -7.89*10^{-4}; \quad \beta_9 = -1.266;$

$\beta_{10} = 0; \beta_{11} = -5.820*10^{-4}; \; \beta_{12} = -1.90*10^{-4}$

$\gamma_1 = -3.532*10^{-4}; \; \gamma_2 = 4.8*10^{-9}; \; \gamma_3 = -4.350*10^{-4}$

$K_1 = -1.4625*10^{-5}; \; K_2 = -7.505*10^{-7};$

$K_3 = 1.7115*10^{-4}; \; K_4 = -2.123*10^{-6};$

$K_5 = -2.046*10^{-5}$

Substituting the coefficients and the constants into eqn.(13) we obtain:

$$11.572V_4'' + 3.532*10^{-4}V_3 = -1.7115*10^{-4} \quad (a)$$
$$-62.625V_2^{iv} + 0.675V_3^{iv} + 50.773V_4^{iv} + 7.89*10^{-4}V_3$$
$$+1.266V_4'' = -2.123*10^{-6} \quad (b) \qquad (21)$$
$$0.675V_2^{iv} - 1.893V_3^{iv} - 2.078V_4^{iv} + 5.820*10^{-4}V_3'' +$$
$$+1.90*10^{-4}V_4'' - 4.35*10^{-4}V_3 = -2.046*10^{-5} \quad (c)$$

Integrating by method of trigonometric series with accelerated convergence we have,

$$V_2(x) = -3.287*10^{-2} Sin\frac{\pi x}{50};$$
$$V_3(x) = 4.282*10^{-2} Sin\frac{\pi x}{50}; \qquad (22)$$
$$V_4(x) = 4.077*10^{-3} Sin\frac{\pi x}{50}$$

8. Discussion of Results

The results of the analyses of the single cell mono symmetric box girder structure are presented in Figs. 2, 3, 4 and 5. Fig. 2 shows the variation of flexural and torsional displacements along the length of the girder in a situation where flexural and torsional strain modes interact with each other alone. The maximum mid span flexural deformation was 80mm and that of torsional deformation was 3mm. Fig. 3 shows the variation of flexural and distortional displacements along the length of the girder where flexural strain mode interacts with distortional strain mode alone. The maximum (mid-span) flexural and distortional deformations were 4mm and 42mm respectively.

By comparing Figs 2 and 3 we noticed that the effect of the interaction between flexural and distortional strain modes was 95% reduction in the flexural deformation obtained in the case of flexural-torsional analysis.

Fig. 4 shows the variation of torsional and distortional displacements along the length of the girder in a situation where torsional strain mode interacts with distortional strain mode alone. The maximum (mid-span) torsional deformation was 3mm and that of distortional deformation was 33mm. By comparing Fig. 4 with Fig. 2 we observed that the maximum (mid-span) torsional deformation remained relatively unchanged in both flexural-torsional analysis and torsional-distortional analysis. However, by comparing Fig .4 with Fig. 3 we discover that the effect of the interaction between torsional and distortional strain modes was 21% reduction in the distortional deformation obtained for flexural and distortional strain modes interaction.

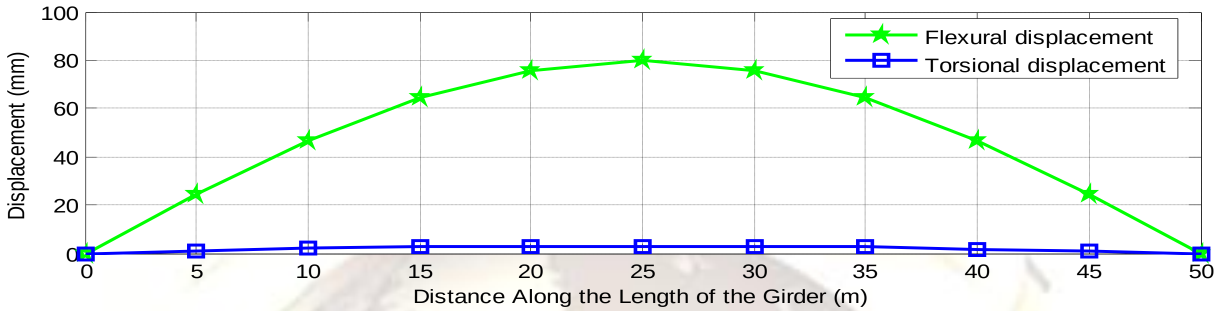


Fig. 2: Variation of flexural and torsional displacements along the length of the girder

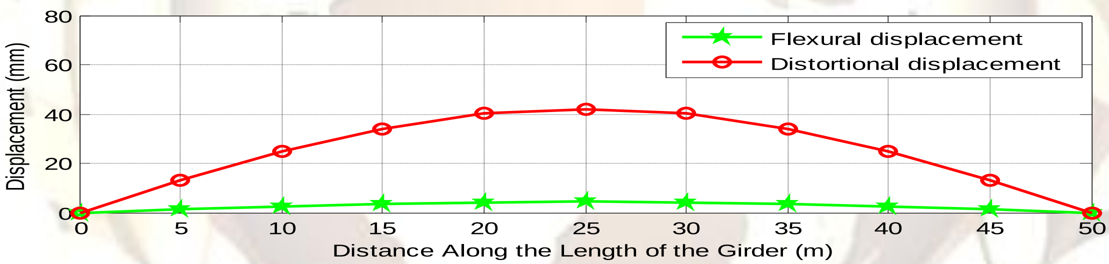


Fig.3 Variation of flexural and distortional displacements along the length of the girder

In a real life situation, flexural-torsional-distortional interactions are inseparable as can be seen from eqns. (13) which are coupled. Fig.5 therefore represents the true behaviour of a real life structure. It shows the variation of flexural displacements, torsional displacements and distortional displacements along the length of the box girder structure. The maximum mid-span flexural deformation was 33mm while those of torsional and distortional deformations were 4mm and 43mm respectively. By comparing Figs. 2, 3 and 4 with Fig. 5, we ascertained that: (a) the results for flexural-torsional analysis of the single cell mono symmetric box girder frame, showed 59% increase in the flexural deformation over the real life situation, (b) for flexural-distortional analysis the results had 88% reduction in the flexural deformation below the real life situation, (c) for torsional-distortional analysis the results confirm 25% reduction in the torsional deformation and 23% reduction in the distortional deformation, all below the real life situation.

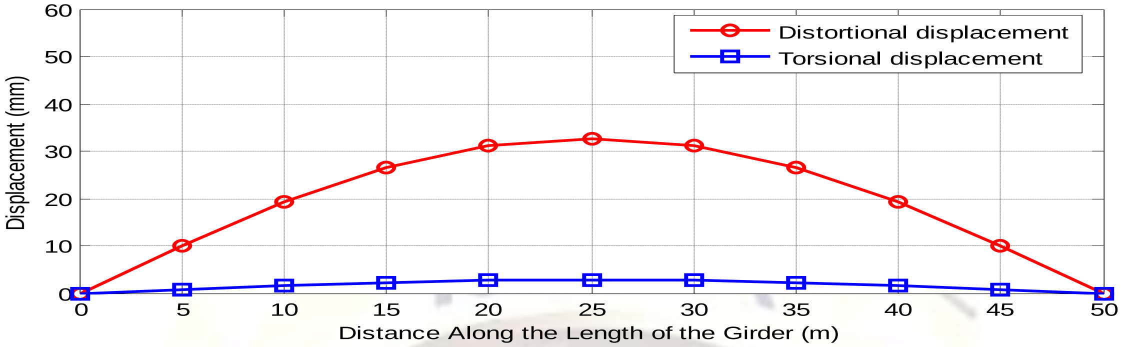
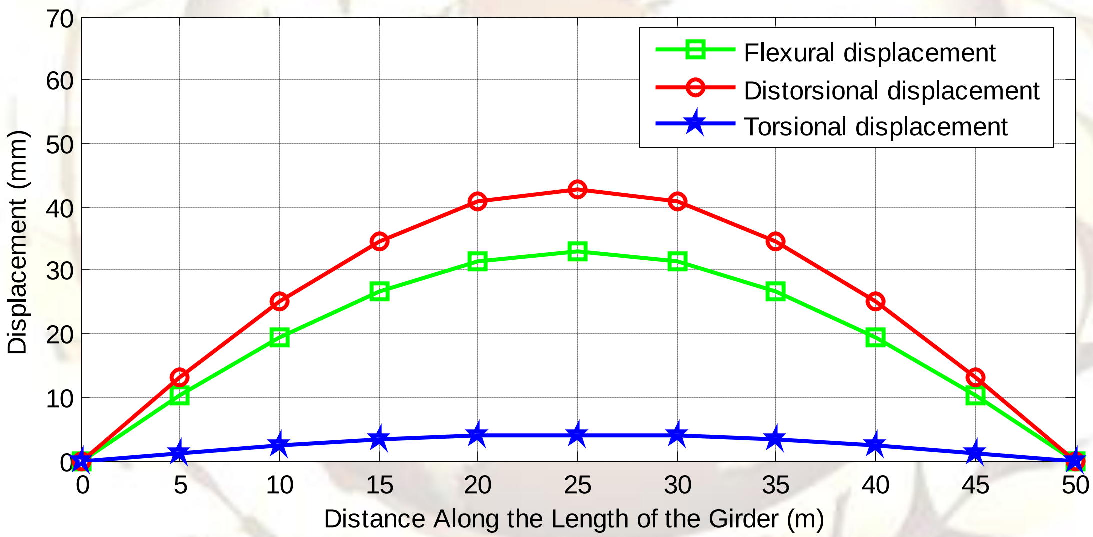


Fig.4 Variation of torsional and distortional displacements along the length of the girder



y
Fig. 5

9. Conclusion

The results obtained from flexural-torsional analysis, flexural- distortional analysis, and torsional-distortional analysis of the mono symmetric box girder frame have inherent errors due to the negligence of some of the interactions between the strain modes of flexure, torsion and distortion. Flexural-torsional-distortional analysis of the frame depicts the real life behaviour of such structures as it makes provisions for all strain modes interactions. The derived equations of flexural-torsional-distortional equilibrium eqn. (13), enables such complete analysis to be easily carried out.

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