Application of Branch And Bound Technique for n*3 Flow Shop Scheduling with Breakdown Interval

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Abstract

This paper deals with minimization of the total elapsed time for nx3 flow shop scheduling problem in which effect of the breakdown interval is being considered. A Branch and Bound technique is given to optimize the objective. The algorithm defined in this paper is very simple and easy to understand and, also provide an important tool for decision makers to design a schedule. The method is clarified with the help of numerical illustration.

Keywords: Flow shop scheduling, Processing time, Transportation time, Branch and Bound Technique, Optimal sequence.

1. Introduction:

Sequencing simply refers to the determination of order in which the jobs are to be processed on various machines. The scheduling /sequencing problems are common occurrence in our daily life e.g. ordering of jobs for processing in a manufacturing plant, waiting air craft for landing clearance, programs to be run in a sequence at a computer center etc. Such problems exist whenever there is an alternative choice in which a no. of jobs can be done. The selection of an appropriate order or sequence in which to receive waiting customer is called sequencing . The research in to flow shop scheduling has drawn a great attention in the last decade with the aim to increase the effectiveness of industrial production. Now-a-days, the decision makers for the manufacturing plant must find a way to successfully manage resources in order to produce products in the most efficient way with minimum total flow time. Various researchers have done a lot of work in this direction. Johnson (1954) first of all gave a method to minimise the makespan for n-job, two-machine scheduling problems. The scheduling problem practically depends upon the important factors namely, Transportation time, break down effect, Relative importance of a job over another job etc. These concepts were separately studied by Ignall and Scharge (1965), Cambell (1970), Maggu and Dass (1981), Heydari (2003), Temiz and Erol(2004),Yoshida and Hitomi (1979), Lomnicki (1965), Palmer (1965) , Bestwick and Hastings (1976), Nawaz et al. (1983) , Sarin and Lefoka (1993) , Koulamas (1998) , Dannenbring (1977) , etc.

Yoshida and Hitomi (1979) considered two stage flow shop problem to minimize the makespan whenever set up times are separated from processing time. The basic concept of equivalent job for a job block has been introduced by Maggu and Dass (1981). Singh T.P. and Gupta Deepak (2005)studied the optimal two stage production schedule in which processing time and set up time both were associated with probabilities including job block criteria. Heydari (2003)dealt with a flow shop scheduling problem where n jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed and other job block in which order of jobs is arbitrary.

Lomnicki (1965) introduced the concept of flow shop scheduling with the help of branch and bound method. Further the work was developed by Ignall and Scharge (1965), Chandrasekharan (1992), Brown and Lomnicki(1966) , with the branch and bound technique to the machine scheduling problem by introducing different parameters. The break down of the machines have significant role in the production concern. The effect of break down interval is important as there are feasible situations where machine during process may get sudden break down due to either failure of any component of machine or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as electric cut/shortage due to government policy. The working of machine no longer remains continuous and is subject to break-down for a certain interval of time. This paper extends the study made by Ignall and Scharge (1965) by introducing the concept of break down interval. Hence the problem discussed here is wider and has significant use of theoretical results in process industries.

2. Notations:

We are given n jobs to be processed on three stage flowshop scheduling problem and we have used the following notations:

A_i	:	Processing time for job i on machine A
B_i	:	Processing time for job i on machine B
C_i	:	Processing time for job i on machine C
C_{ij}	:	Completion time for job i on machines A, B and C
S_k	:	Sequence using johnson's algorithm
L	:	Length of break down interval.
J_r	:	Partial schedule of r scheduled jobs
$J_{r'}$	:	The set of remaining (n-r) free jobs

3. Mathematical Development:

Consider n jobs say i=1, 2, 3 … n are processed on three machines A, B & C in the order ABC. A job i (i=1,2,3…n) has processing time A_i , B_i & C_i on each machine respectively, assuming their respective probabilities p_i , q_i & r_i such that $0 \le p_i \le 1$, $\Sigma p_i = 1$, $0 \le q_i \le 1$, $\Sigma q_i = 1$, $0 \le r_i \le 1$, $\Sigma r_i = 1$. The mathematical model of the problem in matrix form can be stated as :

Jobs	Machine A	Machine B	Machine C
i	A_i	B_i	C_i
1	A_1	B_1	C_1
2	A_2	B_2	C_2
3	A_3	B_3	C_3
4	A_4	B_4	C_4
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n	A_n	B_n	C_n

Tableau – 1

Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time whenever the effect of break down interval (a, b) is given, using branch and bound technique.

4. Algorithm:

Step1: Calculate

(i) $g_1 = t(J_r, 1) + \sum_{i \in J'_r} A_i + \min_{i \in J'_r}(B_i + C_i)$

(ii) $g_2 = t(J_r, 2) + \sum_{i \in j'_r} B_i + \min_{i \in J'_r}(C_i)$

(iii) $g_3 = t(J_r, 3) + \sum_{i \in j'_r} C_i$

Step 2: Calculate

g = max [g_1, g_2, g_3] We evaluate g first for the n classes of permutations, i.e. for these starting with 1, 2, 3………n respectively, having labelled the appropriate vertices of the scheduling tree by these values.

Step 3: Now explore the vertex with lowest label. Evaluate g for the (n-1) subclasses starting with this vertex and again concentrate on the lowest label vertex. Continuing this way, until we reach at the end of the tree represented by two single permutations, for which we evaluate the total work duration. Thus we get the optimal schedule of the jobs.

Step 4: Prepare in-out table for the optimal sequence obtained in step 4 and read the effect of break down interval (a, b) on different jobs.

Step 5: Form a modified problem with processing times A'_i , B'_i , C'_i on machines A, B & C respectively. If the break down interval (a, b) has effect on job i then

$A'_i = A_i + L$, $B'_i = B_i + L$ and $C'_i = C_i + L$ where L = b – a, the length of the break down interval.

If the break down interval (a, b) has no effect on job i then $A'_i = A_i$, $B'_i = B_i$ and $C'_i = C_i$

Step 6: Repeat the procedure to get the optimal sequence for the modified scheduling problem using step1 to step 3. Compute the in-out table and get the minimum total elapsed time.

5. Numerical Example:

Consider 5 jobs 3 machine flow shop problem. processing time of the jobs on each machine is given. Our objective is to obtain the optimal schedule of all jobs which minimize the total elapsed time whenever the effect of break down interval (25, 35) is given.

Jobs	Machine A	Machine B	Machine C
i	A_i	B_i	C_i
1	15	20	16
2	25	10	5
3	10	12	12
4	18	15	18
5	16	25	3

Tableau – 2

Solution:
Step1: Calculate

(i) $g_1 = t(J_r,1) + \sum_{i\in J'_r} A_i + \min_{i\in J'_r}(B_i + C_i)$

(ii) $g_2 = t(J_r,2) + \sum_{i\in j'_r} B_i + \min_{i\in J'_r}(C_i)$

(iii) $g_3 = t(J_r,3) + \sum_{i\in j'_r} C_i$

For J_1 = (1).Then J′(1) = {2,3,4}, we get
g_1 = 43 , g_2 = 37 & g_3 = 43 and g = max(g_1, g_2, g_3) = 43
similarly, we have LB(2)= 51 LB(3)= 52 LB(4)= 58
Step 2 & 3: Now branch from J_1 = (1). Take J_2 =(12).
Then J'_2={3,4} and LB(12) = 51
Proceeding in this way, we obtain lower bound values on the completion time on machine C as shown in the tableau- 3
Step 4 : Therefore the sequence S_1 is 1-3-4-2 and the corresponding in-out table and checking the effect of break down interval (25, 35) on sequence S_1 is as in tableau-4:
*Step 5***:** The modified problem after the effect of break down interval (25,35) with processing times A'_i, B'_i and C'_i on machines A, B & C respectively is as in tableau-5:
*Step 6***:** Now, on repeating the procedure to get the optimal sequence for the modified scheduling problem using step 1 to step 3, we obtain lower bound values on the completion time on machine C as shown in the tableau- 6
we have get the sequence S_2 : 3-1-4-5-2. Compute the in-out table for S_2 and get the minimum total elapsed time as in tableau-7.
Hence the total elapsed time of the given problem is 120 units.

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Table 3: lower bounds for respective jobs are as in tableau-3:

Table 4: In-out table and checking the effect of break down interval (20, 30) on sequence S_1 is as in tableau-4:

Job I	Machine A In-out	Machine B In-out	Machine C In-out
3	0 - 10	10 – 22	**22 - 34**
1	**10 – 25**	**25 – 45**	45 – 61
4	**25 – 43**	45 – 60	61 – 79
5	43 – 59	60 – 85	85 – 88
2	59 – 84	85 – 95	95 – 100

Tableau-4

Node Jr	LB (Jr)
(1)	100
(2)	110
(3)	99
(4)	103
(5)	103
(31)	99
(32)	112
(34)	101
(35)	101
(312)	112
(314)	99
(315)	100
(3142)	112
(3145)	99

Tableau-3

Table 5: Modified problem is as in tableau-5.

Job I	Machine A	Machine B	Machine C
	A'_i	B'_i	C'_i
1	25	30	16
2	25	10	5
3	10	12	22
4	28	15	18
5	16	25	3

Tableau-5

Table 6: lower bounds for respective jobs of the modified problem are as in tableau-6:

Job i	Machine A In-out	Machine B In-out	Machine C In-out
3	0 – 10	10 – 22	22 – 44
1	10 – 35	35 – 65	65- 81
4	35 – 63	65 – 80	81 – 99
5	63 – 79	80 – 105	105 – 108
2	79 - 104	105 - 115	115 - 120

Tableau-6

Table 7: In-out table for S_2 and the minimum total elapsed time as in tableau-7.

Node Jr	LB (Jr)
(1)	120
(2)	132
(3)	119
(4)	123
(5)	119
(31)	119
(32)	122
(34)	121
(35)	119
(312)	132
(314)	119
(315)	119
(3142)	132
(3145)	119

Tableau-7