SELECTION OF MIXED SAMPLING PLAN WITH TNT-$(n;c_1,c_2)$ PLAN AS ATTRIBUTE PLAN INDEXED THROUGH MAPD AND AQL

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ABSTRACT

This paper presents the procedure for the construction and selection of the mixed sampling plan using MAPD as a quality standard with the TNT- $(n;c_1,c_2)$ plan as attribute plan. The plans indexed through MAPD and AQL are constructed and compared for their efficiency. Tables are constructed for easy selection of the plan.

Key words and Phrases: *acceptable quality level, maximum allowable percent defective, operating characteristic, tangent intercept.*

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1. INTRODUCTION

Mixed sampling plans consist of two stages of different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables are used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise, the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of a mixed sampling plan over pure attribute sampling plans is a reduction in sample size for a similar amount of protection.

Schilling (1967) proposed a method for determining the operating characteristics of Mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation. Later Adams and Lamberson (1975) developed a modified combined attribute plan. Adams and Mirkhani (1976) developed mixed plans for the case of unknown standard deviation. Elder and Muse (1982) provided an approximate method for evaluating attribute mixed plans. Devaarul (2003),Sampath Kumar (2007), Radhakrishnan and Sampath Kumar (2006a, 2006b, 2007a, 2007b, 2007c, 2009) and Radhnakrishan, et.al have made contributions to mixed sampling plans for independent case. Radhakrishnan, et.al (2009) studied mixed sampling plan for dependent case.

In this paper, using the operating procedure of mixed sampling plan (independent case) with Tightened – Normal – Tightened plan of the type TNT - $(n;c_1,c_2)$ plan as attribute plan, tables are constructed for the mixed sampling plan indexed through i) MAPD (ii) AQL (acceptable quality level). The plan indexed through MAPD is compared with the plan indexed through AQL. Suitable suggestions are also provided for future research.

2. GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

- p : submitted quality of lot or process
- $P_a(p)$: probability of acceptance for given quality 'p'
- p_1 : the submitted quality such that $P_a(p_1) = 0.95$ (also called AQL)
- p_* : maximum allowable percent defective (MAPD)
- h_* : relative slope at 'p_*
- $n_{1,1}$: sample size for variable sampling plan
- $n_{1,2}$: tightened (larger) sample size for attribute sampling plan
- $n_{2,2}$: normal (smaller) sample size for attribute sampling plan
- s : criterion for switching to tightened inspection
- t : criterion for switching to normal inspection
- β_j : probability of acceptance for lot quality 'p_j'

β_j' : probability of acceptance assigned to first stage for percent defective ‘p_j’
β_j'' : probability of acceptance assigned to second stage for percent defective‘p_j’
z (j) : ‘z’ value for the j^{th} ordered observation
k : variable factor such that a lot is accepted if $\bar{X} \le A = U - k\sigma$

3. OPERATING PROCEDURE OF MIXED SAMPLING PLAN WITH TNT-($n;c_1,c_2$) AS ATTRIBUTE PLAN

In this paper, independent mixed sampling plans are considered. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit ‘U’. By symmetry a parallel discussion can be made use for lower specification limits. It is suggested that the mixed sampling plan with TNT-($n;c_1,c_2$) in the case of single sided specification (U), S.D (σ) known can be formulated by the parameters $n_{1,2}$, $n_{2,2}$, ‘s’ and ‘t’. While giving the values for the parameters an independent plan for single sided specification, S.D known would be carried out as follows:

1. Determine the parameters of the mixed sampling plan $n_{1,2}$, $n_{2,2}$, s and t with reference to OC curves.

2. Take a random sample of size $n_{1,1}$ from the lot assumed to be large
3. If the sample average $\bar{X} \le A = U - k\sigma$, accept the lot
4. If the sample average $\bar{X} > A = U - k\sigma$, take another sample of size $n_{1,2}$

i) inspect using tightened inspection with a larger sample size $n_{1,2}$ and acceptance number c_1.
ii) switch to normal inspection when ‘t’ lots in a row are accepted under tightened inspection.

iii) inspect using normal inspection with smaller sample size $n_{2,2}$ and acceptance number c_2 ($>c_1$)

iv) switch to tightened inspection after a rejection if an additional lot is rejected in the next ‘s’ lots.

When the S.D (σ) is not known, simply substitute the sample standard deviation (s_1) where $s_1 = \sqrt{\dfrac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$ for σ in the known standard deviation procedure by choosing an appropriate value of ‘k’ and sample size ‘n’ for the unknown standard deviation case.

The operation of mixed plans can be assessed if the formula for the ordinates is clearly defined for the known percent defectives. The following formula can be used in determining the operating characteristic curve and associated measures of performance of an independent mixed plan. The probability of acceptance of a lot is

$$P_a(p) = P_{n_{1,2}}(\bar{X} \le A) + P_{n_{1,2}}(\bar{X} > A)\sum_{j=0}^{c} P(j;n_{2,2}) \quad (1)$$

4. CONSTRUCTION OF MIXED SAMPLING PLAN HAVING TNT- ($n;c_1,c_2$) AS ATTRIBUTE PLAN INDEXED THROUGH THE GIVEN POINT ON THE OC CURVE

The procedure for the construction of mixed variables – attributes sampling plans are provided by Schilling (1967) for a given ‘$n_{1,1}$’ and a point ‘p_j’ on the OC curve is given below.

- Assume that the mixed sampling plan is independent
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size $n_{1,1}$ (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as

$A = U - k\sigma = U - [z(p_j) + \{z(\beta_j')/\sqrt{n_{1,1}}\}]\sigma$, where z (t) is the standard normal variate corresponding to ‘t’ such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$

- Determine the sample average $\bar{X}$. If a sample average $\bar{X}$ > A = U - kσ, take a second stage sample of size '$n_{2,2}$' using attribute sampling plan.
- Split the probability of acceptance β_* as β_*' and β_*'', β_j as β_j' and β_j'' such that $\beta_* = \beta_*' + (1-\beta_*')\beta_*''$ and $\beta_j = \beta_j' + (1-\beta_j')\beta_j''$ where β_*' and β_j' are the probability of acceptance assigned to the attribute sampling plan. Fix the values of β_*' and β_j'.
- Determine β_*'' and β_j'' as $\beta_*'' = (\beta_* - \beta_*') / (1-\beta_*')$ and $\beta_j'' = (\beta_j - \beta_j') / (1-\beta_j')$
- Determine the appropriate second stage sample of size '$n_{2,2}$' from (sample size for the attribute plan Pa (p) = β_*'' for p = p_* and Pa (p) = β_j'' for p = p_j.

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with TNT-(n;c_1,c_2) plan as attribute plan indexed through MAPD (p_*) or AQL (p_1).

According to Calvin (1977), the OC function of a TNT scheme is given by

$$P_a(p) = \frac{P_1(1-P_2^s)(1-P_1^t)(1-P_2) + P_2 P_1^t (1-P_1)(2-P_2^s)}{(1-P_1^s)(1-P_1^t)(1-P_2) + P_1^t(1-P_1)(2-P_2^s)} \quad (2)$$

Where P_1 = Probability of acceptance under tightened inspection
P_2 = Probability of acceptance under normal inspection
s = Criteria for switching to tighted inspection
and t = Criteria for switching to normal inspection

Based on the conditions of the application of the Poisson model, when c_1 = 0 the probability of acceptance under tightened inspection becomes

$$P_1 = e^{-n_{1,2}p} \quad (3)$$

When c_2 = 1, the probability of acceptance under normal inspection becomes

$$P_2 = (1+np)e^{-n_{2,2}p} \quad (4)$$

Since $n_{1,2} > n_{2,2}$, we set $n_{1,2}$ equal to some multiple of $n_{2,2}$ say, $mn_{2,2}$. The tables furnished in this paper are for the case when m = 2. Suresh and Balamurali (1996) have constructed TNT- (n;c_1,c_2) plan indexed by MAPD. The paper aims at giving tables and procedures for the selection of the TNT-(n;c_1,c_2) scheme when c_1 = 0 and c_2 = 1 indexed by MAPD and is compared to the TNT- (n;c_1,c_2) scheme indexed by AQL
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5. CONSTRUCTION OF THE PLANS INDEXED THROUGH MAPD

MAPD, introduced by Mayer (1967) and studied by Soundararajan (1975) is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p_*' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. For designing, Soundararajan (1975) proposed a selection procedure for single sampling plan indexed with MAPD and R = $\frac{p_t}{p_*}$

Using the expressions (2) and (3) in the expression (1) we get the plan which is called as TNT-(n;c_1,c_2) plan and the inflection point (p_*) is obtained by using $\frac{d^2 p_a(p)}{dp^2} = 0$ and $\frac{d^3 p_a(p)}{dp^3} \neq 0$. The relative slope of the OC curve $h_* = \left[\frac{-p}{Pa(p)}\right]\frac{dPa(p)}{dp}$ at p = p_*. The inflection tangent of the OC curve cuts the p axis at $p_t = p_* + (p_*/h_*)$.

The values of $n_{2,2}p_*$, h_*, $n_{2,2}p_t$ and R = $\frac{p_t}{p_*}$ are calculated for different values of s and t for β_*' = 0.40 using visual basic program and presented in Table 1.

Table 1: Various characteristics of the mixed sampling plan when $\beta_*' = \beta_1' = 0.40$ and $\beta_1 = 0.95$

s	t	$n_{2,2}p_1$	β_*''	$n_{2,2}p_*$	h_*	$n_{2,2}p_t$	$R = p_t / p_*$
1	2	0.4560	0. 3422	1. 4110	2. 0589	2. 0963	1.4857
1	3	0.4410	0. 3903	1.1640	1. 8048	1. 8089	1..5540
1	4	0.4260	0.4387	0..9920	1..5870	1. 6171	1. 6301
1	5	0. 4090	0. 4817	0. 8670	1. 4201	1.4775	1.7042
1	6	0. 3930	0. 5177	0. 7740	1. 2907	1. 3737	1. 7748
2	3	0. 3880	0. 3717	1.1410	1. 5512	1. 8766	1. 6447
2	4	0. 3640	0. 4068	0. 9940	1. 3217	1. 7461	1. 7566
2	5	0. 3420	0. 4393	0. 8860	1.1453	1. 6596	1.8731
2	6	0. 3240	0. 4682	0. 8040	1. 0104	1. 5997	1. 9897
2	7	0. 3070	0. 4938	0. 7390	0. 9052	1. 5554	2. 1047
3	4	0. 3470	0. 4327	0. 9400	1. 2564	1. 6882	1. 7960
3	5	0. 3260	0. 4628	0. 8390	1. 0932	1. 6065	1. 9148
3	6	0. 3070	0. 4898	0. 7630	0. 9667	1. 5523	2. 0345
3	7	0. 2910	0. 5137	0. 7020	0. 8669	1. 5118	2. 1536
3	8	0. 2760	0. 535	0. 6530	0. 7872	1. 4825	2. 2703
4	5	0. 3180	0. 484	0. 8030	1. 0641	1. 5576	1. 9397
4	6	0. 2990	0. 5098	0. 7290	0. 9428	1. 5022	2. 0606
4	7	0. 2830	0. 5325	0. 6710	0. 8465	1. 4637	2. 1814
4	8	0. 2690	0. 5525	0. 6250	0. 7690	1. 4377	2. 3003
4	9	0. 2560	0. 5707	0. 5850	0. 7052	1. 4146	2. 4181
5	6	0. 2940	0.5263	0.7040	0.9272	1.4633	2.0786
5	7	0.2780	0.5482	0.6470	0.8333	1.4234	2.2000
5	8	0.2640	0.5675	0.6020	0.7573	1.3969	2.3204
5	9	0.2520	0.5847	0.5650	0.6947	1.3783	2.4395
5	10	0.2410	0.6002	0.5330	0.6423	1.3628	2.5568

5.1 Selection of the Plan

For the given values of p_* and p_t, the ratio $R = \frac{p_t}{p_*}$ is found and the nearest value of R is located in Table 1.

The corresponding value of s,t and $n_{2,2}p_*$ values are noted and the value of $n_{2,2}$ is obtained using $n_{2,2} = \frac{n_{2,2}\,p_*}{p_*}$

5.2 Example: Given $p_* = 0.036$, $p_t = 0.074$ and $\beta_*' = 0.40$, the ratio $R = \frac{p_t}{p_*} = 2.0556$. In Table 1, the nearest R value is 2.0606 which is corresponding to s = 4 and t = 6 and the second stage sample size for normal inspection plan is $n_{2,2} = \frac{n_{2,2}\,p_*}{p_*} = \frac{0.7290}{0.036} = 20$. The sample size for the tightened plan $n_{1,2}$ is determined as $n_{1,2} = 2n_{2,2} = 40$. Thus $n_{1,2} = 40$,

$n_{2,2}$ = 20, s = 4 and t = 6 are the parameters selected for the mixed sampling plan having TNT-($n;c_1,c_2$) as attribute plan for a specified p_* = 0.036 and p_t = 0.074.

5.2 Practical application

In a Mobile manufacturing company, for a specified lot quality p_t = 0.074 (74 non conformities out of thousand mobiles), p_* = 0.036(36 non conformities out of thousand mobiles), if the sample average $\bar{X}$ > A = U-kσ (for a known U, k & σ), take a sample of size 40(= $n_{1,2}$) under Tighted inspection with c_1 = 0 and if t = 6 lots in a row are accepted, switch to Normal inspection with a smaller sample size $n_{2,2}$ = 20 with c_2 = 1 and then switch to Tightened inspection after a rejection if an additional lot is rejected in the next s = 4 lots.

6. CONSTRUCTION OF MIXED SAMPLING PLAN WITH TNT- ($n;c_1,c_2$) AS ATTRIBUTE PLAN INDEXED THROUGH AQL

The procedure given in Section 4 is used for constructing the mixed sampling plan having TNT-($n;c_1,c_2$) as attribute plan indexed through AQL (p_1). By assuming the probability of acceptance of the lot be β_1 = 0.95 and β_1' = 0.40 for AQL, the $n_{2,2}p_1$ values are calculated for different values of s and t using visual basic program and is presented in Table 1.

6.1 Selection of the Plan for a given AQL, s and t

Table 1 is used to construct the plans when AQL (p_1), s and t are given. For any given values of p_1, s and t one can determine $n_{2,2}$ value using $n_{2,2} = \frac{n_{2,2}p_1}{p_1}$ and $n_{1,2} = 2n_{2,2}$.

6.2 Example: Let the probability of acceptance of the lot be β_1 = 0.95 and β_1' = 0.40. For the given values of p_1 = 0.009, s = 5 and t = 10 from Table 1, the second stage sample size $n_{2,2} = \frac{n_{2,2}p_1}{p_1} = \frac{0.241}{0.009}$ = 27 and $n_{1,2} = 2n_{2,2}$ = 54.

Thus $n_{1,2}$ = 54, $n_{2,2}$ = 27, s = 5 and t = 10 are the parameters selected for the mixed sampling plan having TNT-($n;c_1,c_2$) as attribute plan for a specified p_1 = 0.009, s = 5 and t = 10.

7. COMPARISON OF TNT-($n;c_1,c_2$) INDEXED THROUGH MAPD AND AQL.

In this section TNT-($n;c_1,c_2$) plan indexed through MAPD is compared with TNT-($n;c_1,c_2$) plan indexed through AQL by fixing the parameters (s, t) and the assumption β_j'.

For a specified values of p_* and p_t with the assumption β_*' = 0.40 one can find the values of s, t and $n_{2,2}$ indexed through MAPD. By fixing the values of s and t, find the value of p_1 by equating Pa (p) = β_1 = 0.95. Using β_1' = 0.40, s, t and p_1 one can find the value of $n_{2,2}$ using $n_{2,2} = \frac{n_{2,2}p_1}{p_1}$ from Table 1. For different combinations of p_*, p_t, s and t the values of $n_{1,2}$, $n_{2,2}$ (indexed through MAPD) and $n_{1,2}$, $n_{2,2}$ (indexed through AQL) are calculated and presented in Table 2.

Table 2: Comparison of plans indexed through MAPD and AQL

p_*	p_t	s	t	INDEXED THROUGH MAPD		INDEXED THROUGH AQL	
				$n_{1,2}$	$n_{2,2}$	$n_{1,2}$	$n_{2,2}$
0.010*	0.017	1	5	174	87	216	108
0.017	0.042	5	9	66	33	78	39
0.022	0.050	3	8	60	30	70	35
0.039	0.082	2	7	38	19	46	23

***OC curve is drawn**

7.1 Construction of OC curve

The OC curves for the plan $n_{2,2}$ = 87,s =1,t = 5,c_1 = 0,c_2 =1 (indexed through MAPD) and $n_{2,2}$ = 108, s =1, t = 5, c_1 = 0, c_2 = 1 (indexed through AQL) based on the values different values of $n_{2,2}p$ and $P_a(p)$ are presented in Figure 1.

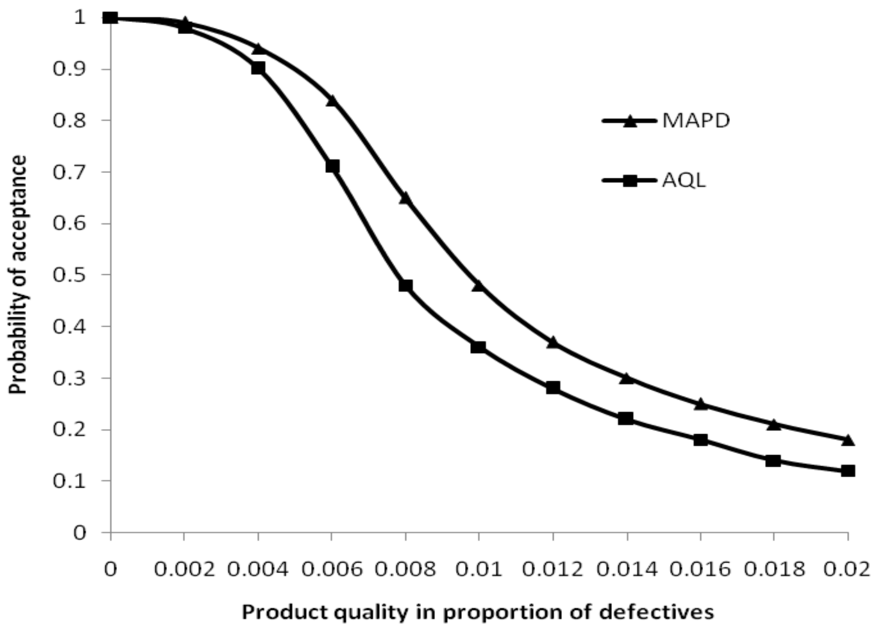

Figure 1: OC curves for TNT-(87;0,1) & (108;0,1)

8. CONCLUSION

It is concluded from the study that the second stage sample size required for TNT-($n;c_1,c_2$) plan indexed through MAPD is less than that of the second stage sample size of the TNT-($n;c_1,c_2$) plan indexed through AQL justified by Sampath Kumar (2007). Examples are provided for a specified value of $\beta_1' = 0.40$. If the floor engineers know the levels of MAPD or AQL, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*', β_1') and can also be compared for their efficiency.

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