

Object tracking in Curvelet Domain with dominant Curvelet Subbands

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Abstract— In this paper authors describe the Curvelet representation of image (object) with dominant angular subbands in Curvelet domain and analyse about the energy distribution for each subbands at different angles. Curvelet transform is localized not only in position (the spatial domain) and scale (the frequency domain), but also in orientation. Here energy of dominant orientations (angles) in a given scale is used as a measure for object tracking and found better results as described in paper.

Keywords— Curvelet transform, Ridgelet transform,

I. INTRODUCTION

Curvelet transform is a new multi scale transform used as an effective tool in image denoising, image decomposition, texture classification, image deconvolution, astronomical imaging and contrast enhancement, etc. Compare to wavelet Curvelet can represent any discontinuity in image more effectively with very few amounts of non-zero coefficients. This is because of wavelet transform generate non-zero value for three discontinuities: Horizontal, Vertical and diagonal. But for any curve discontinuity it will generate three different non-zero coefficients for each discontinuity. Discontinuities across a simple curve affect all the wavelets coefficients on the curve. So wavelet can only capture limited directional information. Compare to that Curvelet transform can handle such discontinuity more precisely because it localized in scale, position and orientation. Compare to wavelet, Curvelet pyramid contains elements with a very high degree of directional specificity [1]. First generation Curvelet transform is based on ridgelet. Ridgelet is effective in representing functions that have discontinuities along straight lines. This characteristic we have used in Curvelet transformation. A second generation Curvelet transform is much simpler structure and easier manipulation than the first generation. Second generation Curvelet can obtain by mainly two methods: USFFT and wrapping based Curvelet transform. Curvelet coefficients have different scales and angles. Energy of these coefficients is different for different coefficients based on angle and scale. Here we have represent the image using dominant directional subbands of particular scale and then after we will describe the effect of rotation on such dominant coefficients.

The rest of paper is organized as follow: In section II we describe the concept of First and Second generation of

Curvelet transform. In section III we have describe concept of represent the image using dominant angles, In section IV we presented experiment results about dominant subbands concept followed by tracking a object using such dominant angles and in section V we presents conclusion.

II. CURVELET TRANSFORM

Curvelet transform is one of new anisotropic directional wavelet transforms that allow an optimal sparse representation of objects. The initial approach of Curvelet transform implements the concept of discrete ridgelet Transform [2]. Since its creation in 1999, ridgelet based Curvelet transform has been successfully used as an effective tool in image denoising, image decomposition, texture classification, image deconvolution, astronomical imaging and contrast enhancement, etc. But ridgelet based Curvelet transform is not efficient as it uses complex ridgelet transform.

The definition of ridgelet transform, given an image $f(x, y)$, the continuous ridgelet coefficients are expressed as:

$$R_f(a, b, \Theta) = \iint \psi_{a,b,\Theta}(x, y) f(x, y) dx dy$$

'a' is scale, 'b' is position and 'Θ' is orientation.

A ridgelet can be defined as:

$$\psi_{a,b,\Theta}(x, y) = a^{1/2} \psi\left(\frac{x \cos \Theta + y \sin \Theta - b}{a}\right)$$

The ridgelet based Curvelet transform can be obtain by using following steps:

1. Sub band Decomposition
2. Smooth partition
3. Renormalization
4. Ridgelet transformation

Curvelet transform can handle Curve discontinuities very well compare to wavelet. In ridgelet based Curvelet it divide any curve in small partition so that small part can be consider as a straight line. Ridgelet transform can handle such line discontinuity well. Fig 1 shows the concept of the process. After applying ridgelet to all small portions it integrate all of them and result is Curvelet coefficients. In 2005, Candès et al. proposed two new forms of Curvelet transform based on different operations of Fourier samples, namely, unequally-spaced fast Fourier transform (USFFT) and wrapping based

fast Curvelet transform which are simpler, faster, and less redundant than existing technique[3].

Fast discrete Curvelet transform based on the wrapping of Fourier samples has less computational complexity as it uses

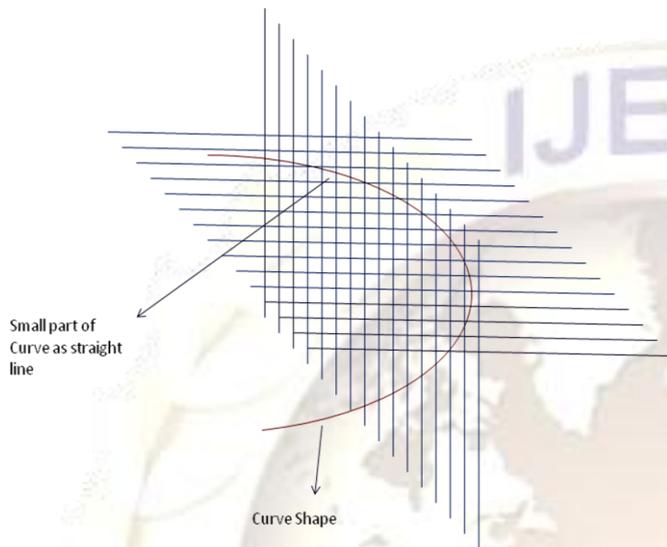


Fig 1 Ridgelet based Curvelet method

fast Fourier transform instead of complex ridgelet transform. Curvelet transform based on wrapping of Fourier samples takes a 2-D image as input in the form of a Cartesian array $f[m, n]$ such that $0 \leq m < M, 0 \leq n < N$ and generates a number of Curvelet coefficients indexed by a scale j , an orientation l and two spatial location parameters (k_1, k_2) as output. Discrete Curvelet coefficients can be defined by [3]:

$$C^D(j, l, k_1, k_2) = \sum_{\substack{0 \leq m < M \\ 0 \leq n < N}} f[m, n] \phi_{j, l, k_1, k_2}^D[m, n]$$

Basically, wrapping based Curvelet transform is a multiscale transform with a pyramid structure consisting of many orientations at each scale. This pyramid structure consists of several sub bands at different scales in the frequency domain. For lowest scale image contain low frequency and for highest scale image contain only high frequency component. To achieve higher level of efficiency, Curvelet transform is usually implemented in the frequency domain. That is, both the Curvelet and the image are transformed and are then multiplied in the Fourier frequency domain. The product is then inverse Fourier transformed to obtain the Curvelet coefficients. The process can be described as

$$\text{Curvelet transform} = \text{IFFT} [\text{FFT} (\text{Curvelet}) \times \text{FFT} (\text{Image})]$$

III. IMAGE REPRESENTATION WITH DOMINANT CURVELET SUBBANDS.

Curvelet representation of image contain very less non zero coefficients. Curvelet can handle curve discontinuity well compare to wavelet transform because it can generate very less number of non-zero coefficients for such curve. Curvelet transform can represent image in different number of scale with different number of orientation at each scale. The height of scale is depends on size of image. For any image $f(x, y)$ with size $n \times n$ can have a Curvelet transform of $(\log_2(N) - 3)$ level, for $n = 128$ level of Curvelet transform will be 4.

For any Curvelet decomposed image at any scale with different direction two peak energy level always remain with each other. This is because the energy of the dominant orientation of an image usually spreads between two neighboring subbands [4].

Curvelet Energies of real valued for Lena image with size 512×512 , at scale 4 and orientation 32, are shown in table 1.

Table 1 Curvelet Energy for each angle with scale 4

Angle	Energy	Angle	Energy
1	574.52	17	567.03
2	642.17	18	588.9
3	480.95	19	461.82
4	346.93	20	348.84
5	489.4	21	492.42
6	423.84	22	396.35
7	458.61	23	452.38
8	327.41	24	326.8
9	356.53	25	357.85
10	746.82	26	759.73
11	970.13	27	977.74
12	1381.5	28	1368.6
13	1536.4	29	1530.8
14	1251.4	30	1341.8
15	1001.5	31	988.03
16	775.86	32	775.77

Energy for any image $f(x,y)$ can calculated by formula

$$E = \sum_{(i,j) \in \text{bounding box}} |\text{coef}_{i,j}|^2$$

Here $\text{coef}_{i,j}$ are the Curvelet coefficients.

Fig 2 shows first 16 angle Curvelet coefficients for scale 4. From the above table it can be seen that at angle '13' energy is 1536.4 which is the peak energy. Next 1 distance from the peak energy is also have a high energy, for above case at angle '12' energy is 1381.5. As object changes its angular position then its peak energy will be shifted according to rotation. This can be seen from the Experiment results.

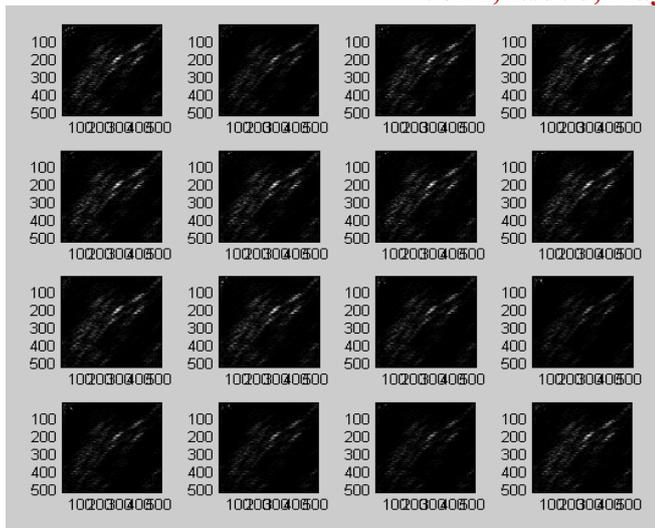


Fig 2 Curvelet coefficients with scale 4 angle 1 to 16

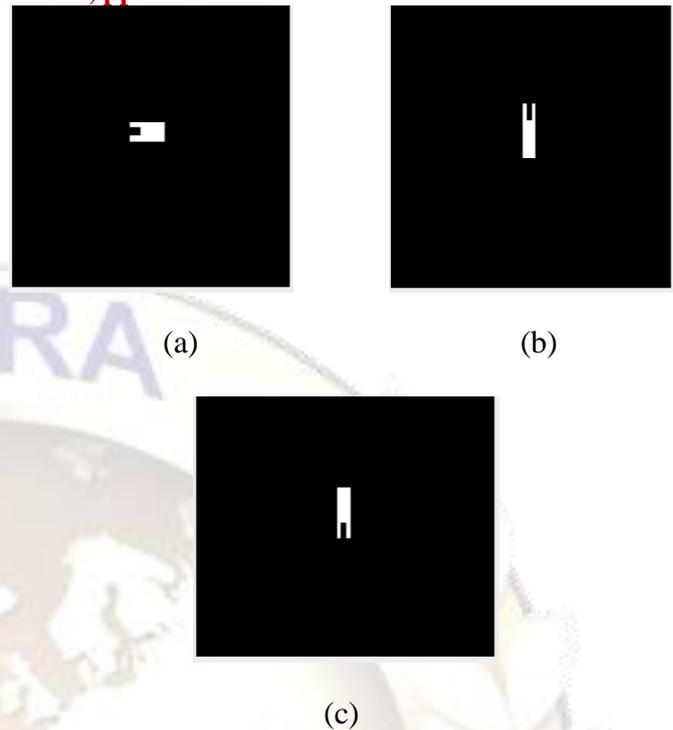


Fig 3 Three images with orientation (a) 0, (b) 90 clockwise and (c) 90 anticlockwise

IV. EXPERIMENT RESULTS

In Fig 3, consider fig (a), fig (b) and (c) are the 90° rotated images in clockwise and anticlockwise respectively.

For Fig 3 we have applied Curvelet decomposition and calculate energy at 5 scale and 64 directions. Fig 4 shows the graph of distribution of energy comparison for these three figures. From table it can be seen that for fig (a), peak energy is at '8' and '9' angle. $S5W8 = 240.4551$ and $S5W9 = 286.7565$. For the fig (b) peak energy is at angle 24 and 25 which is $S5W24 = 435.8872$ and $S5W25 = 392.1196$. From the figure it can be seen that for fig (b) is actually 90 clock wise rotated image of figure (a) has a peak energy shifted from angle '89' to '2425'. For scale 5 has total number of orientations are 64 which is divided in 360 so for 90 clockwise rotated image has a peak energy shifted to angle 24 from angle 8. In case of fig (c) which is a 90 anti clock rotated version of fig (a) has peak energy at angle 56, $S5W56 = 405.8018$ and at angle 57, $S5W57 = 293.6732$. For fig (c) peak energy is shifted to angle 56 from 8.

Table 2 Curvelet Energy for dominant angle for fig (1)

	1 st Peak Energy	Subband	2nd Peak Energy	Subband
Fig (a)	286.7565	S5W9	240.4551	S5W8
Fig (b)	435.8872	S5W24	392.1196	S5W25
Fig (c)	405.8018	S5W56	293.6732	S5W57

SxWy – Scale x and Orientation y

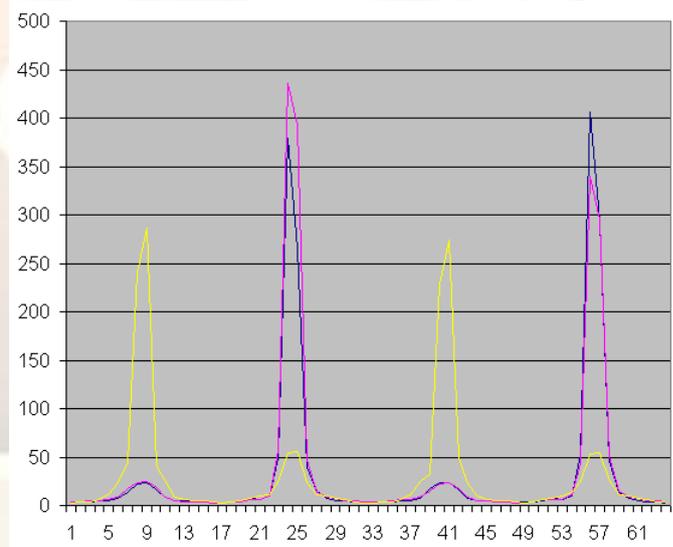


Fig 4 Energy distribution for fig (3) ((a), (b) and (c))

This property of Curvelet can be used for many applications like for tracking object using energy matching algorithm as in [1] we can take some of Dominant subbands for particular scale like for above experiment it can be seen that in table 1 some subbands have high energy compare to others so instead of taking all subbands if we take Dominant subbands out of all for tracking a moving object which has a constant shape without rotations, then information for tracking become more

accurate. If we take all the subbands then information (energy) becomes more generalized. This can be seen from the below examples of red ball tracking (database ref [5]).

For tracking an object in video sequence using Curvelet transform algorithm has been developed by Nguyen Thanh Binh [1]. For tracking object we have search the object in successive frame with fixed distance. In our algorithm we have not predict about future location of object as in [1] so it required more searching area for object in next frames. Object is manually selected in first frame. We have used Curvelet energy as a parameter for searching the location of object as in [1].

$$E = \sum_{(i,j) \in \text{bounding_box}} |\text{coef}_{i,j}|^2$$

Here $\text{coef}_{i,j}$ are the Curvelet coefficients.

Before applying Curvelet decomposition we have applying Sobel edge detection on each frame. Here we have search the object in all the direction equally for future frame from the present position of object in current frame for tracking.

We have used complex Curvelet coefficients for tracking. Here we consider two cases for tracking. In first case tracking is done from the Curvelet coefficients at scale 5 with first 16 peak dominant angles for first frame. From that for next frames we have calculate energy for only those 16 subbands and as in [1] exact position of object is track from minimum energy changing position in next frames.

For Second case we have used peak 8 subbands at scale 5 for tracking. From the results it can seen that in frame object is symmetric so its 4 dominant angles (peak energy subbands) are remain almost same. This can be seen from following example: We have calculated energy distribution for 10 frames. For all 10 frames except 6 have 4 dominant subbands are S5W57, S5W25, S5W56 and S5W24 and for frame no 6, S5W57, S5W25, S5W40 and S5W8 due to changed background. So if we consider only peak subbands of first frame for calculate energy then for such symmetric object this parameter becomes constant and hence tracking become more accurate. However if object changed its (angular) position

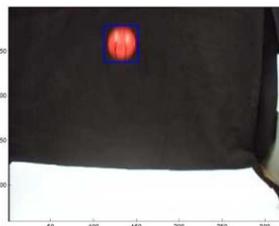
then we can measure the peak angle by measuring that what amount of object is rotated (this is proved in previous section using three figure example). For measuring accuracy we have used Centroid of tracked object. Centroid is a geometric centre of object. Here we have compare the actual centroid of object that we have measured manually, with practical centroid of tracked object. Table 3 shows the centroid values for all three cases. X is for column and y is for row.

Fig 5 shows tracked frames for both cases. Size of the input image is of 320 x 240. We have applied the algorithm for 15 frames out of these 10 frames shown in figure 5. Here experiment for tracking is done many times and best results are used.

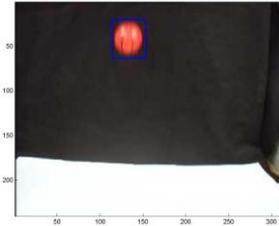
Fig 6 shows analysis of the tracked results. It shows Comparison of centroid for 2 to 15 frames. (a) Comparison of actual centroid Vs Centroid for dominant 16 angles. (b) Comparison of actual centroid Vs Centroid for dominant 8 angles. From the graph it can see that object is tracked using dominant angle concept.

Table 3 Centroid Values for Different cases.

Frame No.	Actual Value		Dominant 16 sub bands		Dominant 8 sub bands	
	X	Y	X1	Y1	X2	Y2
2	133	65.5	143	65.5	129.5	68.5
3	133.5	95	135	94.5	127.5	100.5
4	132	128	127	138.5	134.5	135.5
5	130.5	162	128	158.5	132.5	158.5
6	132.5	173	126	154.5	130.5	154.5
7	132	152.5	130	156.5	131.5	144.5
8	133	130.5	128	143.5	129.5	143.5
9	132	116	138	112.5	133.5	127.5
10	132.5	104	133	108.5	131.5	114.5
11	134	97	131	98.5	135.5	113.5
12	133.5	95	132	91.5	133.5	109.5
13	132.5	97.5	133	102.5	134.5	108.5
14	134.5	103.5	131	107.5	132.5	116.5
15	133.5	114	138	124.5	136.5	124.5



Frame 1

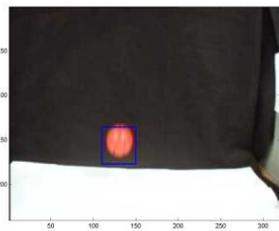


Frame 1

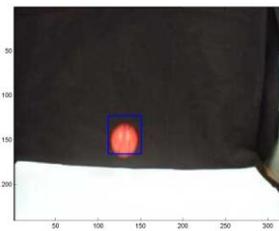
Fig 5 results of tracking with Curvelet transform (a) Dominant 16 sub bands, (b) Dominant 8 sub bands [Frame ref. 5].



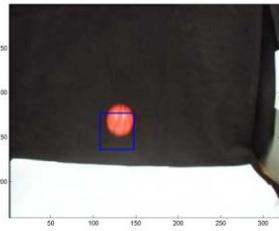
Fig 5 results of tracking with Curvelet transform (a) Dominant 16 sub bands, (b) Dominant 8 sub bands [Frame ref. 5].



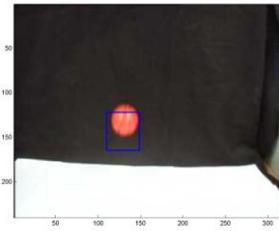
Frame 7



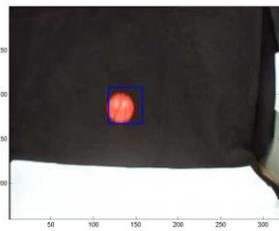
Frame 7



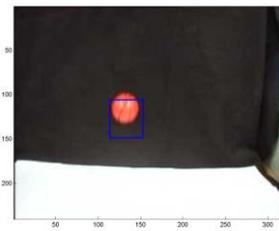
Frame 8



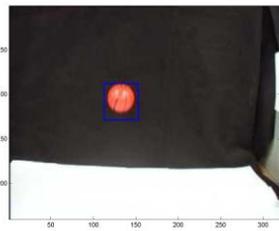
Frame 8



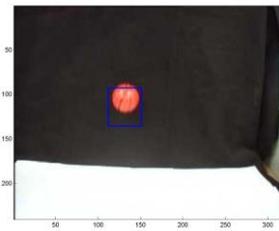
Frame 9



Frame 9

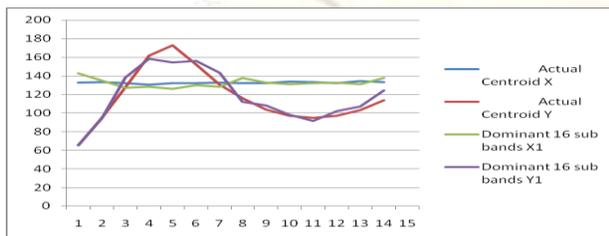


Frame 10

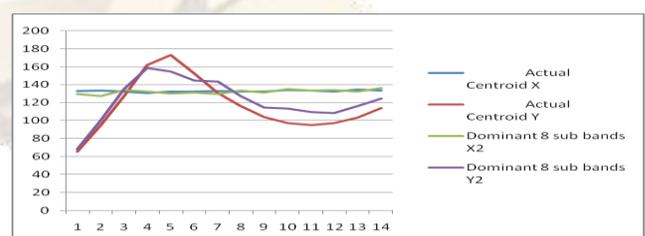


Framer 10

Fig 5 results of tracking with Curvelet transform (a) Dominant 16 sub bands, (b) Dominant 8 sub bands [Frame ref. 5].



(a) Actual Vs Dominant 16 angle based tracking centroids



(b) Actual Vs Dominant 8 angle based tracking centroids

Fig 6 Comparison of Centroids value for different frames

V. CONCLUSION

From the above experiment we can conclude that Dominant subbands concept is very useful to represent object. Energy distribution for Curvelet subbands has a fixed peak energy level for object and if object is rotated then this peak energy subbands also shifted at appropriate position according to direction of rotation. We have also use tracking example to shows the representation of symmetric object using dominant subbands. If object is rotated then by measuring the amount of rotation we can recalculate the energy for those subbands which are shifted because of rotation.

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