ISOMORPHISM AND ANTI-ISOMORPHISM IN Q-FUZZY TRANSLATION OF Q-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT: In this paper, we made an attempt to study the algebraic nature of Q- fuzzy subhemirings of a hemiring.
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KEY WORDS: Q-Fuzzy set, Q-fuzzy subhemiring and Q- fuzzy translation.

INTRODUCTION: There are many concepts of universal algebras generalizing an associative ring (R ; + ; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (R ; + ; .) share the same properties as a ring except that (R ; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b + c) = a. b + a. c and (b + c) .a = b. a + c. a for all a, b and c in R. A semiring R is said to be additively commutative if a + b = b + a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0 + a = a = a + 0 and a. 0 = 0 = 0. a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets **by** L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. Osman Kazanci , Sultan yamark and serife yilmaz in [6] have introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear-rings) in a near-ring and investigated some related properties. Solairaju.A and R.Nagarajan, have given a new structure in the construction of Q-fuzzy groups and subgroups [13] , [14] and [15].In this paper, we introduce the some Theorems in Q- fuzzy subhemirings of a hemiring.

1.PRELIMINARIES

1.1 Definition: Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is a function A : XxQ → [0, 1].

1.2 Definition: The **union** of two Q-fuzzy subsets A and B of a set X is defined by $(A\cup B)(x, q) = \max\{ A(x, q), B(x, q) \}$, for all x in X and q in Q.

1.3 Definition: The **intersection** of two Q-fuzzy subsets A and B of a set X is defined by $(A \cap B)(x, q) = \min\{ A(x, q), B(x, q) \}$, for all x in X and q in Q.

1.4 Definition: Let (R, +, .) be a hemiring. A Q-fuzzy subset A of R is said to be a Q-fuzzy subhemiring (QFSHR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y, q) \geq \min\{ \mu_A(x, q), \mu_A(y, q)\}$,

(ii) $\mu_A(xy, q) \geq \min\{\mu_A(x, q), \mu_A(y, q)\}$, for all x and y in R and q in Q.

1.5 Definition: Let (R, +, .) be a hemiring. A Q-fuzzy subhemiring A of R is said to be a Q-fuzzy normal subhemiring (QFNSHR) of R if $\mu_A(xy, q) = \mu_A(yx, q)$, for all x and y in R and q in Q.

1.6 Definition: Let (R, +, .) and (R^l, +, .) be any two hemirings. Then the function $f : R \to R^l$ is called a **hemiring homomorphism** if it satisfies the following axioms:

(i) f(x+y) = f(x) + f(y),

(ii) f(xy) = f(x) f(y), for all x and y in R.

1.7 Definition: Let (R, +, .) and (R^l, +, .) be any two hemirings. Then the function $f : R \to R^l$ is called a **hemiring anti-homomorphism** if it satisfies the following axioms:

(i) f(x + y) = f(y) + f(x),

(ii) f(xy) = f(y) f(x), for all x and y in R.

1.8 Definition: Let (R, +, .) and (R^l, +, .) be any two hemirings. Then the function $f : R \to R^l$ be a hemiring homomorphism. If f is one-to-one and onto, then f is called a **hemiring isomorphism**

1.9 Definition: Let (R, +, .) and (R^I, +, .) be any two hemirings. Then the function $f : R \to R^I$ be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a **hemiring anti-isomorphism**.

1.10 Definition: Let A be a Q-fuzzy subset of X and $\alpha \in [\,0, 1– \text{Sup}\{ A(x, q) : x\in X, 0 < A(x, q) < 1 \}]$. Then T = T_α^A is called a **Q-fuzzy translation** of A if T(x, q) = A(x, q)+α, for all x in X.

1.1 Example: Consider the set X = { 0, 1, 2, 3, 4} and Q = {q}. Let A = { ((0, q), 0.5), ((1, q), 0.4), ((2, q), 0.6), ((3, q), 0.45), ((4, q), 0.2) } be a Q-fuzzy subset of X and α = 0.25. The Q-fuzzy translation of A is T = $T^A_{0.25}$ = { ((0, q), 0.75), ((1, q), 0.65), ((2, q), 0.85), ((3, q), 0.7), ((4, q), 0.45)}.

2. ISOMORPHISM AND ANTI-ISOMORPHISM IN Q-FUZZY TRANSLATION OF Q-FUZZY SUBHEMIRINGS OF A HEMIRING

2.1 Theorem: Let (R, +, .) and (R^I, +, .) be any two hemirings. The Q-fuzzy normal subhemiring of f (R) = R^I under the anti-homomorphic preimage is a Q-fuzzy normal subhemiring of R.

Proof: Let (R, +, .) and (R^I, +, .) be any two hemirings and $f : R \to R^I$ be an anti-homomorphism. Then, f (x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V be a Q-fuzzy normal subhemiring of a hemiring f (R) = R^I and A be an anti-homomorphic pre-image of V under f. We have to prove that A is a Q-fuzzy normal subhemiring of a hemiring R. Let x and y in R and q in Q, then,
clearly A is a Q-fuzzy subhemiring of the hemiring R, since V is a Q-fuzzy subhemiring of a hemiring R^I.Now, $\mu_A(xy, q) = \mu_v(f(xy), q)$, since $\mu_A(x, q) = \mu_v(f(x), q)$ $= \mu_v(f(y)f(x), q)$, as f is an anti-homomorphism $= \mu_v(f(x)f(y), q) = \mu_v(f(yx), q)$, as f is an anti-homomorphism $= \mu_A(yx, q)$, since $\mu_A(x, q) = \mu_v(f(x), q)$ which implies that $\mu_A(xy, q) = \mu_A(yx, q)$,for all x and y in R and q in Q.Hence A is a Q-fuzzy normal subhemiring of the hemiring R.

In the following Theorem ◦ is the composition operation of functions:

2.2 Theorem: Let A be a Q-fuzzy subhemiring of a hemiring H and f is a isomorphism from a hemiring R onto H. If A is a Q-fuzzy normal subhemiring of the hemiring H, then A◦f is a Q-fuzzy normal subhemiring of the hemiring R.

Proof: Let x and y in R and q in Q and A be a Q-fuzzy normal subhemiring of a hemiring H. Then we have, clearly A◦f is a Q-fuzzy subhemiring of the hemiring R. Now, $(\mu_A \circ f)(xy, q) = \mu_A(f(xy), q) = \mu_A(f(x)f(y), q)$, as f is an isomorphism
$= \mu_A(f(y)f(x), q) = \mu_A(f(yx), q)$, as f is an isomorphism$= (\mu_A \circ f)(yx, q)$,which implies that $(\mu_A \circ f)(xy, q) = (\mu_A \circ f)(yx, q)$, for all x and y in R and q in Q.Hence A◦f is a Q-fuzzy normal subhemiring of the hemiring R.

2.3 Theorem: Let A be a Q-fuzzy subhemiring of a hemiring H and f is an anti- isomorphism from a hemiring R onto H. If A is a Q-fuzzy normal subhemiring of the hemiring H, then A◦f is a Q-fuzzy normal subhemiring of the hemiring R.

Proof: Let x and y in R and q in Q and A be a Q-fuzzy normal subhemiring of a hemiring H. Then we have, clearly A◦f is a Q-fuzzy subhemiring of the hemiring R. Now, $(\mu_A \circ f)(xy, q) = \mu_A(f(xy), q) = \mu_A(f(y)f(x), q)$, as f is an anti-isomorphism $= \mu_A(f(x)f(y), q) = \mu_A(f(yx), q)$, as f is an anti-isomorphism$= (\mu_A \circ f)(yx, q)$,which implies that $(\mu_A \circ f)(xy, q) = (\mu_A \circ f)(yx, q)$, for all x and y in R and q in Q.Hence A◦f is a Q-fuzzy normal subhemiring of the hemiring R.

2.4 Theorem: If M and N are two Q-fuzzy translations of Q-fuzzy normal subhemiring A of a hemiring (R, +, .), then their intersection $M \cap N$ is a Q-fuzzy translation of A.

Proof: It is trivial.

2.5 Theorem: The intersection of a family of Q-fuzzy translations of Q-fuzzy normal subhemiring A of a hemiring (R, +, .) is a Q-fuzzy translation of A.

Proof: It is trivial.

2.6 Theorem: If M and N are two Q-fuzzy translations of Q-fuzzy subhemiring A of a hemiring (R, +, .), then their union $M \cup N$ is a Q-fuzzy translation of A.

Proof: It is trivial.

2.7 Theorem: The union of a family of Q-fuzzy translations of Q-fuzzy normal subhemiring A of a hemiring (R, +, .) is a Q-fuzzy translation of A.

Proof: It is trivial.

2.8 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set. If f : R → R^l is a homomorphism, then the Q-fuzzy translation of a Q-fuzzy normal subhemiring A of R under the homomorphic image is a Q-fuzzy normal subhemiring of f (R) = R^l.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set and f : R → R^l be a homomorphism. That is f(x+y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let T= T_α^A be the Q-fuzzy translation of a Q-fuzzy normal subhemiring A of R and V be the homomorphic image of T under f. We have to prove that V is a Q-fuzzy normal subhemiring of R^l. Now, for f(x) and f(y) in R^l and q in Q, clearly V is a Q-fuzzy subhemiring of R^l.We have V(f(x)f(y), q)) = V(f(xy), q) ≥ T(xy, q) = A(xy, q) + α = A(yx, q) + α = T(yx, q)
≤ V(f(yx), q) = V(f(y) f(x), q), which implies that V(f(x)f(y), q)=V(f(y)f(x), q), for all f(x) and f(y) in R^l and q in Q. Therefore, V is a Q-fuzzy normal subhemiring of the hemiring R^l.

2.9 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set. If f : R → R^l is a homomorphism, then Q-fuzzy translation of a Q-fuzzy normal subhemiring V of f(R) = R^l under the homomorphic pre-image is a Q-fuzzy normal subhemiring of R.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set and f : R → R^l be a homomorphism. That is f (x+y) = f(x)+f(y) and f(xy)= f(x)f(y), for all x and y in R. Let T = T_α^V be the Q-fuzzy translation of Q-fuzzy normal subhemiring V of R^l and A be the homomorphic pre-image of T under f. We have to prove that A is a Q-fuzzy normal subhemiring of R. Let x and y be in R and q in Q. Then, clearly A is a Q-fuzzy subhemiring of R,A(xy, q) = T(f(xy), q) = V(f(xy), q) + α = V(f(x)f(y), q) + α = V(f(y)f(x), q) + α = V(f(yx), q) + α = T(f(yx), q) = A(yx, q),which implies that A(xy, q) = A(yx, q), for all x and y in R and q in Q.Therefore, A is a Q-fuzzy normal subhemiring of R.

2.10 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set. If f : R → R^l is an anti-homomorphism, then the Q-fuzzy translation of a Q-fuzzy normal subhemiring A of R under the anti-homomorphic image is a Q-fuzzy normal subhemiring of R^l.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set and f : R → R^l be an anti-homomorphism. That is f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x), for all x and y in R and q in Q. Let T_α^A be the Q-fuzzy translation of a Q-fuzzy normal subhemiring A of R and V be the anti-homomorphic image of T_α^A under f. We have to prove that V is a Q-fuzzy normal subhemiring of f(R) = R^l.
Now, for f(x) and f(y) in R^l and q in Q, clearly V is a Q-fuzzy subhemiring of R^l.
We have, V(f(x)f(y), q) = V(f(yx), q) ≥ T(yx, q) = A(yx, q) + α = A(xy, q) + α
= T(xy, q) ≤ V(f(xy), q)) = V(f(y) f(x), q) which implies that V(f(x)f(y), q)= V(f(y)f(x), q)), for f(x) and f(y) in R^l and q in Q. Therefore, V is a Q-fuzzy normal subhemiring of the hemiring R^l.

2.11 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set. If f : R → R^l is an anti-homomorphism, then the Q-fuzzy translation of a Q-fuzzy normal subhemiring V of f(R) = R^l under the anti-homomorphic pre-image is a Q-fuzzy normal subhemiring of R.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and Q be a non-empty set and f : R → R^l be an anti-homomorphism. That is f(x+y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let T=T_α^V be the Q-fuzzy translation of a Q-fuzzy normal subhemiring V of R^l and A be the anti-homomorphic pre-image of T under f. We have to prove that A is a Q-fuzzy normal subhemiring of R. Let x and y be in R and q in Q. Then, clearly A is a Q-fuzzy subhemiring of R,A(xy, q) = T(f(xy), q) = V(f(xy), q) + α = V(f(y)f(x), q) + α = V(f(x)f(y), q) + α = V(f(yx), q) + α = T(f(yx), q) = A(yx, q),which implies that A(xy, q) = A(yx, q), for all x and y in R and q in Q.Therefore, A is a Q-fuzzy normal subhemiring of R.

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