To Minimize The Rental Cost For 3 - Stage Specially Structured Flow Shop Scheduling with Job Weightage

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Abstract
This article describe the development of a new heuristic algorithm which guarantees an optimal solution for specially structured flow shop problem with n-job, 3-machines in which processing times are associated with their probabilities including weightage of jobs, to minimize the rental cost of machines. Further the expected processing times are not random but bear a well defined relationship to one another. Many heuristic approaches have already been discussed in literature to minimize the makespan. But it is not necessary that minimization of makespan always lead to minimize the rental cost of machines. Objective of this work is to minimize the rental cost of machines under a specified rental policy. A numerical illustration is followed to support the algorithm.

Keywords: Flow Shop Scheduling, Rental Cost, Weightage of job, Utilization Time.

Mathematical Subject Classification: 90B30, 90B35

1. Introduction
Scheduling can be defined as the allocation of resources over a period of time to perform a collection of tasks. The goal is to specify a schedule that specify when and on which machine each job is to be executed. All the scheduling models beginning from Johnson's work in 1954 on words, upto the 1980 there is no reference of job weightage in the literature. The scheduling problem with weights arises when inventory costs for jobs are involved. The weights of a job show its relative priority over some other jobs in a scheduling model. The first research concerned to the flow shop scheduling problem was proposed by Johnson [1]. Johnson described an exact algorithm to minimize makespan for the n-jobs 2 stage flow shop scheduling problem. Smith [12] considered minimization of min flow time and maximum tardiness Yoshida and Hitomi [7] further considered the problem with set up times. Gupta, J.N.D. [4] gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Gupta, Sharma and Bala Shashi [18] studied specially structured two stage flow shop problem to minimize the rental cost of the machines under pre-defined rental policy in which the probabilities have been associated with processing time. The work was developed by Bagga and Bhambani [16], Gupta Deepak et.el [17], Maggu & Dass [6], Chader Sekharan [9] by considering various parameters.

The paper is an extension of the study made by Gupta, Sharma & Bala Shashi [18] by introducing weight in jobs on three machines. Thus making the problem wider and more practical in process/ production industry. The work can further be extended by introducing parameters like job block, transportations time and break in interval etc.

2. Practical Situation
Manufacturing industries are the backbone in the economic structure of a nation, as they contribute to increasing G.D.P. / G.N.P. and providing employment. Productivity can be maximized. If the available resources are utilized in an optimized manner. Optimized utilization of resources can only be possible if there is a proper scheduling system making scheduling a highly important aspect of a manufacturing system. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various qualities of paper, sugar and oil are produced with relative importance i.e. weight in jobs, hence weightage of jobs is significant. Due to unavaibility of funds in real life in his starting career one has to be taken the machines on rent. Renting enables saving working capital, gives option for having the equipment and allows up-gradation to new technology.

3. Notations

S : Sequence of jobs 1, 2, 3,….,n

S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2, 3, ------ r.
M_j : Machine j= 1,2,3
M : Minimum makespan
a_{ij} : Processing time of i^{th} job on machine M_j
p_{ij} : Probability associated to the Processing time a_{ij}
A_{ij} : Expected processing time of i^{th} job on machine M_j.
w_i : weight of i^{th} job.
$G_i`$: Expected flow time of i^{th} job on machine M_j.
G_i'' : Weighted flow time of i^{th} job on machine M_j.
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$U_j(S_k)$: Utilization time for which machine M_j is required
$R(S_k)$: Total rental cost for the sequence S_k of all machine
C_j : Renal cost of j^{th} machine.
$CT(S_i)$: Total completion time of the jobs for sequence S_k

4. Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as:
$t_{ij} = max\,(t_{i-1,j}\,,\, t_{i,j-1}) + a_{ij} \times p_{ij}$ for $j \geq 2$.
$= max\,(t_{i-1,j}\,,\, t_{i,j-1}) + A_{i,j}$, where A_{ij}= Expected processing time of i^{th} job on j^{th} machine.

5. Rental Policy (P)

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine.

6. Problem Formulation

Let some jobs i(1,2,……..,n) are to be processed on three machines M_j (J= 1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij}. Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine. Satisfying the structural relationship

either $\mathrm{Min}(A_{i1}) \geq \mathrm{Max}\,(A_{i2})$
or $\mathrm{Min}(A_{i3}) \geq \mathrm{Max}\,(A_{i2})$ or both

Let w_i be the weight of i^{th} job. Our objective is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of machines while minimizing total utilization time.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1		Machine M_2		Machine M_3		Weight of job
i	a_{i1}	a_{i1}	p_{i2}	p_{i2}	a_{i3}	p_{i3}	w_i
2	a_{11}	a_{11}	p_{12}	p_{12}	a_{13}	p_{13}	w_1
2	a_{21}	a_{21}	p_{22}	p_{22}	a_{23}	p_{23}	w_2
3	a_{31}	a_{31}	p_{32}	p_{32}	a_{33}	p_{33}	w_3
4	a_{41}	a_{41}	p_{42}	p_{42}	a_{43}	p_{43}	w_4
-	-	-	-	-	-	-	-
n	a_{n1}	a_{n1}	p_{n2}	p_{n2}	a_{n3}	p_{n3}	w_n

Tableau -1

Mathematically, the problem is stated as: Minimize $U_j\,(S_k)$ and
Minimize

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2\,(S_k) \times C_2 + U_3(S_k) \times C_3$$

Subject to constraint: Rental Policy (P)
Our objective is to minimize rental cost of machines.

7. Assumptions

1. Two jobs cannot be processed on a single machine at a time.

2. The 2nd job will be proposed on a machine when first job is completed
3. $\sum p_{ij}=1,\ 0 \le p_{ij} \le 1$,
   $i = 1, 2, \ldots, n$
   $j = 1, 2, 3$.
4. Jobs are independent to each other.
5. Machine break down is not considered.
6. Jobs may be held in inventory before going to machine.

8. Algorithm

To obtain optimal schedule, we proceed as:

Step 1: Define expected processing time A_{ij} on machine M_{ij} as follow:

$A_{ij} = a_{ij} \times p_{ij}$, $j=1, 2, 3$.

Step 2: Check the condition

either $A_{i1} \ge A_{i2}$
or $A_{i3} \ge A_{i2}$
or Both for all i.

If the conditions are satisfied then go to Step3. Else data is not in standard form.

Step 3: Define two fictious machines G and H with processing time G_i and H_i for job *i* on machine G and H respectively as:

$G_i = A_{i1}+A_{i2}$
$H_i = A_{i2}+A_{i3}$

Step 4: Compute G_i and H_i as follow:

i. If min $(G_i, H_i) = G_i$
   then $G_i` = G_i + w_i$
   and $H_i` = H_i$.

ii. If min $(G_i, H_i) = H_i$
   then $G_i` = G_i$
   and $H_i` = H_i + w_i$

Step 5: Calculated weighted flow time of job *i* of machine G and H respectively as follow:

$G_i'' = G_i' / w_i$
$H_i'' = H_i' / w_i$

Step 6: Obtain the sequence S_1 (say) by applying Johnson's algorithm on machine G and H.

Step 7: Obtain other sequence by putting 2nd, 3rd,…..nth jobs of sequence S_i in the 1st position respectively and all other jobs of S_i in same order. Let these sequences be

$S_2, S_3 \ldots S_{n-1}$.

Step 8: Compute the total completion time $CT_2(S_k)$ & $CT_3(S_k)$ k=1, 2…r. by computing in – out table for sequences S_k (K= 1, 2, ...r.).

Step 9: Calculate $\sum A_{i1}$

Step 10: Calculate utilization time $U_2(S_k)$ of 2nd & $U_3(S_k)$ of 3rd machine where

$U_2(S_k) = CT_2(S_k) – A_{11}(S_k)$ and
$U_3(S_k) = CT_3(S_k) – (A_{11}(S_k) + A_{12}(S_k))$; $k=1,2,\ldots r$.

Step 11: Calculate

$$R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

Where C_1, C_2 and C_3 are the rental cost per unit time of 1st, 2nd and 3rd machine respectively.

Step 12: Find min $R(S_k)$; k= 1,2,….,n. let it be minimum for the sequence S_p, then sequence S_p will be the optimal sequence with rental cost $R(S_p)$.

9. Numerical Illustration

Consider 5 jobs and 3 machines flow shop problem with weights of jobs, Processing Time associated with their respective probabilities as given in the following table the rental cost per unit time for machine M_1, M_2 and M_3

are 5 units, 7 units and 10 units respectively. Our objective is to obtain optimal schedule to minimize the total production/total elapsed time subject to minimization of total rental cost of the machines, under the specified rental policy P.

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Jobs	Machine M_1		Machine M_2		Machine M_3		Weight
i	a_{i1}	a_{i1}	p_{i2}	p_{i2}	a_{i3}	p_{i3}	w_i
1	100	.2	50	.2	35	.3	3
2	120	.3	40	.3	110	.2	2
3	140	.1	65	.2	75	.2	4
4	130	.2	80	.1	125	.1	2
5	90	.2	55	.2	85	.2	1

Table :2

Solution : As per step 1: the expected processing time for machines M_1, M_2 and M_3 is as follow:

Jobs	Machine M_1	Machine M_2	Machine M_3	Weight
i	A_{i1}	A_{i2}	A_{i3}	w_i
1	20.0	10.0	15.5	3
2	36.0	12.0	22.	2
3	14.0	13.0	15.0	4
4	26.0	8.0	12.5	2
5	18.0	11.0	17.0	1

Table : 3

Each $A_{i1} \geq A_{i2}$ for all i,j.

As per step 3reduced problem two factious machines G and H having G_i and H_i as their processing times is as follow:

Jobs	Machine G	Machine H	weight
i	G_i	H_i	w_i
1	30.0	20.5	3
2	48.0	34.0	2
3	27.0	28.0	4
4	34.0	20.5	2
5	29.0	28.0	1

Table - 4

As per step 4 & 5 reduced problem with weighted flow time for two machine G and H is as follow

Jobs	Machine G	Machine H
i	G''_i	H''_i
1	10.0	7.8
2	24.0	18.0
3	7.31	7.0
4	17.0	11.25
5	29.0	29.0

Table - 5

As per step 6 the sequences with minimum makespan is

$S_1 = 5 – 2 – 4 – 1 – 3$

As per step 7: other feasible sequences which may correspond to minimum rental cost are:

$S_2 = 2 – 5 – 4 – 1 – 3$

$S_3 = 4 – 5 – 2 – 1 – 3$

$S_4 = 1 – 5 – 2 – 4 – 3$

$S_5 = 3 – 5 – 2 – 4 – 1$

Form in – out tables for these sequences, we have:

For S_1: $\sum A_{i1} = 114$; $U_2(S_1) = 109$; $U_3(S_1) = 113$; $R(S_1) = 2463$

For S_2: $\sum A_{i1} = 114$; $U_2(S_2) = 91$; $U_3(S_2) = 94$; $R(S_2) = 2463$

For S_3: $\sum A_{i1} = 114$; $U_2(S_3) = 101$; $U_3(S_3) = 108$; $R(S_3) = 2463$

For S_4: $\sum A_{i1} = 114$; $U_2(S_4) = 107$; $U_3(S_4) = 112$; $R(S_4) = 2463$

For S_5: $\sum A_{i1} = 114$; $U_2(S_5) = 110$; $U_3(S_5) = 107.5$; $R(S_5) = 2463$

There for min $R\{S_K\} = 2147$ and is for the sequence S_2. Hence $S_2 = 2 – 5 – 4 – 1 – 3$ is the optimal sequence with minimum rental cost 2147 units, although total elapsed time for S_2 is not minimum.

10. Conclusion

The algorithm proposed in this paper minimizes the rental cost of machines given an optimal sequence having minimum rental cost of machines irrespective of total elapsed time. The algorithm proposed by Maggu & Dass [1982] to find an optimal to minimize the makespan/ total elapsed time is not always corresponds to minimum rental cost of machines. Hence proposed algorithm is more effective to minimize the rental cost of machine under a specified rental policy.

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