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TORSIONAL-DISTORTIONAL RESPONSE OF THIN-WALLED MONO SYMMETRIC BOX GIRDER STRUCTURES

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ABSTRACT
In this paper the differential equations for torsional-distortional analysis of mono symmetric box girder structure were derived using Vlasov's theory. The application of the derived equations was demonstrated using a single cell mono symmetric box girder structure.

The evaluation of the coefficients (Vlasov's coefficients) of the governing equations of equilibrium which form a major part of the work was accomplished using the strain mode diagrams and Morh's integral for displacement computation. Torsional-distortional equations of equilibrium for the analysis of the single cell mono symmetric box girder structure were obtained by substituting the Vlasov's coefficients into the generalized differential equations of equilibrium. A single span simply supported mono symmetric box girder structure was considered in the analysis.

The derived equations are fourth order coupled ordinary differential equations of equilibrium which were integrated by method of trigonometric series with accelerated convergence. The coupling of the equations show that there is strong interaction between torsional strain mode and distortional strain mode such that torsional analysis of a mono symmetric box girder structure can not be carried out independent of distortional analysis without introducing errors in the analysis.

Keywords: Box girder, distortion, mono- symmetric, torsion, Vlasov's theory.

1. Introduction
A thin-walled structure is one which is made from thin plates joined along their edges. The plate thickness however is small compared to other cross sectional dimensions which are in turn often small compared with the overall length of the member or structure. Thin walled structures have gained special importance and notably increased application in recent years. The wide use of these thin walled structures is due to their great carrying capability and reliability and to the economic advantage they have over solid (column and beam) structures.

Initially design of box girder bridges is related to the design of plate girder bridges. However, such design knowledge does not contain important primary conditions of cross sectional deformations such as warping torsion and distortion. The application of cross sectional deformation equations formulated by Vlasov [1], Dabrowski [2], and Varbanov [3], has opened a new way to analyze the torsional and distortional effects of loads on such girders.

The objectives of this study are to derive a set of differential equations governing the torsional-distortional behaviour of thin-walled mono symmetric box girder structures on the basis of Vlasov's theory and to apply the obtained differential equations in the analysis of single cell mono symmetric box girder section to obtain torsional and distortional deformations.

Generally, a thin walled structure can be of open cross section e.g., channel and prismatic sections, or a closed cross section such as rectangular and trapezoidal sections. Thin walled structures of closed sections are generally referred to as 'box structures'. Thus a girder structure cellular in section can be called box girder structure. These find their uses in different fields of civil engineering such as box culverts, and box girders in highway and bridge engineering.

Thin walled structures of open cross section are subjected only to bending stresses, no axial load or torsion, Heins [4]. Because of their thin wall thicknesses, the shearing resistances are constant across the thickness of the plate. On the other hand thin walled box structures may be subjected to bending, torsional and distortional stresses. Distortion alters the geometry of the cross section and generates some additional stresses thereby reducing the bearing capability of the box structural component.

2. Review of Past Work
The curvilinear nature of box girder bridges along with their complex deformation patterns and stress fields have led designers to adopt approximate and conservative methods for their analysis and design. Recent literatures: Hsu et al [5], Fan and Helwig [6], Sennah and Kennedy [7], on straight and curved box girder bridges, deal with analytical formulations to better understand the behaviour of these complex structural systems. Few authors: Okil and El-tawil [8], Sennah and Kennedy [7], have undertaken experimental studies to investigate the accuracy of existing methods. Before the advent of Vlasov's theory of thin-walled beams [1], the conventional method of predicting

warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure

Several investigators; Bazant and El-Nimeiri [9], Zhang and Lyons [10], Boswell and Zhang [11], Usuki [12], Waldron [13], Paavola [14], Razaqpur and Li [15], Fu and Hsu [16], Tesar [17], have combined thin-walled beam theory of Vlasov and the finite element technique to develop a thin walled box element programs for elastic analysis of straight and curved cellular bridges.

Thus, various theories were postulated by different authors examining methods of analysis, both classical and numerical. A few others however carried out tests on prototype models to verify the authenticity of the theories. At the end of it all, the authors concluded that Vlasov's theory captures all peculiarities of cross sectional deformations such as warping, torsion, distortion etc, and therefore adopted the method

3. Energy Formulation of the Equilibrium Equations

The longitudinal warping and transverse (distortional) displacements given by Vlasov [1] are:

$$u(x,s)=U(x)\varphi(s), \quad v(x,s)=V(x)\psi(s) \tag{1}$$

These displacements may be represented in series form as:

$$u(x,s)=\sum_{i=1}^{m}U_i(x)\varphi_i(s)$$
$$v(x,s)=\sum_{k=1}^{n}V_k(x)\psi_k(s) \tag{2}$$

where, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the box girder frame. $\varphi_i(s)$ and $\psi_k(s)$ are elementary displacements of the strip frame, respectively out of the plane (m displacements) and in the plane (n displacements). These displacements are chosen among all displacements possible, and are called the generalized strain coordinates of a strip frame.

From the theory of elasticity the strains in the longitudinal and transverse directions are given by:

$$\frac{\partial u(x,s)}{\partial x}=\sum_{i=1}^{m}U_i'(x)\varphi_i(s)$$
$$\frac{\partial v(x,s)}{\partial x}=\sum_{k=1}^{n}V_k'(x)\psi_k(s) \tag{3}$$

The expression for shear strain is: $\gamma(x,s)=\dfrac{\partial u}{\partial s}+\dfrac{\partial v}{\partial x}$ or

$$\gamma(x,s)=\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x) \tag{4}$$

Using φ_i *and* ψ_i displacement fields, and basic stress-strain relationships of the theory of elasticity the expressions for normal and shear stresses become:

$$\sigma(x,s)=E\frac{\partial u(x,s)}{\partial x}=E\sum_{i=1}^{m}\varphi_i(s)U_i'(x) \tag{5}$$

and $\tau(x,s)=G\gamma(x,s)$

$$=G\left[\sum_{i=1}^{m}\varphi_i'(s)U_i(x)+\sum_{k=1}^{n}\psi_k(s)V_k'(x)\right] \tag{6}$$

Transverse bending moment generated in the box structure due to distortion is given by:

$$M(x,s)=\sum_{k=1}^{n}M_k(s)V_k(x) \tag{7}$$

where $M_k(s)$ = bending moment generated in the cross sectional frame of unit with due to a unit distortion, V(x) = 1

The potential energy of the box structure under the action of a distortional load of intensity q is given by:

$$\Pi=U+W_E \tag{8}$$

where,

Π = the total potential energy of the box structure,

U = Strain energy,

W_E = External potential or work done by the external loads.

From strength of material, the strain energy of the box structure is given by

$$U=\frac{1}{2}\int_L\int_S\left[\left(\frac{\sigma^2(x,s)}{E}+\frac{\tau^2(x,s)}{G}\right)t(s)+\frac{M^2(x,s)}{EI_{(s)}}\right]dxds \tag{9}$$

The work done by external load is given by;

$$W_E=-qv(x,s)dxds$$
$$=-\int_s\int_x q\sum V_h(x)\varphi_h(s)dsdx$$
$$=-\int_x\sum q_hV_hdx \tag{10}$$

Substituting eqns.(9 and 10) into eqn.(8) we obtain:

$$\Pi = \frac{1}{2}\left[\int_L \int_S \begin{matrix} \left[\frac{\sigma^2(x,s)}{E} + \frac{\tau^2(x,s)}{G}\right] t(s) \\ + \frac{M^2(x,s)}{EI_s} - 2qv(x,s) \end{matrix}\right] dxds \qquad (11)$$

where,

$\sigma(x,s)$ = Normal stress

$\tau(x,s)$ = Shear stress

$M(x,s)$ = Transverse distortional bending moment

q = Line load per unit area applied in the plane of the plate

$I_s = \frac{t^3(s)}{12(1-\nu^2)}$ = Moment of inertia

E = Modulus of elasticity

G = Shear modulus

ν = poisson ratio

t = thickness of plate

Substituting eqns (1), (5), (6) and (7) into eqn.(11), and simplifying noting that $t(s)ds = dA$, we obtain the potential energy of the box structure as follows:

$$\Pi = \frac{E}{2}\sum a_{ij}U_i\,'(x)U_j\,'(x)dx +$$
$$\frac{G}{2}\left[\sum b_{ij}U_i(x)U_j(x) + \sum c_{kj}U_k(x)V_j\,'(x)\right]dx +$$
$$+\frac{G}{2}\left[\sum c_{ih}U_i(x)V_h\,'(x) + \sum r_{kh}V_k\,'(x)V_h\,'(x)\right]dx +$$
$$+\frac{E}{2}\sum s_{hk}V_k(x)V_h(x)dx - \sum q_h V_h dx \qquad (12)$$

where the (Vlasov's) coefficients are defined as follows.

$$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA \qquad (a)$$
$$b_{ij} = b_{ji} = \int \varphi_i'(s)\varphi_j'(s)dA \qquad (b)$$
$$c_{kj} = c_{jk} = \int \varphi_k'(s)\psi_j(s)dA \qquad (c)$$
$$c_{ih} = c_{hi} = \int \varphi_i'(s)\psi_k(s)dA \qquad (d) \qquad (13)$$
$$r_{kh} = r_{hk} = \int \psi_k(s)\psi_h(s)dA; \qquad (e)$$
$$s_{kh} = s_{hk} = \frac{1}{E}\int \frac{M_k(s)M_h(s)}{EI_{(s)}}ds \qquad (f)$$
$$q_h = \int q\psi_h ds$$

The governing equations of torsional-distortional equilibrium are obtained by minimizing the energy functional eqn.(12), with respect to its functional variables u(x) and v(x) using Euler Lagrange technique, Elgolts [18].

Minimizing with respect to u(x) we obtain:

$$k\sum_{i=1}^{m} a_{ij}U_i\,''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k\,'(x) = 0 \qquad (14)$$

Minimizing with respect to v(x) we have:

$$\sum c_{ih}U_i\,'(x) - \kappa\sum s_{hk}V_k(x) +$$
$$+\sum r_{kh}V_k\,''(x) + \frac{1}{G}\sum q_h = 0 \qquad (15)$$

where $\kappa = \frac{E}{G} = 2(1+\nu)$

Equations (14) and (15) are Vlasov's generalized differential equations of distortional equilibrium for a box girder. They are presented in matrix notation as eqns.[16 (a) and (b)].

$$\kappa\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{Bmatrix} U_1'' \\ U_2'' \\ U_3'' \end{Bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} -$$

$$-\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{bmatrix}\begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{Bmatrix} = 0 \qquad (16a)$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{bmatrix}\begin{Bmatrix} U_1' \\ U_2' \\ U_3' \end{Bmatrix} - \kappa\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{41} & s_{43} & s_{44} \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} +$$

$$+\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}\begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4'' \end{Bmatrix} + \frac{1}{G}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0 \qquad (16b)$$

4. Generation of Strain Modes and Evaluation of Vlasov's Coefficients

From the energy formulation of the equilibrium equation we noted that φ and ψ represent generalized warping and distortional strain modes respectively, and that $\varphi_i(s)$ and $\psi_k(s)$ are elementary displacements of the strip frame out of the plane (m displacements) and in the plane (n displacements) respectively. Thus, Vlasov's coefficients of differential equations of equilibrium, eqn.(13), which involve a combination of these elementary

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displacements and their derivatives may be obtained by consideration of the box girder cross section as a strip frame, and then applying unit displacement one after the other at the nodal points of the frame in longitudinal direction, to determine the corresponding out of plane displacement of every joint regarded as fixed) on the frame. By applying another set of unit displacements at the joints in **n** possible transverse directions, the corresponding transverse (in-plane) displacements can also be obtained. The first order derivatives of these displacement functions may be obtained by numerical differentiation and used for computation of the coefficients with the aid of Morh's integral for displacement computations.

A consideration of the single cell mono-symmetric strip frame in Fig.1(a) shows that it has four degrees of freedom in the longitudinal direction and four in the transverse direction. From eqn.(2), where in this case m = 4 and n = 4, it follows that we have sixteen displacement quantities to compute and hence, sixteen differential equations of distortional equilibrium will be required. For multi-celled profiles the number of degrees of freedom will increase and hence the number of independent displacement quantities (m + n) will require 2(m x n) differential equations to solve for the displacement quantities and this can be quite cumbersome. The application of Vlasovs generalized strain modes as modified by Varbanov [3], reduces the number of displacement quantities and hence the differential equations of equilibrium required to solve for them to seven, irrespective of the number of degrees of freedom possessed by the structure.

In the generalized strain modes, there are three strain fields in the longitudinal direction, φ_1, φ_2 and φ_3 and four, ψ_1, ψ_2, ψ_3, ψ_4 in the transverse direction. Thus, from eqn (2) we have,

$$u(x,s) = U_1(x)\varphi_1(s) + U_2(x)\varphi_2(s) + \\ + U_3(x)\varphi_3(s) \qquad (17)$$

$$v(x,s) = V_1(x)\psi_1(s) + V_2(x)\psi_2(s) + V_3(x)\psi_3(s) + \\ + V_4(x)\psi_4(s) \qquad (18)$$

where

φ_1 = o ut of plane displacement parameter when the load is acting (vertically) normal to the top flange of the girder, i.e., bending is about horizontal axis.

φ_2 = out of plane displacement parameter when the load is acting tangential to the plane of the flanges, i.e., bending is about vertical axis.

φ_3 = out of plane displacement parameter due to distortion of the cros section, i.e., the warping function .

ψ_1 = in-plane displacement parameter due to the load giving rise to φ_1

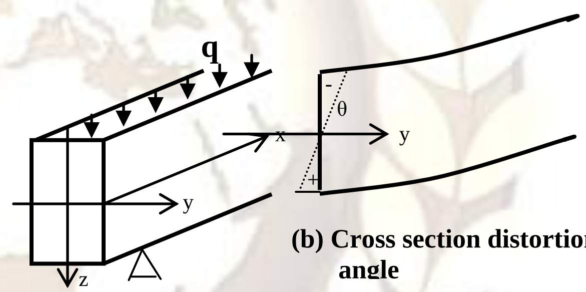
ψ_2 = in-plane displacement parameter due to the load giving rise to φ_2

ψ_3 = in-plane displacement parameter due to the distortion of the cross section, i.e., non uniform torsion.

ψ_4 = in-plane displacement function due to pure rotation or Saint Venant torsion of the cross section.

5. Strain Mode Diagrams

Considering a simply supported girder loaded as shown in Fig.1(a), if we assume the normal beam theory, i.e., neutral axis remaining neutral before and after bending, then the distortion angle of the cross section will be as shown in Fig.1(b) where, θ is the distortion angle (rotation of the vertical axis). The displacement φ_1 (strain mode 1) at any distance R, from the centroid is given by $\varphi_1 = R\theta$. If we assume a unit rotation of the vertical (z) axis then $\varphi_1 = R$, at any point on the cross section. Thus, φ_1 is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian.



(b) Cross section distortion angle

(a) Simply supported girder under vertical line load

Fig. 1 Simply supported girder and cross section distortion

Similarly, if the load is acting in horizontal (y- y) direction, normal to the x-z plane then the bending is in x-z plane and y axis is rotated through angle θ_2 giving rise to φ_2 (strain mode 2) displacement out of plane. The values of φ_2 are obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian. The warping function φ_3 (strain mode 3), of the beam cross section is the out of plane displacement of the cross section when the beam is twisted about its axis through the pole, one radian per unit length without bending in either z or y direction and without longitudinal extension. ψ_1 and ψ_2 are in-plane displacements of the cross section in x-z and x-y planes respectively while ψ_3 is the distortion of the cross section.

In an un published work the authors have shown that these in-plane displacement quantities ψ_1, ψ_2 and ψ_3 are the same as the derivatives of their corresponding out of plane displacements. Consequently, ψ_1, ψ_2 and ψ_3 are obtained by numerical differentiation of φ_1, φ_2 and φ_3 diagrams respectively. In strain mode 4, ψ_4 is the displacement diagram of the beam cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section. It is assumed to be positive if the member moves in the positive directions of the coordinate axis and negative otherwise.

Fig.2 shows a single cell mono symmetric box girder section and its strain mode diagrams used for numerical example. The coefficients $a_{ij}, b_{ij}, c_{kj}, c_{ih}$ and r_{kh}, of the governing equations of equilibrium are computed with the aid of Morh's integral chart using the strain mode diagrams.

6. Formulation of Differential Equations of Equilibrium for Mono Symmetric Box Girder Structure

The relevant coefficients for torsional-distortional equilibrium (strain modes 3 and 4), are a_{33}, b_{33}, c_{33}, c_{34}, r_{33}, r_{34}, r_{44} and s_{33}. Substituting these into the matrix notation eqns.(16a) and (16b) and multiplying out we obtain:

$$ka_{33}U_3'' - b_{33}U_3 - c_{33}V_3' - c_{34}V_4' = 0 \qquad (19)$$

$$c_{33}U_3' - ks_{33}V_3 + r_{33}V_3'' + r_{34}V_4'' = -\frac{q_3}{G} \qquad (20)$$

$$c_{43}U_3' + r_{43}V_3'' + r_{44}V_4'' = -\frac{q_4}{G} \qquad (21)$$

Simplifying further we obtain the coupled differential equations of torsional-distortional equilibrium for mono symmetric sections as follow:

$$\begin{aligned} &\beta_1 V_4'' - \gamma_1 V_3 = K_1 \quad (a) \\ &V_3^{iv} + \alpha_2 V_4^{iv} - \beta_2 V_4'' = K_2 \quad (b) \end{aligned} \qquad (22)$$

where, $\alpha_2 = \dfrac{r_{44}}{c_{43}}$, $\beta_1 = r_{34}c_{43} - c_{33}r_{44}$; (23)

$$\beta_2 = \frac{b_{33}r_{44} - c_{34}c_{43}}{ka_{33}c_{43}}, \quad \gamma_1 = c_{43}ks_{33}; \qquad (24)$$

$$K_1 = c_{33}\frac{\bar{q}_4}{G} - c_{43}\frac{\bar{q}_3}{G}; \quad K_2 = \left(\frac{b_{33}}{ka_{33}c_{43}}\right)\frac{\bar{q}_4}{G} \qquad (25)$$

7. Torsional-Distortional Analysis of Mono Symmetric Box Girder Structure

In this section the solutions of the differential equations of equilibrium eqns.[16(a) and (b)] are obtained for the single cell mono symmetric box girder section shown in Fig.2(a). Live loads are considered according to AASHTO-LRFD [19], following the HL-93 loading. Uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110 KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A three span simply supported bridge deck structure, 50m per span, was considered.

The obtained torsional loads are as follows;

$\bar{q}_3 = 157.16KN, \quad \bar{q}_4 = 1446.505KN$.

The governing equations of equilibrium are

$$\begin{aligned} &\beta_1 V_4'' - \gamma_1 V_3 = K_1 \\ &\alpha_1 V_3^{iv} + \alpha_2 V_4^{iv} - \beta_2 V_4'' = K_2 \end{aligned} \qquad (26)$$

The values of the relevant coefficients are,

$$a_{33} = 0.750; \; b_{33} = c_{33} = r_{33} = 1.407$$

$$c_{34} = c_{43} = r_{34} = r_{43} = 1.265; \; r_{44} = 14.616$$

$$s_{33} = 0.261*6.9712*10^{-4} = 1.8195*10^{-4}$$

The parameters for the governing equations are,

$$\alpha_2 = ka_{33}r_{44} = 27.405$$

$$\beta_1 = r_{34}c_{43} - c_{33}r_{44} = -18.964$$

$$\beta_2 = b_{33}r_{44} - c_{34}c_{43} = 18.964$$

$$\gamma_1 = c_{43}ks_{33} = 5.503*10^{-4}$$

$$K_1 = c_{33}\frac{\bar{q}_4}{G} - c_{43}\frac{\bar{q}_3}{G} = 1.9163*10^{-4}$$

$$K_2 = \left(\frac{b_{33}}{ka_{33}c_{43}}\right)\frac{\bar{q}_4}{G} = 2.120*10^{-4}$$

$E = 24*10^9 N/m^2, G = 9.6*10^9 N/m^2, k = 2.5$ Substituting these parameters into eqn.(26) we obtain:

$$\begin{aligned} &-18.964V_4^{iv} - 5.503*10^{-4}V_3 = 1.9163*10^{-4} \\ &2.371V_3^{iv} + 27.405V_4^{iv} - 18.963V_4'' = 2.120*10^{-6} \end{aligned} \qquad (27)$$

Integrating by method of trigonometric series with accelerated convergence we have

$$\begin{aligned} &V_3(x) = 3.268*10^{-2} Sin(\pi x/50) \\ &V_4(x) = 2.80*10^{-3} Sin(\pi x/50) \end{aligned} \qquad (28)$$

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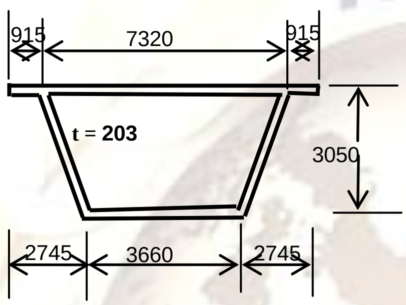
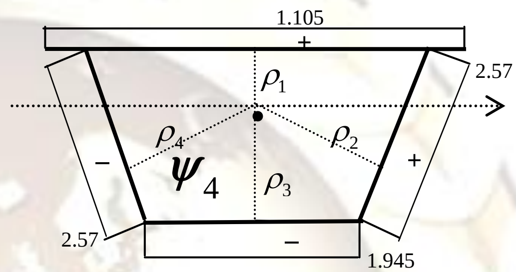
8. Discussion of Results

The governing differential equations of torsional-distortional equilibrium for mono symmetric box girder structures are given by eqn.(22). They are applicable to both single cell and multi-cell mono symmetric profiles. For the single cell mono symmetric box girder section, the torsional and distortional deformations obtained by integration of eqn.(27) are given by eqn.(28) and graphically presented in Fig.3.

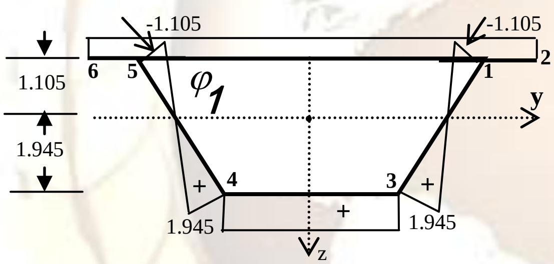
The maximum (mid span) torsional displacement was 3mm while the mid span deformation was 33mm. Thus, the maximum distortional deformation is eleven times that of torsional deformation.



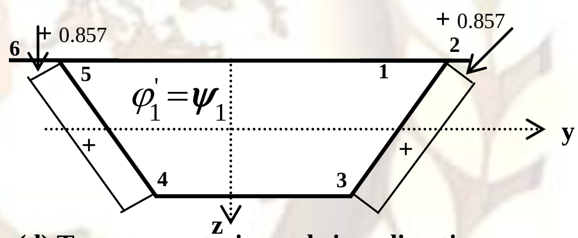
(a) Single cell box girder section



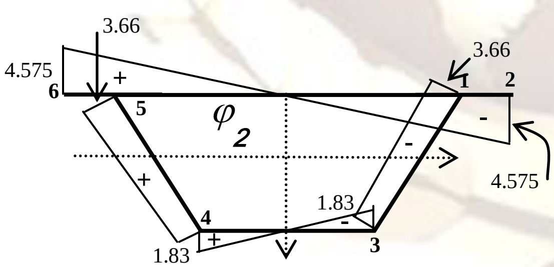
(b) Pure Rotation Diagram



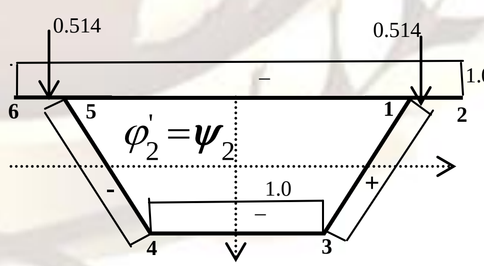
(c) Longitudinal Strain Mode Diagram
(Bending about o-y axis)



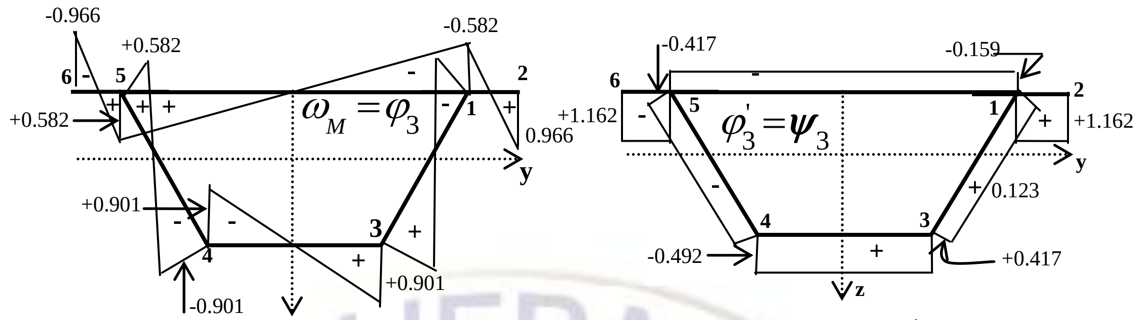
(d) Transverse strain mode in z-direction



(e)Longitudinal Strain Mode Diagram
(Bending about z-z axis)



(f) Transverse strain mode in z-direction



(g) Warping Function Diagram (h) Distortion Diagram $\varphi_3'(s) = \psi_3(s)$

Fig.2 Generalized strain modes for single cell mono-symmetric box girder frame

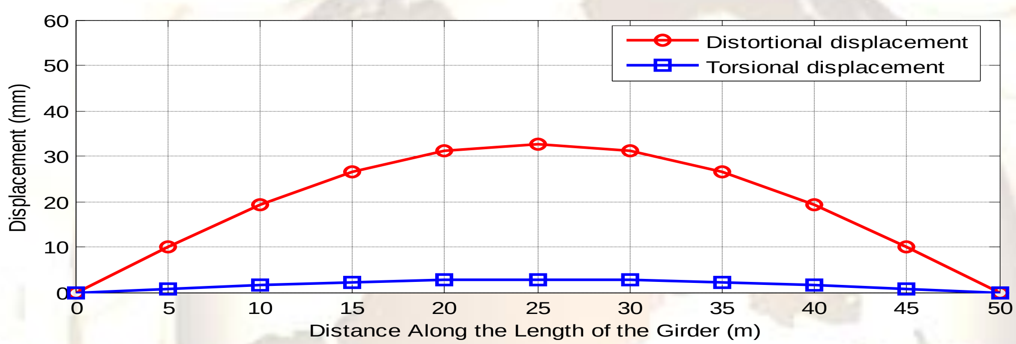


Fig.3 Variation of torsional and distortional displacements along the length of the girder

9. Conclusions

The obtained governing differential equations of torsional-distortional equilibrium are fourth order coupled linear differential equations.

The coupling of the equations of torsional-distortional equilibrium reveal a strong interaction between torsional strain mode and distortional strain mode such that torsional analysis of a mono symmetric box girder structure can not be carried out independent of distortional analysis without introducing errors in the analysis.

The derived equations of equilibrium are suitable for torsional-distortional analysis of all mono symmetric thin-walled box girder structures with single cell or multiple cell profiles.

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