# Capacity rates of erasure channels in Single Hop Networks

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**ABSTRACT :**

**This paper deals with finding the capacity rates for single hop networks in erasure channels. As the importance of erasure channels are well identified, this paper finds the capacity rates for complex multi sender relay network by using the concept of mutual information with respect to information theory concepts in terms of erasure probabilities. Even a comparative analysis is done between the capacities of degraded and non degraded channels in case of single relay network.**

***Keywords*–**Capacity rates, Degraded, Erasure Channel, Erasure Probabilities, Mutual Information

## 1. INTRODUCTION

Generally, a relay network is a network with one sender, one receiver and a number of intermediate nodes that participate in the communication by relaying packets from source to receiver. Here, we consider a single hop network in which communication from source to relay takes place over a channel, the channel connecting these nodes is supposed to be an erasure channel. Erasure channels are those in which the symbols are received correctly without any error, or lost due to buffer overflows or excessive delays. Erasure channels are chosen as the work can be continued easily to another channels basing on this. In this paper, we are going to derive the capacity rates of various complex relay networks in terms of erasure probabilities. The binary erasure channel with erasure probability $p_e$ is shown in the fig.1

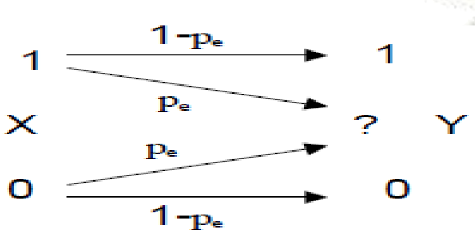


Fig.1: Binary Erasure Channel with Erasure Probability $p_e$

## 2. CAPACITY BOUNDS

This section will present the capacity rates of erasure relay channel by using the concept of mutual information. For two random variables *X* and *Y*, the mutual information is given by

$$I(X;Y) = H(X)H(X|Y)$$

$$= \sum_{x,y} p(x,y) \log\left(\frac{p(x,y)}{p(x)p(y)}\right) \quad (1)$$

We first begin with the simple single relay case and extend the analysis to multi sender situations.

### 2.1. SINGLE RELAY CASE WITH SINGLE SENDER:

We consider the simplest case of an erasure relay channel which is a network composed of one sender, one receiver and one intermediate or relay node shown in fig. 2. In this case, the relay channel can be described with five random variables $X^0$, $X^1$, $Y^0$, $Y^1$ and $Y^2$ and a conditional probability density function $p(y^0, y^1, y^2|x^0, x^1)$ .This function gives the probability that when $x^0$ is sent by sender and $x^1$ by the relay, $y^0$and $y^2$ are received at the receiver and the relay receives $y^1$. We further define $Y = (Y^0, Y^2)$ as the received variable from sender and relay at the receiver. Let's consider a special case of relay network where the relay channel consists of two separate and independent (non interfering) erasure broadcast channels: from sender to all relays and the receiver $((X^0 ; Y^0 ,Y^1))$ and from relay to all other relays and the receiver $(X^1 ;Y^2 )$ (in the single relay case this broadcast channel is only point to point). We further assume that the erasure probabilities are p, p1 and p2, where p, p1 and p2 are the packet loss rate of sender-receiver, sender-relay and relay-receiver links as shown in Fig.2 .We assume that no interference occurs between the relay to receiver and the sender to receiver transmissions. This separation can be achieved through using different physical channels, for example by making the sender operating in frequency f1 and the relay sending over frequency f2.

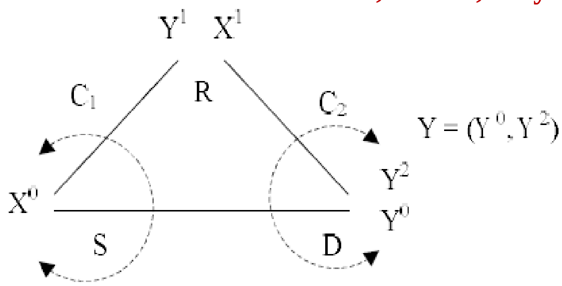


Fig .2 : Single Relay Channel

Even, we consider the same single relay channel whose relay and sender information are interfering i.e. for degraded case. The capacity rates can be obtained by mutual information concept using shearer theorem.

**Theorem 1(Capacity region):** The capacity region of the relay channel in Fig.2 is bounded by

$$R = \sup_{p(x^1,x^2)} \min \{I(X^0; Y^0, Y^1) - I(X^1; Y^0, Y^1), \; I(X^0; Y^0) + I(X^1; Y^2|X^0)\}$$

Moreover, if the sender to relay channel is more capable than the sender to receiver channel, i.e. $I(X^0; Y^0) < I(X^0; Y^1)$, the bound is simplified as:

$$R = \sup_{p(x^1,x^2)} \min\{ I(X^0; Y^1) - I(X^1; Y^1), \; I(X^0; Y^0) + I(X^1; Y^2|X^0)\}$$

**Theorem 2** The capacity region over an erasure degraded relay channel is bounded as below:

$$R \leq \max_{\alpha} \min\{(1 - p.p1), (1 - p) + \alpha(1 - p2)\} \quad (3)$$

Under the situation that the sender to relay channel is more capable than the sender to receiver channel the bound can be tightened and become:

$$R \leq \max_{\alpha} \min\{(1 - p1), (1 - p) + \alpha(1 - p2)\} \quad (4)$$

Where p1 is the loss probability between sender and relay, p2 is the loss probability between relay and receiver and p is the loss probability between sender and receiver. α is a coupling parameter 0 to 1.

### 2.2.MULTI SENDER RELAY CHANNEL:

The specific multi-sender relay network shown in the Fig.3 is a network composed of two senders (S1 and S2) and two receivers (D1 and D2) .The sender Si, i = 1, 2 sends information to the two receivers Dj , j = 1, 2. Simultaneously each sender might acts as a relay for the other sender. The Multi-sender relay channel can be described with 8 random variables Xi, i = 1, 2 representing the symbols sent by the sender, Yij , i, j = 1, 2 representing symbols received from each sender by each receiver and $Yi^s$ representing the symbol received by sender i from the other sender.

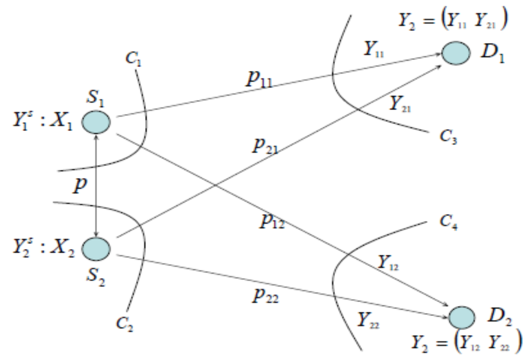


Fig 3: Multi sender relay network

**Theorem 3 (Capacity region)** Under the hypothesis that X1 and X2 being independent the capacity region of multiple-Input relay channel in Fig. 3 is bounded by :

$$\begin{aligned} R_1^* &\leq I(X_1; Y_2^s Y_{11}, Y_{12}) \\ R_2^* &\leq I(X_2; Y_1^s Y_{21}, Y_{22}) \quad (5) \\ R_1^* + R_2^* &\leq I(X_1; Y_{11}) + I(X_2, Y_{21}) \\ R_1^* + R_2^* &\leq I(X_1; Y_{12}) + I(X_2, Y_{22}) \end{aligned}$$

**Theorem 4**: The capacity region bound over a multiple-Input erasure relay channel is bounded as:

$$\begin{aligned} R_1^* &\leq (1-p) \\ R_2^* &\leq (1-p) \quad (6) \\ R_1^* + R_2^* &\leq (1-p_{11}) + (1-p_{21}) \\ R_1^* + R_2^* &\leq (1-p_{12}) + (1-p_{22}) \end{aligned}$$

The first two bounds in this theorem are constraints bounding the rate available for collaboration between the two senders. The two last bounds are bounding the amount of information coming in the receiver.
So, now we are extending the analysis to the different cases of multiple sender relay networks.

**Case 1:** Consider the network having n senders and 2 receivers as shown in the Fig.4

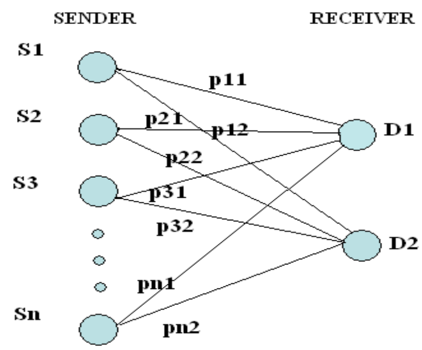


Fig 4: Multiple senders and 2 receivers relay network

The network composed of n senders (S1,S2, S3….Sn) and two receivers (D1 and D2).The sender Si, i = 1,2,3,…n sends information to the two receivers Dj , j = 1, 2.Simultaneously each sender might acts as a relay. The erasure probabilities of senders Si, i=1,2,3,---n to receiver D1 are given by p11,p21,p31…pn1 and the corresponding probabilities from senders Si to receiver D2 are given by p12,p22,p32…pn2.

**Theorem 5:** The capacity rate of erasure multi sender relay network shown in Fig.4 is given by
$R1^*+R2^*+\text{-------}+Rn^* \leq \min\{(1-p11)+(1-p21)+(1-p31)+\text{-------}+(1-pn1),(1-p12)+(1-p22)+(1-p32)+\text{-------}+(1-pn2)\}$ (7)

Where $R_i^*$ represents the rate at which the common information is broadcasted by the senders to all receivers. Here i give the index of the sender.

**Case2:** Consider another case of relay network having single sender and n receivers as shown in Fig.5. The network composed of single sender S and multiple receivers Di where i=1,2,3…n. The sender S sends the information to the multiple receivers Di. The erasure probabilities for sender to receiver link are given by p11, p12, p13….p1n.

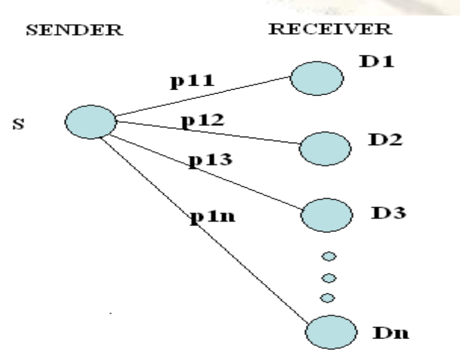


Fig 5: Single sender and Multiple receivers relay network

**Theorem 6:** The capacity region for the multi sender relay network shown in the Fig.5 is given by

$R1^* \leq \{(1-p11), (1-p21),(1-p31),\ldots\ldots..,(1-pn1)\}$ (8)

**Case3**: Consider a complex multi sender relay network having a relay channel between the two senders and two receivers. It is shown in the Fig.6

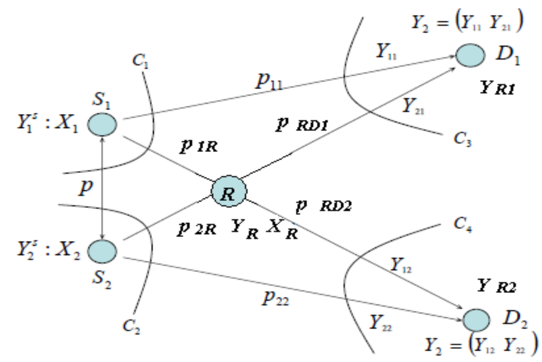


Fig.6: 2 senders and 2 receivers network with relay channel

R represents the relay network and $p_{RD1}$ and $p_{RD2}$ give the erasure probabilities between the relay and receiver links.$p_{1R}$ and $p_{2R}$ denotes erasure probabilities of source to relay links.

**Theorem 7:** The capacity region of a multiple input relay channel which consists of a relay channel between the senders and receivers is given by

$R_1^* \leq \sup\ p(X_1,X_2)\min\{I(X_1;Y_{11},Y_R,Y_2^s)-I(X_R;Y_{11},Y_R,Y_2^s),\ I(X_1;Y_{11})+I(X_R,Y_{R1}/X_1)\}$

$R_2^* \leq \sup\ P(X_1,X_2)\ \min\{I(X_2;Y_{22},Y_R,Y_1^s)-I(X_R;Y_{22},Y_R,Y_1^s),\ I(X_2;Y_{22})+I(X_R,Y_{R2}/X_2)\}$

$R_1^* + R_2^* \leq I(X_1;Y_{11}) + I(X_R,Y_{21}/X_2)$ (9)

$R_1^* + R_2^* \leq I(X_2;Y_{22}) + I(X_R,Y_{22}/X_1)$

It is simpler if we consider the capacity rates equations in terms of erasure probabilities

For maximizing,

$I(X_R;Y_{11},Y_R,Y_2^s)=0$ and $I(X_R;Y_{22},Y_R,Y_1^s)=0$

$I(X_1;Y_{11},Y_R,Y_2^s)=(1-p_{11}.p_{1R}.p)H(X_1)$

$I(X_1;Y_{11})=(1-p_{11})H(X_1)$

$I(X_R, Y_{22}/X_1)=(1-p_{RD1})\alpha H(X_1)$

So the capacity rate of sender 1 is given by

$R_1^* \leq \min \{(1-p_{11}.p_{1R}.p), (1-p_{11})+\alpha(1-p_{RD1})\}$

Similarly, the capacity rate for sender 2 is given by

$R_1^* \leq \min\{(1-p_{22}.p_{2R}.p), (1-p_{22})+\alpha(1-p_{RD2})\}$

$$R_1^* + R_2^* \leq (1-p_{11}) + \alpha(1-p_{RD1}) \qquad (10)$$

$$R_1^* + R_2^* \leq (1-p_{22}) + \alpha(1-p_{RD2})$$

$$R_1^* + R_2^* \leq \min\{(1-p_{11}) + \alpha(1-p_{RD1}), (1-p_{22}) + \alpha(1-p_{RD2})\}$$

## 3. SIMULATED RESULTS

Obtained capacity regions are simulated using MATLAB software by taking erasure probabilities on X-axis and capacity rates on Y-axis. Comparative analysis between degraded and non degraded cases of single relay network is done. Results are drawn by varying erasure probabilities and coupling parameter. Results show that non-degraded case is better than degraded case in single relay networks. Even the capacity rates of multi sender relay networks discussed are also plotted.

### 3.1. Comparison of capacity region under degraded and non degraded hypothesis for a relay channel:

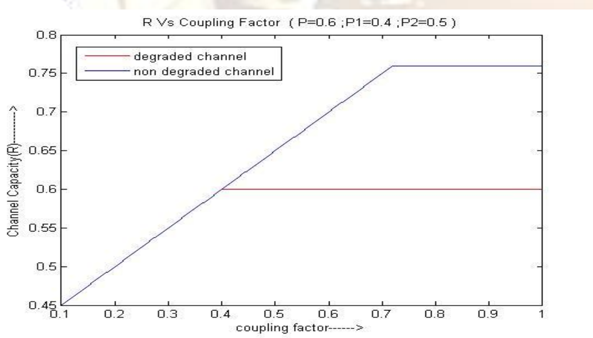


Fig. 7.Capacity region of erasure relay channel with parameters , Coupling parameter varies from 0 to 1; p=0.6; p1=0.4; p2=0.5; Non degraded channel has the maximum capacity rate with 0.76 than degraded channel with 0.6 with coupling factor =1

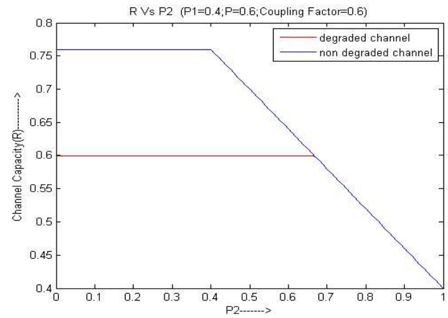


Fig. 8.Capacity region of erasure relay channel with parameters ,Coupling parameter=0.6; p=0.6; p1=0.4; p2=varies from 0 to 1; Non degraded channel has the maximum capacity rate with 0.76 than degraded channel with 0.6

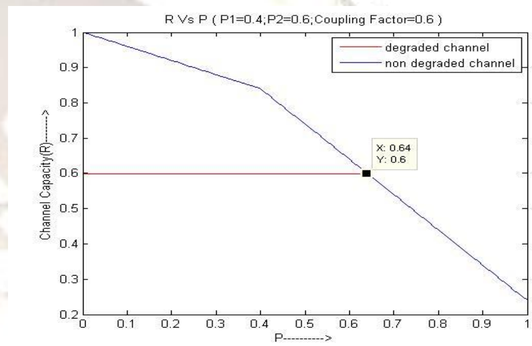


Fig.9. Capacity region of erasure relay channel with parameters Coupling parameter=0.6; p1=0.4; p2=0.6; p varies from 0 to 1; Non degraded channel capacity reaches maximum value as p decreases

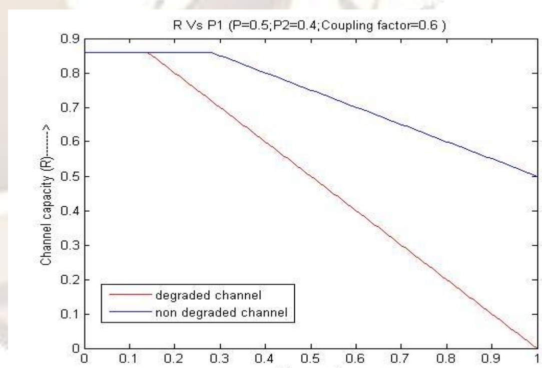


Fig.10 .Capacity region of erasure relay channel with parameters Coupling parameter=0.6; p=0.5; p2=0.4; p1 varies from 0 to 1; Non degraded case have maximum capacity rate when compared to degraded case as p1 increases to 1

**3.2.Capacity region of Multi Sender Relay Networks:**

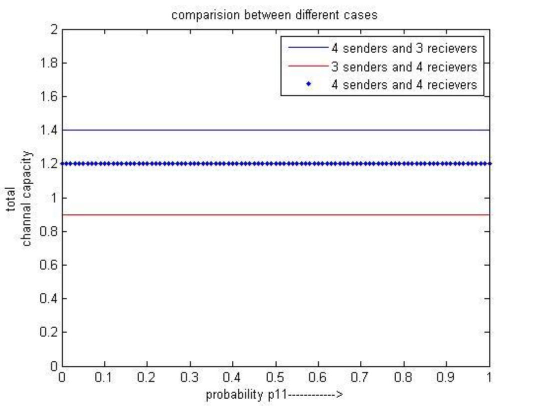


Fig.11: Comparision between capacity rates for multiple senders and multiple receivers

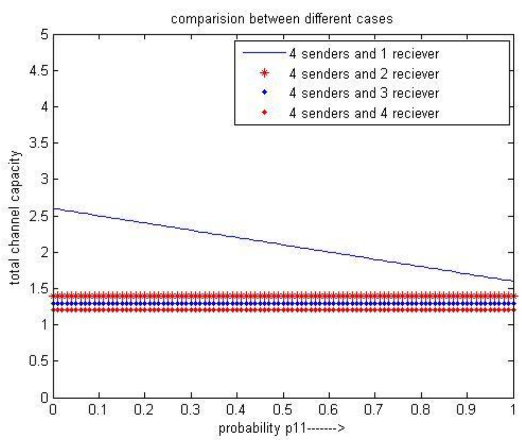


Fig.12.Capacity rates of erasure relay network having 4 senders and different number of receivers

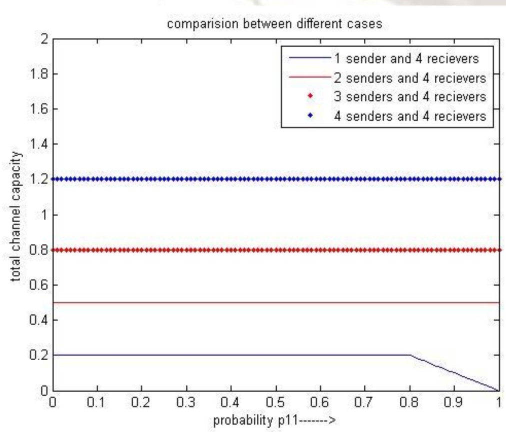


Fig.13.Capacity region of erasure relay network having single sender and different number of receivers

## 4. CONCLUSION

We have derived the capacity rate for complex multi sender relay networks by considering different cases by using the concept of mutual information. Even comparative analysis is done for degraded and non degraded cases in single relay network. In addition to single relay network, channel capacities of multi sender relay networks are also plotted .It is observed that the capacity rates of non degraded channel are higher than the degraded relay channel. Later we can extend the work to many more complex networks having multiple relays.

## 5. ACKNOWLEDGMENT

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