A NEW METHOD OF ORDER REDUCTION FOR HIGH ORDER INTERVAL SYSTEMS USING LEAST SQUARES METHOD

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Abstract: The paper presents the application of Least Squares moment matching method about a general point 'a' for the reduction of high order Interval systems. A heuristic criteria have been employed for selecting the linear shift point 'a' based upon the harmonic mean of real parts of the poles of four high order fixed systems obtained by Kharitonov's Theorem. The denominator polynomials are obtained by least square method and the numerator are obtained using the moment matching .A numerical example illustrates the proposed procedure.

Keywords: Least Squares Mean, Large scale Interval system, order reduction.

1. Introduction

Shoji et al [1] proposed an idea of using least squares moment matching techniques for fixed linear time invariant .This method was refined by Lucas and Beat [1] and was extended to include the use of Markov parameters [1] .If the system transfer function contains a pole of magnitude less than one, then numerical problems can arise owing to a rapid increase in the magnitude of successive time moments .This gives an ill conditioned set of linear equations to solve for the reduced denominator .To overcome this problem ,it is sometimes possible to use a linear shift s→ (s+a)

such that the pole of smallest magnitude has the modulus of approximately one, this tends to reduce the sensitivity of the method . However, the focus of the work so far appears to concentrate mainly on the basic idea of extending this technique for order reduction of fixed parameter systems. In this paper this method is extend for order reduction of high order Interval systems.

2. Main procedure

Let the transfer function of a high order interval systems be represented

$$H(s) = \frac{[a_0^-, a_0^+] + [a_1^-, a_1^+]s + \cdots + [a_{n-1}^-, a_{n-1}^+]s^{n-1}}{[b_0^-, b_0^+] + [b_1^-, b_1^+]s + \cdots + [b_n^-, b_n^+]s^n} \quad \ldots (1)$$

Where $[a_i^-, a_i^+]$ for i = 0 to n-1 and $[b_i^-, b_i^+]$ for i = 0 to n are the interval parameters.
Consider now the set δ(s) of real polynomials of degree 'n' of the form

$$\delta(s) = \delta_0 + \delta_1 s + \cdots + \delta_n s^n \ \ldots (2)$$

Where the coefficients lie within given ranges

$$\delta_0 \in [x_o, y_o], \delta_1 \in [x_1, y_1], \ldots \delta_n \in [x_n, y_n]$$

Write $\delta = [\delta_0, \delta_1, \ldots\ldots\ldots\ldots, \delta_n]$

using the Kharitonov's theorem the following four extreme polynomials are derived

$$K_1(s) = x_0 + x_1 s + y_2 s^2 + y_3 s^3 + \cdots$$
$$K_2(s) = x_0 + y_1 s + y_2 s^2 + x_3 s^3 + \cdots$$
$$K_3(s) = y_0 + x_1 s + x_2 s^2 + y_3 s^3 + \cdots$$
$$K_4(s) = y_0 + y_1 s + x_2 s^2 + x_3 s^3 + \cdots$$

From the above equations the numerator and the denominator polynomials are obtained.

Thus the four nth order system transfer functions are obtained each defined as

$$G_p(s) = \frac{A_{P0} + A_{P1}s + A_{P2}s^2 + \cdots + A_{Pn-1}s^{n-1}}{B_{P0} + B_{P1}s + B_{P2}s^2 + \cdots + B_{pn}s^n} \quad \ldots\ldots\ldots\ldots (3)$$

Where p = 1, 2, 3, 4. and
n = order of the original system.

Replace the $G_p(s)$ by $G_p(s+a)$ where the value of 'a' obtained by harmonic mean.

Let the nth order system transfer function of $G_p(s)$ is given by:

$$G_p(s) = k\frac{\prod_{i=1}^{m}(s+Z_i)}{\prod_{i=1}^{n}(s+P_i)}$$

Where, P_i and Z_i are the poles and zeros of the system, respectively. For this system, 'a' is given by the Harmonic mean (H.M) of $|P_i|$, as:

$$\frac{1}{a} = \sum_{i=1}^{n}\left[\frac{1}{|P_i|}\right]\Big/ n$$

The above equation gives value for the linear shift point 'a'. If $G_p(s+a)$ is expanded about s=0, then the time moment proportionals, c_i are obtained by:

$$G_p(s+a) = \sum_{i=0}^{\infty} c_i s^i \qquad \text{........ (4)}$$

Similarly, if $G_p(s+a)$ is expanded about $s=\infty$, then the Markov parameters m_j are obtained by:

$$G_p(s+a) = \sum_{j=1}^{\infty} m_j s^{-j} \qquad \text{......... (5)}$$

The four reduced r^{th} order models obtained as

$$R_p(s) = \frac{d_{p0} + d_{p1}s + d_{p2}s^2 + \cdots + d_{pr-1}s^{r-1}}{e_{p0} + e_{p1}s + e_{p2}s^2 + \cdots + e_{pr}s^r} \qquad \text{......... (6)}$$

Which retains 't' Time moments and 'm' Markov parameters, the coefficients e_{pk}, d_{pk} in (6) are derived from following set of equations

$$\begin{aligned}
d_{p0} &= e_{p0}c_0 \\
d_{p1} &= e_{p1}c_0 + e_{p0}c_1 \\
&\vdots \\
d_{pr-1} &= e_{pr-1}c_0 + \cdots + e_{p0}c_{r-1} \\
-c_0 &= e_{pr}c_0 + \cdots + e_{p0}c_r \\
-c_1 &= e_{pr-1}c_1 + \cdots + e_{p0}c_{r+1} \\
&\vdots \\
-c_t &= e_{pr-1}c_t + \cdots + e_{p0}c_{r+t-1}
\end{aligned} \qquad \text{.... (7)}$$

$$\begin{aligned}
d_{pr-1} &= m_1 \\
d_{pr-2} &= m_1 e_{pr-1} + m_2 \\
&\vdots \\
d_{pt} &= m_1 e_{pt+1} + m_2 e_{pt+2} + \ldots + m_{r-t}
\end{aligned} \qquad \text{..(8)}$$

Where the c_j and m_k are the Time moment proportional and Markov parameters of the system, such that j=(0,1,…..t-1) and k=(1,2,…..m) respectively. The denominator coefficients of the reduced model are obtained by substituting (8) in (7) and are given by the solution set.

$$\begin{bmatrix}
c_{t-1} & c_{t-2} & c_{t-3} & \cdots & \cdots & \cdots & c_{t-r} \\
c_{t-2} & c_{t-3} & \cdots & \cdots & \cdots & \cdots & c_{t-r-1} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c_{r-1} & c_{r-2} & \cdots & \cdots & \cdots & c_1 & c_0 \\
c_{r-2} & c_{r-3} & \cdots & \cdots & \cdots & c_0 & -m_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-m_{m-r} & \cdots & \cdots & \cdots & \cdots & -m_{m-2} & -m_{m-1}
\end{bmatrix}$$

$$x \begin{bmatrix} e_{p0} \\ e_{p1} \\ e_{p2} \\ \vdots \\ \vdots \\ \vdots \\ e_{pr-2} \\ e_{pr-1} \end{bmatrix} = \begin{bmatrix} c_{t-r-1} \\ c_{t-r-2} \\ \vdots \\ \vdots \\ -m_1 \\ \vdots \\ \vdots \\ -m_m \end{bmatrix} \qquad \text{.. (9)}$$

The k[th] order reduced interval model $R_k(s)$ is constructed using $R_1(s)$,......$R_4(s)$ such that:

$$[dp_j^-, dp_j^+] = [\mathrm{Min}(dpj), \mathrm{Max}(dpj)] \quad \text{and}$$
$$[ep_j^-, ep_j^+] = [\mathrm{Min}(epj), \mathrm{Max}(epj)] \qquad \text{... (3.100b)}$$

or, the above equation can be represented as H e = m in matrix vector form and 'e' can be calculated from,

$$e = \left(H^T H\right)^{-1} H^T m \qquad \text{.. (10)}$$

are the coefficients of the reduced model denominator. If this estimate still does not yield a stable reduced denominator then H and m in (10) are extended by another row, which corresponds to using the next markov parameter from the full system in least squares match. Once the reduced denominator obtained, formed by 'e', apply the inverse shift s→(s-a) to this reduced denominator. Later calculate the reduced numerator as before by matching proper number of time moments of $G_p(s+a)$ to that of reduced model.

3. Illustrative Example

Example: Consider the 6^{th} order interval system given by its transfer function:

$$G(s)=\frac{[2\ \ 3]s^5+[70\ \ 71]s^4+[762\ \ 763]s^3+[3610\ \ 3611]s^2+[7700\ \ 7701]s+[6000\ 6001]}{[1\ \ 2]s^6+[41\ \ 42]s^5+[571\ \ 572]s^4+[3491\ \ 3492\ ]s^3+[10060\ \ 10061]s^2+[13100\ 13101]s+[6000\ \ 6001]}$$

The four transfer functions obtained using the Kharitonov's theorem

$$G_1(s)=\frac{2s^5+70s^4+763s^3+3611s^2+7700s+6000}{2s^6+41s^5+571s^4+3492s^3+10061s^2+13100s+6000}$$

$$G_2(s)=\frac{3s^5+70s^4+762s^3+3611s^2+7701s+6000}{2s^6+42s^5+571s^4+3491s^3+10061s^2+13101s+6000}$$

$$G_3(s)=\frac{2s^5+71s^4+763s^3+3610s^2+7700s+6001}{s^6+41s^5+572s^4+3492s^3+10060s^2+13100s+6001}$$

$$G_4(s)=\frac{3s^5+71s^4+762s^3+3610s^2+7701s+6001}{s^6+42s^5+572s^4+3491s^3+10060s^2+13101s+6001}$$

The reduced models obtained using four time moments

$$R_1(s)=\frac{4.507090+2.092549s}{4.507090+6.148930s+s^2} \quad \text{H.M =2.36654}$$

$$R_2(s)=\frac{7.694403+2.050627s}{7.694403+8.975589s+s^2} \quad \text{H.M=2.401805}$$

$$R_3(s)=\frac{10.422039+2.048517s}{10.422039+11.426788s+s^2} \quad \text{H.M=2.7283}$$

$$R_4(s)=\frac{21.027405+1.050062s}{21.027405+20.780373s+s^2} \quad \text{H.M=2.72918}$$

Thus the transfer function of the reduced interval model obtained as

$$R(s)=\frac{[4.507090,21.027405]+[1.858862\ ,2.092549]s}{[4.507090,21.027405]+[6.148930,20.780373]s+[1\ ,1]s^2}$$

The step responses of the reduced model are compared with the original interval system for lower bound and upper bound in Fig.1 and Fig.2.respectively.

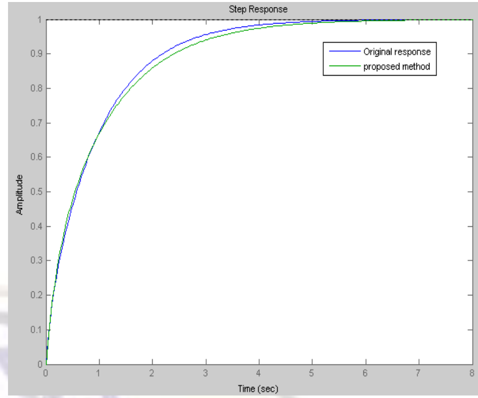

Fig: 1

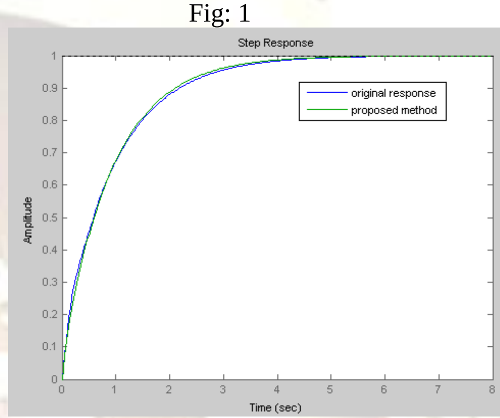

Fig: 2

Conclusions:

A new method is suggested for the order reduction of high order Interval system based on Least Square moment matching method. The proposed criterion leads to good and stable reduced models for linear time invariant interval systems.

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