NOTES ON (Q, L)-FUZZY SUBNEARRINGS OF A NEARRING

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ABSTRACT: In this paper, we study some of the properties of (Q, L)-fuzzy subnearring of a nearring and prove some results on these.

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KEY WORDS: (Q, L)-fuzzy subset, (Q, L)-fuzzy subnearring, (Q, L)-fuzzy relation, Product of (Q, L)-fuzzy subsets.

INTRODUCTION: After the introdution of fuzzy sets by L.A.Zadeh[19], several researchers explored on the generalization of the notion of fuzzy set. Azriel Rosenfeld[4] defined a fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups and A.Solairaju and R.Nagarajan[16, 17, 18] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of (Q, L)-fuzzy subnearring of a nearring and established some results.

1.PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q, L)-fuzzy subset** A of X is a function A: XxQ → L.

1.2 Definition: Let (R, +, ·) be a nearring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-**fuzzy subnearring(QLFSNR)** of R if the following conditions are satisfied:

(i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,
(ii) $A(-x, q) \geq A(x, q)$,
(iii) $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

1.3 Definition: Let A and B be any two (Q, L)-fuzzy subsets of sets R and H, respectively. The product of A and B, denoted by AxB, is defined as AxB ={ ⟨ ((x, y), q), AxB((x, y), q) ⟩ / for all x in R and y in H and q in Q }, where AxB((x, y), q) = A(x, q)∧ B(y, q).

1.4 Definition: Let A be a (Q, L)-fuzzy subset in a set S, the **strongest (Q, L)-fuzzy relation** on S, that is a (Q, L)-fuzzy relation V with respect to A given by $V((x, y), q) = A(x, q) \wedge A(y, q)$, for all x and y in S and q in Q.

2 – PROPERTIES OF (Q, L)-FUZZY SUBNEARRINGS:

2.1 Theorem: If A is a (Q, L)-fuzzy subnearring of a ring (R, +, ·), then $A(x, q) \leq A(e, q)$, for x in R, the identity e in R and q in Q.

Proof: For x in R, q in Q and e is the identity element of R. Now, $A(e, q) = A(x-x, q) \geq A(x, q) \wedge A(-x, q) = A(x, q)$. Therefore, $A(e, q) \geq A(x, q)$, for x in R and q in Q.

2.2 Theorem: If A is a (Q, L)-fuzzy subnearring of a ring (R, +, ·), then $A(x-y, q) = A(e, q)$ gives $A(x, q) = A(y, q)$, for x and y in R , e in R and q in Q.

Proof: Let x and y in R, the identity e in R and q in Q. Now, $A(x, q) = A(x-y+y, q) \geq A(x-y, q) \wedge A(y, q) = A(e, q) \wedge A(y, q) = A(y, q) = A(x-(x-y), q) \geq A(x-y, q) \wedge A(x, q) = A(e, q) \wedge A(x, q) = A(x, q)$. Therefore, $A(x, q) = A(y, q)$, for x and y in R and q in Q.

2.3 Theorem: A is a (Q, L)-fuzzy subnearring of a ring (R, +, ·) if and only if $A(x-y, q) \geq A(x, q) \wedge A(y, q)$ and $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

Proof: Let A be a (Q, L)-fuzzy subnearring of a nearring (R, +, ·) and x, y in R, q in Q. Then, $A(x-y, q) \geq A(x, q) \wedge A(-y, q) \geq A(x, q) \wedge A(y, q)$. Therefore, $A(x-y, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q and $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q. Conversely, if $A(x-y, q) \geq A(x, q) \wedge A(y, q)$, replace y by x, then $A(x, q) \leq A(e, q)$, for all x in R and q in Q. Now, $A(-x, q) = A(e-x, q) \geq A(e, q) \wedge A(x, q) = A(x, q)$. Therefore, $A(-x, q) \geq A(x, q)$, for all x in R and q in Q. It follows that, $A(x+y, q) = A(x-(-y), q) \geq A(x, q) \wedge A(-y, q) \geq A(x, q) \wedge A(y, q)$. Therefore, $A(x+y, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q and clearly $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q. Hence A is a (Q, L)-fuzzy subnearring of R.

2.4 Theorem: Let A be a (Q, L)-fuzzy subset of a nearring (R, +, ·). If $A(e, q) = 1$ and $A(x-y, q) \geq A(x, q) \wedge A(y, q)$, $A(xy, q) \geq A(x, q) \wedge A(y, q)$, then A is a (Q, L)-fuzzy subnearring of R, for all x and y in R and q in Q, where e is the identity element of R.

Proof: Let x and y in R, e in R and q in Q. Now, $A(-x, q) = A(e-x, q) \geq A(e, q) \wedge A(x, q) = 1 \wedge A(x, q) = A(x, q)$. Therefore, $A(-x, q) \geq A(x, q)$, for all x in R and q in Q. Now, $A(x+y, q) = A(x-(-y), q) \geq A(x, q) \wedge A(-y, q) \geq A(x, q) \wedge A(y, q)$. Therefore, $A(x+y, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q and clearly $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q. Hence A is a (Q, L)-fuzzy subnearring of R.

2.5 Theorem: If A is a (Q, L)-fuzzy subnearring of a nearring (R, +, ·), then $H = \{ x / x \in R : A(x, q) = 1\}$ is either empty or is a subnearring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $A(x-y, q) \geq A(x, q) \wedge A(-y, q) \geq A(x, q) \wedge A(y, q) = 1 \wedge 1 = 1$. Therefore, $A(x-y, q) = 1$.
We get x−y in H. And $A(xy, q) \geq A(x, q) \wedge A(y, q) = 1 \wedge 1 = 1$. Therefore, $A(xy, q) = 1$. We get xy in H. Therefore, H is a subnearring of R. Hence H is either empty or is a subnearring of R.

2.6 Theorem: If A is a (Q, L)-fuzzy subnearring of a ring (R, +, ·), then $H = \{ x \in R: A(x, q) = A(e, q) \}$ is a subnearring of R.

Proof: Let x and y be in H. Now, $A(x-y, q) \geq A(x, q) \wedge A(-y, q) \geq A(x, q) \wedge A(y, q) = A(e, q) \wedge A(e, q) = A(e, q)$. Therefore, $A(x-y, q) \geq A(e, q)$ ----------- (1). And, $A(e, q) = A((x-y) - (x-y), q) \geq A(x-y, q) \wedge A(-(x-y), q) \geq A(x-y, q) \wedge A(x-y, q) = A(x-y, q)$.
Therefore, $A(e, q) \geq A(x-y, q)$ ---------- (2). From (1) and (2), we get $A(e, q) = A(x-y, q)$.
Therefore, x−y in H. Now, $A(xy, q) \geq A(x, q) \wedge A(y, q) = A(e, q) \wedge A(e, q) = A(e, q)$. Therefore, $A(xy, q) \geq A(e, q)$ ------------- (3). And clearly, $A(e, q) \geq A(xy, q)$ --------- (4).
From (3) and (4), we get $A(e, q) = A(xy, q)$. Therefore, xy in H. Hence H is a subnearring of R.

2.7 Theorem: Let A be a (Q, L)-fuzzy subnearring of a ring (R, +, ·). If $A(x-y, q) = 1$, then $A(x, q) = A(y, q)$, for x and y in R and q in Q.

Proof: Let x and y in R and q in Q. Now, $A(x, q) = A(x-y+y , q) \geq A(x-y, q) \wedge A (y, q) = 1 \wedge A(y, q) = A(y, q) = A(-y, q) = A(-x+x-y, q) \geq A(-x, q) \wedge A(x-y, q) = A(-x, q) \wedge 1 = A(-x, q) = A(x, q)$. Therefore, $A(x, q) = A(y, q)$, for x and y in R, q in Q.

2.8 Theorem: Let A be a (Q, L)-fuzzy subnearring of a nearring (R, +, ·). If $A(x-y, q) = 0$, then either $A(x, q) = 0$ or $A(y, q) = 0$, for all x and y in R and q in Q.

Proof: Let x and y in R and q in Q. By the definition $A(x-y, q) \geq A(x, q) \wedge A(y, q)$ which implies that $0 \geq A(x, q) \wedge A(y, q)$. Therefore, either $A(x, q) = 0$ or $A(y, q) = 0$.

2.9 Theorem: Let (R, +, ·) be a nearring and Q be a non-empty set. If A is a (Q, L)-fuzzy subnearring of R, then $A(x+y, q) = A(x, q) \wedge A(y, q)$ with $A(x, q) \neq A(y, q)$, for each x and y in R and q in Q.

Proof: Let x and y belongs to R and q in Q. Assume that $A(x, q) > A(y, q)$. Now, $A(y, q) = A(-x + x + y, q) \geq A(-x, q) \wedge A(x + y, q) \geq A(x, q) \wedge A(x + y, q) \geq A(y, q) \wedge A(x + y, q) = A(y, q)$. And $A(y, q) = A(x, q) \wedge A(x+y, q) = A(x+y, q)$. Therefore, $A(x+y, q) = A(y, q) = A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

2.10 Theorem: If A and B are two (Q, L)-fuzzy subnearrings of a nearring R, then their intersection A∩B is a (Q, L)-fuzzy subnearring of R.

Proof: Let x and y belong to R and q in Q, A = { ⟨ (x, q), A(x, q) ⟩ / x in R and q in Q } and B = { ⟨ (x, q), B(x, q) ⟩ / x in R and q in Q }. Let C =A∩B and C = {⟨ (x, q), C(x, q) ⟩ / x in R and q in Q }. (i) C(x+y, q) = A(x+y, q) ∧ B(x+y, q) ≥ {A(x, q) ∧ A(y, q)} ∧ { B(x, q) ∧ B(y, q) }≥ {A(x, q) ∧ B(x, q) }∧{A(y, q) ∧ B(y, q)}= C(x, q) ∧C(y, q). Therefore, C(x+y, q) ≥ C(x, q) ∧ C(y, q), for all x and y in R and q in Q. (ii) C(−x, q) = A(−x, q)∧B(−x, q) ≥A(x, q)∧ B(x, q) = C(x, q). Therefore, C(−x, q) ≥ C(x, q) , for all x in R and q in Q. (iii) C(xy, q)= A(xy, q)∧B(xy, q) ≥{A(x, q)∧A(y, q)}∧{ B(x, q)∧ B(y, q)} ≥ {A(x, q) ∧ B(x, q)} ∧{ A(y, q) ∧ B(y, q)}= C(x, q) ∧ C(y, q). Therefore, C(xy, q) ≥ C(x, q) ∧ C(y, q), for all x and y in R and q in Q. Hence A∩B is a (Q, L)-fuzzy subnearring of the nearring R.

2.11Theorem: The intersection of a family of (Q, L)-fuzzy subnearrings of a nearring R is a (Q, L)-fuzzy subnearring of R.

Proof: Let $\{A_i\}_{i\in I}$ be a family of (Q, L)-fuzzy subnearrings of a nearring R and $A=\bigcap_{i\in I} A_i$.

Then for x and y belongs to R and q in Q, we have (i) A(x+y, q) = $\inf_{i\in I} A_i(x+y,q) \geq \inf_{i\in I}$ { A_i(x, q) ∧ A_i(y, q)}≥ $\inf_{i\in I} (A_i(x,q)) \wedge \inf_{i\in I} (A_i(y,q))$ = A(x, q) ∧ A(y, q). Therefore, A(x+y, q) ≥ A(x, q) ∧ A(y, q), for all x and y in R and q in Q. (ii) A(−x, q) = $\inf_{i\in I} A_i(-x,q) \geq \inf_{i\in I} A_i(x,q)$ = A(x, q). Therefore, A(−x, q)≥ A(x, q), for all x in R and q in Q. (iii) A(xy, q) = $\inf_{i\in I} A_i(xy,q) \geq \inf_{i\in I}$ { A_i(x, q) ∧ A_i(y, q) } ≥ $\inf_{i\in I} (A_i(x,q)) \wedge \inf_{i\in I} (A_i(y,q))$ = A(x, q) ∧ A(y, q). Therefore, A(xy, q) ≥ A(x, q) ∧ A(y, q), for all x and y in R and q in Q. Hence the intersection of a family of (Q, L)-fuzzy subnearrings of the nearring R is a (Q, L)-fuzzy subnearring of R.

2.12 Theorem: Let A be a (Q, L)-fuzzy subnearring of a nearring R. If A(x, q) < A(y, q), for some x and y in R and q in Q, then A(x+y, q) = A(x, q) = A(y+x, q), for all x and y in R and q in Q.

Proof: Let A be a (Q, L)-fuzzy subnearring of a nearring R. Also we have A(x, q) < A(y, q), for some x and y in R and q in Q, A(x+y, q) ≥ A(x, q) ∧ A(y, q) = A(x, q); and A(x, q) = A(x + y − y, q) ≥ A(x + y, q) ∧A(−y, q) ≥ A(x + y, q) ∧ A(y, q) }= A(x+y, q). Therefore, A(x+y, q) = A(x, q), for all x and y in R and q in Q. Hence A(x + y, q) = A(x, q) = A(y + x, q), for all x and y in R and q in Q.

2.13 Theorem: Let A be a (Q, L)-fuzzy subnearring of a nearring R. If A(x, q) > A(y, q), for some x and y in R and q in Q, then A(x + y, q) = A(y, q) = A(y + x, q), for all x and y in R and q in Q.

Proof: It is trivial.

2.14 Theorem: Let A be a (Q, L)-fuzzy subnearring of a nearring R such that Im A={α}, where α in L. If A=B∪C, where B and C are (Q, L)-fuzzy subnearrings of R, then either B⊆C or C⊆B.

Proof: Let A = B ∪ C = { ⟨ (x , q), A(x, q) ⟩ / x in R and q in Q }, B = {⟨ (x, q), B(x, q) ⟩ / x in R and q in Q } and C = { ⟨ (x, q), C(x, q) ⟩ / x in R and q in Q }. Suppose that neither B⊆ C nor C ⊆ B. Assume that B(x, q) > C(x, q) and B(y, q) < C(y, q), for some x and y in R and q in Q. Then, α = A(x, q) = (B∪C)(x, q) = B(x, q) ∨ C(x, q) = B(x, q) > C(x, q). Therefore, α > C(x, q). And, α = A(y, q) = (B∪C)(y, q) = B(y, q) ∨ C(y, q) = C(y, q) > B(y, q). Therefore, α > B(y, q). So that, C(y, q) > C(x, q) and B(x, q) > B(y, q).
Hence B(x+y, q) = B(y, q) and C(x+y, q) = C(x, q), by Theorem 2.12 and 2.13.
But then, α = A(x+y, q) = (B∪C)(x+y, q) = B(x+y, q) ∨ C(x+y, q) }= B(y, q) ∨ C(x, q) < α ---------(1). It is a contradiction by (1). Therefore, either B ⊆ C or C ⊆ B is true.

2.15 Theorem: If A and B are (Q, L)-fuzzy subnearrings of the nearrings R and H, respectively, then AxB is a (Q, L)-fuzzy subnearring of RxH.

Proof: Let A and B be (Q, L)-fuzzy subnearrings of the nearrings R and H respectively. Let x_1 and x_2 be in R, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in RxH and q in Q. Now, AxB $[(x_1, y_1) + (x_2, y_2), q] =$ AxB$((x_1 + x_2, y_1 + y_2), q) = A(x_1+x_2, q) \wedge B(y_1+y_2, q) \geq \{A(x_1, q)\wedge A(x_2, q)\} \wedge\{B(y_1, q) \wedge B(y_2, q)\}= \{A(x_1, q) \wedge B(y_1, q)\}\wedge \{A(x_2, q) \wedge B(y_2, q) \}$ = AxB$((x_1, y_1), q) \wedge$AxB$((x_2, y_2), q)$. Therefore, AxB$[(x_1, y_1)+(x_2, y_2), q] \geq$AxB$((x_1, y_1), q) \wedge$ AxB$((x_2, y_2), q)$. And AxB$[-(x_1, y_1) , q]$ = AxB$((-x_1, -y_1), q)= A(-x_1, q) \wedge B(-y_1, q) \geq A(x_1, q) \wedge B(y_1, q)$ = AxB$((x_1, y_1), q)$. Therefore, AxB$[-(x_1, y_1), q] \geq$ AxB$((x_1, y_1), q)$. Now, AxB$[(x_1, y_1)(x_2, y_2), q]$ = AxB$((x_1x_2, y_1y_2), q)= A(x_1x_2, q) \wedge B(y_1y_2, q) \geq \{A(x_1, q)\wedge A(x_2, q)\}\wedge\{ B(y_1, q)\wedge B(y_2, q) \}= \{A(x_1, q) \wedge B(y_1, q) \}\wedge\{ A(x_2, q) \wedge B(y_2, q) \}$= AxB$((x_1, y_1), q) \wedge$ AxB$((x_2, y_2), q)$. Therefore, AxB$[(x_1, y_1)(x_2, y_2), q] \geq$ AxB$((x_1, y_1), q)\wedge$AxB$((x_2, y_2), q)$. Hence AxB is a (Q, L)-fuzzy subnearring of RxH.

2.16 Theorem: Let A and B be (Q, L)-fuzzy subsets of the nearrings R and H, respectively. Suppose that e and e^{l} are the identity element of R and H, respectively. If AxB is a (Q, L)-fuzzy subnearring of RxH, then at least one of the following two statements must hold.

(i) $B(e^{l} , q) \geq A(x, q)$, for all x in R and q in Q,

(ii) $A(e, q) \geq B(y, q)$, for all y in H and q in Q.

Proof: Let AxB be a (Q, L)-fuzzy subnearring of RxH.
By contra positive, suppose that none of the statements (i) and (ii) holds. Then we can find a in R and b in H such that $A(a, q) > B(e^{l}, q)$ and $B(b, q) > A(e, q)$, q in Q. We have, AxB$((a, b), q) = A(a, q) \wedge B(b, q) > A(e, q) \wedge B(e^{l} , q)$ = AxB$((e, e^{l}), q)$. Thus AxB is not a (Q, L)-fuzzy subnearring of RxH. Hence either $B(e^{l} , q) \geq A(x, q)$, for all x in R and q in Q or $A(e, q) \geq B(y, q)$, for all y in H and q in Q.

2.17 Theorem: Let A and B be (Q, L)-fuzzy subsets of the nearrings R and H, respectively and AxB is a (Q, L)-fuzzy subnearring of RxH. Then the following are true:

(i) if $A(x, q) \leq B(e^{l}, q)$, then A is a (Q, L)-fuzzy subnearring of R.

(ii) if $B(x, q) \leq A(e, q)$, then B is a (Q, L)-fuzzy subnearring of H.

(iii) either A is a Q-fuzzy subnearring of R or B is a Q-fuzzy subnearring of H.

Proof: Let AxB be a (Q, L)-fuzzy subnearring of RxH, x and y in R and q in Q. Then (x, e^{l}) and (y, e^{l}) are in RxH. Now, using the property $A(x, q) \leq B(e^{l}, q)$, for all x in R and q in Q, we get, $A(x-y, q) = A(x-y, q) \wedge B(e^{l}e^{l}, q)$ = AxB$(((x-y), (e^{l}e^{l})), q)$ = AxB$[(x, e^{l}) + (-y, e^{l}), q] \geq$ AxB$((x, e^{l}), q) \wedge$ AxB$((-y, e^{l}), q) = \{A(x, q) \wedge B(e^{l} ,q)\} \wedge\{A(-y, q) \wedge B(e^{l}, q)\}= A(x, q) \wedge A(-y, q) \geq A(x, q) \wedge A(y, q)$. Therefore, $A(x-y, q) \geq A(x, q) \wedge A(y, q)$, for all x, y in R and q in Q. And, $A(xy, q)= A(xy, q)\wedge B(e^{l}e^{l}, q)\}$= AxB$(((xy), (e^{l}e^{l})), q)$ = AxB$[(x, e^{l})(y, e^{l}), q] \geq$AxB$((x, e^{l}), q) \wedge$ AxB$((y, e^{l}), q) = \{A(x, q) \wedge B(e^{l} ,q)\}\wedge A(y, q) \wedge B(e^{l}, q)\}= A(x, q) \wedge A(y, q) \geq A(x, q) \wedge A(y, q)$. Therefore, $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x, y in R and q in Q. Hence A is a (Q, L)-fuzzy subnearring of R. Thus (i) is proved. Now, using the property $B(x, q) \leq A(e, q)$, for all x in H and q in Q, we get, $B(x-y, q) = B(x-y, q) \wedge A(ee, q)$ = AxB$(((ee) ,(x-y)), q)$=AxB$[(e, x)+(e, -y), q] \geq$ AxB$((e, x), q) \wedge$ AxB$((e, -y), q) =\{ B(x, q)\wedge A(e, q)\}\wedge\{ B(-y, q) \wedge A(e, q)\}= B(x, q) \wedge B(-y, q) \geq B(x, q) \wedge B(y, q)$. Therefore, $B(x-y, q) \geq B(x, q) \wedge B(y, q)$, for all x and y in H and q in Q. And, $B(xy, q) = B(xy, q) \wedge A(ee, q)$ = AxB$(((ee) ,(xy)), q)$ = AxB$[(e, x)(e, y), q] \geq$ AxB$((e, x), q) \wedge$ AxB$((e, y), q) =\{ B(x, q) \wedge A(e, q) \}\wedge\{B(y, q) \wedge A(e, q)\}= B(x, q)\wedge B(y, q) \geq B(x, q) \wedge B(y, q)$. Therefore, $B(xy, q) \geq B(x, q) \wedge B(y, q)$, for all x and y in H and q in Q. Hence B is a (Q, L)-fuzzy subnearring of H. Thus (ii) is proved. (iii) is clear.

2.18 Theorem: Let A be a (Q, L)-fuzzy subset of a nearring R and V be the strongest (Q, L)-fuzzy relation of R with respect to A. Then A is a (Q, L)-fuzzy subnearring of R if and only if V is a (Q, L)-fuzzy subnearring of RxR.

Proof: Suppose that A is a (Q, L)-fuzzy subnearring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR and q in Q. We have, $V(x-y, q) = V [(x_1, x_2)-(y_1, y_2), q] =V((x_1-y_1 , x_2-y_2), q) = A((x_1-y_1), q) \wedge A((x_2-y_2), q) \geq\{A(x_1, q) \wedge A(-y_1, q)\}\wedge \{A(x_2, q) \wedge A(-y_2, q)\}=\{A(x_1, q) \wedge A(x_2, q)\}\wedge\{A(-y_1, q) \wedge A(-y_2, q)\}=\{A(x_1, q) \wedge A(x_2, q) \}\wedge \{A(y_1, q) \wedge A(y_2, q)\}= V((x_1, x_2), q) \wedge V((y_1, y_2), q) = V (x, q)\wedge V (y, q)$. Therefore, $V((x-y), q) \geq V(x, q) \wedge V(y, q)$, for all x and y in RxR and q in Q. And we have, $V(xy, q) = V [(x_1, x_2)(y_1, y_2), q] = V((x_1y_1 , x_2y_2), q) = A((x_1y_1), q) \wedge A((x_2y_2), q) \geq \{A(x_1, q) \wedge A(y_1, q) \}\wedge\{A(x_2, q) \wedge A(y_2, q) \}= \{A(x_1, q) \wedge A(x_2, q)\}\wedge\{A(y_1, q) \wedge A(y_2, q)\}= V((x_1, x_2), q) \wedge V($

$(y_1, y_2), q) = V(x, q) \wedge V(y, q)$. Therefore, $V((xy), q) \geq V(x, q) \wedge V(y, q)$, for all x and y in RxR and q in Q. This proves that V is a (Q, L)-fuzzy subnearring of RxR. Conversely, assume that V is a (Q, L)-fuzzy subnearring of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR, we have $A(x_1-y_1, q) \wedge A(x_2-y_2, q) = V((x_1-y_1, x_2-y_2), q) = V[(x_1, x_2) - (y_1, y_2), q] = V(x-y, q) \geq V(x, q) \wedge V(y, q) = V((x_1, x_2), q) \wedge V((y_1, y_2), q) = \{A(x_1, q) \wedge A(x_2, q)\} \wedge \{A(y_1, q) \wedge A(y_2, q)\}$. If we put $x_2 = y_2 = e$, where e is the identity element of R. We get, $A((x_1-y_1), q) \geq A(x_1, q) \wedge A(y_1, q)$, for all x_1 and y_1 in R and q in Q. And $A(x_1y_1, q) \wedge A(x_2 y_2, q) \} = V((x_1y_1, x_2 y_2), q) = V[(x_1, x_2)(y_1, y_2), q] = V(xy, q) \geq V(x, q) \wedge V(y, q) = V((x_1, x_2), q) \wedge V((y_1, y_2), q) = \{A(x_1, q) \wedge A(x_2, q)\} \wedge \{A(y_1, q) \wedge A(y_2, q)\}$. If we put $x_2 = y_2 = e$, where e is the identity element of R. We get, $A((x_1y_1), q) \geq A(x_1, q) \wedge A(y_1, q)$, for all x_1 and y_1 in R and q in Q. Hence A is a (Q, L)-fuzzy subnearring of R.

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