LEFT AND RIGHT COMPLETE NEAR RINGS WITH ARBITRARY CYCLIC GROUP

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Abstract:
Near rings $(S,+,\bullet)$ (either left or right near rings) tell us the operation in the semi group $(S,\bullet)$ is either left distributive or right distributive over the group $(S,+)$ .This paper describes the Near rings $(S,+,\bullet)$ (either left or right near rings) with one more property : the operation in the group $(S,+)$ is distributive over the operation in the semi group $(S,\bullet)$.This kind of Near rings are known as complete near rings (either left complete near rings or right complete near rings according as the near ring of type) . This paper contain a few examples for complete near rings and some interesting results on these structures if the group $(S,+)$ is cyclic : A left near ring becomes a right near ring if it is either left or right complete near ring but the converse need not be true.

Key words: Right (left) complete near ring, commutative right (left) complete near ring.

1 Introduction:

The aim of this paper is to establish some of the theorems that specify the nature of the complete near rings (i.e., whether it is either left or right complete near rings) whenever it occurs.

2. Our contribution:

A left (right) - near ring $(S,+,\bullet)$ is called a left (right)-complete near ring if the operation $+$ in the group $(S,+)$ is distributive over the operation $\bullet$ in the semigroup $(S,\bullet)$.

A left (right)-complete near ring $(S,+,\bullet)$ with commutative property with respect to both operations $+$ and $\bullet$ is known as commutative left (right)-complete near ring.

2.1 Illustrations:

1. Let $Z_{18}=\{0,1,...,17\}$. Define two binary operations $+$ and $\bullet$ on Z_{18} as follows: $+$ is the usual addition modulo 18 so that $(Z_{18},+)$ is a group; $a\bullet b=a$ for all $a,b\in Z_{18}$. Clearly $(Z_{18},+,\bullet)$ is a right - complete near ring.
2. An algebraic structure $(M_{n\times n},+,\bullet)$, where $M_{n\times n}=\{(a_{ij})/a_{ij}\in Z\}$ is a left complete near ring with respect to usual addition of matrices and $A\bullet B=B$ for all $A,B\in M_{n\times n}$.
3. Let Z be a set of integers. Define two binary operations $+$ and $\bullet$ on Z as follows: $+$ is usual addition and $a\bullet b=a$ for all $a,b\in Z$ is a right complete near ring.

Theorem 1: Every left (right) complete near ring $(S,+,\bullet)$ is commutative if $(S,+)$ is a cyclic group

Let g be a generator of the cyclic group $(S,+)$.

If $a,b\in S$, there exist integers m,n such that $a=mg$,$b=ng$

Then $a \bullet b = mg \bullet ng = ng \bullet mg = b \bullet a$

& $a + b = mg + ng = g(m+n) = g(n+m) = ng + mg = b + a$ for all $a, b \in S$.

Theorem 2: Every left complete near ring $(S,+,\bullet)$ is a right complete near ring and vice versa if $(S,+)$ be a cyclic group.

Let g be a generator of the cyclic group $(S,+)$.

If $a,b,c \in S$, there exist integers m_1, m_2, m_3 such that $a = m_1 g, b = m_2 g, c = m_3 g$

Suppose $(S,+,\bullet)$ is a left complete near ring

To show that $(a+b)\bullet c = a \bullet c + b \bullet c \quad \forall a,b,c \in S$

$(a+b)\bullet c = (m_1 g + m_2 g)\bullet m_3 g = (m_1 + m_2) m_3 g \bullet g = (m_1 m_3 + m_2 m_3) g \bullet g =$
$m_1 m_3 g \bullet g + m_1 m_3 g \bullet g = a \bullet c + b \bullet c$

Thus, $(S,+,\bullet)$ is a right complete near ring.

Converse of this theorem is easy to prove.

Note that If $(S,+)$ be a cyclic group, and then every commutative left (right) complete near rings $(S,+,\bullet)$ is also a commutative right (left) complete near rings and vice versa.

Also, note that every left (right) complete near ring $(S,+,\bullet)$ is becomes a ring if $(S,+)$ is a cyclic group.

Corollary: Every left(right) near ring $(S,+,\bullet)$ becomes a right(left) near ring if $(S,+)$ is a cyclic group such that the operation $+$ is distributive over $\bullet$

Theorem 3: If $(S,+)$ be a cyclic group such that every generator g satisfies the condition $g \bullet g \neq 0$ then the left(right) complete near ring $(S,+,\bullet)$ satisfies the cancellation law with respect to operation $\bullet$ in a semi group $(S,\bullet)$.

Let g be a generator of a cyclic group $(S,+)$.

Therefore,

If $a,b,c \in S$, there exist integers m_1, m_2, m_3 such that $a = m_1 g, b = m_2 g, c = m_3 g$

Suppose $a \bullet b = a \bullet c \Leftrightarrow m_1 g \bullet m_2 g = m_1 g \bullet m_3 g$

$\Leftrightarrow m_1 g \bullet g(m_2 - m_3) = 0 \Leftrightarrow m_2 = m_3 \Leftrightarrow m_2 g = m_3 g \Leftrightarrow b = c$

It is easy to show that $b \bullet a = c \bullet a \Leftrightarrow b = c$

Reference

[1] James R.Clay, The near rings on a finite cyclic group , the mathematical monthly, vol 71,no1(jan.,1964),pp 47-50.