# ORDER REDUCTION OF LINEAR DYNAMIC SYSTEMS USING IMPROVED GENERALISE LEAST-SQUARES METHOD AND PSO

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**Abstract—The authors present an algorithm for order reduction of linear dynamic SISO systems using the combined advantages of the improved generalise Least squares method and error minimization by Particle Swarm Optimization technique (PSO). The denominator of the reduced order model is obtained by improved generalise least squares method and PSO is employed for determining numerator coefficients by minimizing the integral square error between the transient responses of original and reduced order models, pertaining to unit step input. The reduction procedure is simple, efficient and computer oriented. The algorithm is illustrated with the help of two numerical examples to highlight the advantages of the approach and the results are compared with the other existing techniques.**

***Keywords:*** Improved Least Squares, Integral square error, Order Reduction, Particle Swarm Optimization.

## I.INTRODUCTION

Scientists and Engineers are confronted with the analysis, design and synthesis of real life problems. The first step in such studies is the development of a ‘mathematical model’ which can be substitute for the real problem. The mathematical procedure of system modelling often leads to comprehensive description of a process in the form of high order differential equations which are difficult to use either for analysis (or) controller synthesis. It is hence useful and sometimes necessary to find the possibility of finding some equation of the same type but of lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration. Numerous methods are available in the literature for order reduction of linear continuous systems in time domain as well as in frequency domain [1]-[7]. Basing on the simplicity and amicability, the frequency domain dependent methods have become more prominent. Transfer function reduction methods are one of the important groups in the frequency domain category. Inspite of the significant number of methods available, no approach always gives the best results for all systems. Almost all methods, however aim at accurate reduced models for a low computation cost. In addition it is desired to preserve the stability of the original model; i.e., given a stable high order model, the reduced order model should also be stable.

A popular approach, known as Pade approximation method for deriving reduced order models has been based on matching of the time moments of original and reduced order Systems [8-9]. This technique has a number of useful Properties, such as, computational simplicity, fitting of the Initial time moments and the steady state values of the output of original and reduced order systems being the same for input of the form $\sum \alpha_i t^i$.This simple technique usually gives good results and is not computationally demanding. A well-known drawback of this method, however, is that an unstable reduced model might arise from a stable model. To remedy this situation, several variants of the method have been proposed. One such technique [10] suggests using a least-squares time moment fit to obtain a reduced transfer function denominator, and the numerator by exact time moment matching. Further, the method of model order reduction by least squares moment matching was generalised [12] by including the Markov parameters in the process to cope with a wider class of transfer functions. On the other hand, Aguirre [13] has argued that one of the chief advantages of the least squares Pade (LS-Pade) method is that additional information concerning the original system over the mid-frequency range is included in the simplified model, and consequently better approximations are often obtained.

Numerous methods of order reduction are also available in the literature [14]-[17],based on the minimization of the integral square error, However a common feature in these methods [16]-[17] is that the values of the denominator coefficients of the reduced order model are chosen arbitrarily by some stability preserving methods and the numerator coefficients of the reduced order model are determined by minimizing of ISE.

Recently, Particle Swarm Optimization(PSO) Technique appeared as a Promising algorithm for handling the optimization problem. PSO is a population based stochastic optimization technique, inspired by social behaviour of bird flocking (or) fish schooling [18]. However unlike Genetic algorithm (GA), PSO has no evolution operators such as crossover and mutation. One of the most promising advantage of PSO over GA is its algorithmic simplicity, as it uses a few parameters and easy to implement. In PSO the potential solutions, called particles flies through the search space with an adaptable velocity and is dynamically modified according to its own flying experience and also to the flying experience of other particles.

In the present work, the authors present an algorithm for order reduction of Single Input Single Output (SISO) dynamic systems, which combines the advantages of the improved generalise Least squares method and error minimization by PSO. In this method the reduced denominator is obtained by the improved least squares method and the numerator of the reduced model is determined by minimizing the Integral Square Error between the transient responses of original and reduced model by PSO pertaining to a unit step input. The relative mapping errors between the original and reduced models are also determined and plotted with respect to time for both unit step and impulse inputs. The comparison between the proposed and the other well known existing order reduction techniques is also shown in the present work. In the following Sections the algorithm is described in detail and the same is used in solving two numerical examples.

## II DESCRIPTION OF ALGORITHM

Let the transfer function of the original high order linear dynamic SISO system of order 'n' be:

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + \cdots + b_{n-1}s^{n-1}}{a_0 + a_1 s + \cdots + a_{n-1}s^{n-1} + a_n s^n} \quad \ldots (1)$$

and Let the corresponding $r^{th}$ order reduced model is synthesized as:

$$R_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_0 + d_1 s + \cdots + d_{r-1}s^{r-1}}{e_0 + e_1 s + \cdots + e_{r-1}s^{r-1} + s^r} \quad \ldots (2)$$

Further, the method consists of the following steps.

**Step 1: Determination of the denominator coefficients of reduced order model using generalised least squares method:**

Expand $G_n(s)$ about s=0, to obtain the time moment proportional $(C_i)$ are given by

$$G_n(s) = \sum_{i=0}^{\infty} C_i s^i \quad \cdots (3)$$

Evaluating Eq.(2) and(3) to retain 't' Time moments of the original model gives the following set of equations:

$$\left.\begin{aligned}
d_0 &= e_0 c_0 \\
d_1 &= e_1 c_0 + e_0 c_1 \\
&\vdots \\
d_{r-1} &= e_{r-1}c_0 + \cdots\cdots + e_0 c_{r-1} \\
-c_0 &= e_{r-1}c_1 + \cdots\cdots + e_1 c_{r-1} + e_0 c_r \\
-c_1 &= e_{r-1}c_2 + \cdots\cdots + e_1 c_r + e_0 c_{r+1} \\
&\vdots \\
-c_{t-r-1} &= e_{r-1}c_{t-r} + \cdots\cdots + e_1 c_{t-2} + e_0 c_{t-1}
\end{aligned}\right\} \cdots (4)$$

Similarly if $G_n(s)$ is expanded about $s = \infty$, to obtain the markov parameters $(M_j)$ are given by:

$$G_n(s) = \sum_{i=1}^{\infty} \frac{M_j}{s^j} \quad \cdots (5)$$

Evaluating Eq.(2) and(5) to retain 'm' Markov parameters of the original model gives the following set of equations:

$$\left.\begin{aligned}
d_{r-1} &= M_1 \\
d_{r-2} &= M_1 e_{r-1} + M_2 \\
&\vdots \\
d_0 &= M_1 e_1 + M_2 e_2 + \cdots + M_r \\
M_1 e_0 + M_2 e_1 + \cdots + M_r e_{r-1} &= -M_{r+1} \\
&\vdots \\
M_{m-r} e_0 + M_{m-r-1} e_1 + \cdots + M_{m-1} e_{r-1} &= -M_m
\end{aligned}\right\} \cdots (6)$$

**Step 2:** Elimination of $d_j$ (j=0,1,......r-1) in Eq.(4) by substituting Eq.(6) gives the reduced denominator coefficients as the solution of:

$$\begin{bmatrix} c_{t-1} & c_{t-2} & \cdots & c_{t-r+1} & c_{t-r} \\ c_{t-2} & c_{t-3} & \cdots & \cdots & c_{t-r-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{r-1} & c_{r-2} & \cdots & c_1 & c_0 \\ c_{r-2} & c_{r-3} & \cdots & c_0 & -M_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -M_{m-r} & -M_{m-r-1} & \cdots & -M_{m-2} & -M_{m-1} \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ \vdots \\ \vdots \\ e_{r-2} \\ e_{r-1} \end{bmatrix} = \begin{bmatrix} -c_{t-r-1} \\ -c_{t-r-2} \\ \vdots \\ M_1 \\ M_2 \\ \vdots \\ M_m \end{bmatrix} \quad \cdots (7)$$

(or) Pe = q in matrix vector form

Eq. (7) is equivalent to equating all the significant time moments $(C_i)$ and markov parameters $(M_j)$ where $i = 0,1,\cdots,t-1$ and $j = 1,2,\cdots,m$.

**Step 3:** Calculation of 'e' from this non square system of equation(7) can only done in the least square sense, i.e.:

$$e = [P^T P]^{-1} P^T q \quad \cdots (8)$$

**Step 4:** Finally the $r^{th}$ order reduced denominator is obtained as: $D_r(s) = e_0 + e_1 s + \cdots + e_{r-1}s^{r-1} + s^r$

**Step 5: Determination of the numerator coefficients of the reduced order model by PSO technique:**

The PSO method is population based search algorithm where each individual solution [coefficients of reduced numerator] is referred to as one particle and Each particle flies through the bounded search space with an adaptable velocity that is dynamically modified according to its own flying experience and also the flying experience of the other particles. In PSO, each particle strives to improve itself by imitating traits from their successful peers. Further each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness[ISE] of a particle is known as p_best(personal best) and overall best of all particles in the population is called g_best(global best).

In a d-dimensional search space, the best particles updates its velocity and positions with following equations:

$$\left.\begin{aligned} v_{j,g}^{t+1} &= W * v_{j,g}^{t} + c_{1*}\gamma_1() * \left(pbest_{j,g} - X_{j,g}^{t}\right) \\ &\quad + c_{2*}\gamma_2() * \left(gbest_g - X_{j,g}^{t}\right) \\ X_{j,g}^{t+1} &= X_{j,g}^{t} + v_{j,g}^{t+1} \end{aligned}\right\} \cdots (9)$$

$with\ j = 1,2,\cdots,n\ \ and\ \ g = 1,2,\cdots,d;$
Where
$n$ = no. Of particles in the swarm
$d$ = Vector dimension of the particle $X_j$ and its velocity $v_j$.
$t$ = number of iteration.
$W$ = inertia weight factor
$W = W_{max} - [W_{max} - W_{min}] * \frac{K-1}{N-1};$
Where 'K' is the current iteration and 'N' is the maximum iteration.
$C_1, C_2 =$ Cognitive and social acceleration factors respectively.
$\gamma_1(), \gamma_2() =$ Random numbers uniformly distributed in the range (0,1).
The $j^{th}$ particle in the swarm is represented by a $d$-dimensional vector $X_j = (X_{j1}, X_{j2}, \cdots, X_{jd})$ and its velocity is denoted by another d-dimensional vector $v_j = (v_{j1}, v_{j2}, \cdots, v_{jd})$. The best previous visited position of the $j^{th}$ particle is represented by $pbest_j = (pbest_{j1}, pbest_{j2}, \cdots, pbest_{jd})$.The best particle among all of the particles in the swarm is represented by $gbest_g = (gbest_1, gbest_2, \cdots, gbest_d)$.

In PSO each particle moves in a search space with a velocity according to its owns previous best solution ($pbest_{j,g}$) and its groups previous best solution ( $gbest_g$). The velocity update in the particle swarm consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm[19]. The parameter $C_1$ *and* $C_2$ determine the relative pull of pbest and gbest and parameters $\gamma_1(), \gamma_2()$ help in stochastically varying these pulls. The position and velocity updates of a particle in PSO for a two dimensional parameter space shown in fig 1.

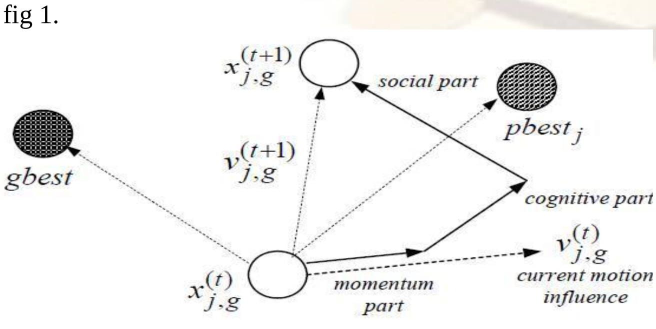


Fig.1.Description of velocity and position updates in particle swarm optimization for a two dimensional parameter space

In the present study, PSO is employed to minimize the objective function 'E' which is the integral square error in between the transient response of original and reduced model is given by:

$$E = \int_0^\infty [y(t) - y_r(t)]^2 . dt \qquad \cdots (10)$$

Where $y(t)$ and $y_r(t)$ are the unit step responses of original $[G_n(s)]$ and reduced $[G_r(s)]$ order systems, and the parameters to be determined are the numerator coefficients of the reduced order model as given in Eq.(2). To eliminate any steady state error in the approximation, the condition is:

$$d_0 = \frac{b_0}{a_0} c_0 \qquad \cdots (11)$$

In Table I , the specified parameters of the PSO algorithm used in the present study are given. The computational flow chart of the proposed algorithm is shown in Figure.2.

TABLE I
PARAMETERS USED FOR PSO ALGORITHM

| **Parameters** | **Value** |
|---|---|
| Swarm Size | 20 |
| Max. Iteration | 50 |
| $C_1, C_2$ | 2.0,2.0 |
| $W_{max}, W_{min}$ | 0.9,0.4 |

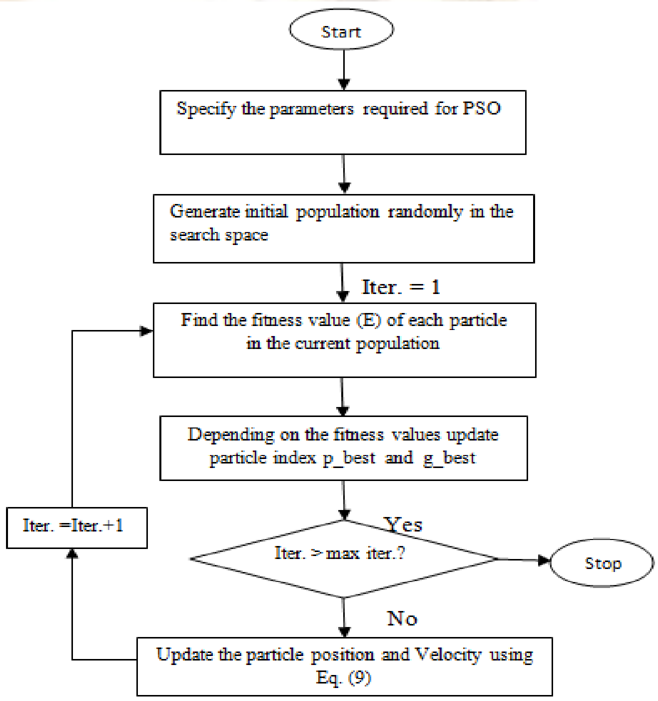


Fig. 2 Flow chart of PSO algorithm

## III. RELATIVE MAPPING ERRORS

The relative mapping errors of the original model relative to its Reduced model are expressed by means of the relative integral square error criterion, which are given by [20] :

$$I = \int_0^\infty [H(t) - H_r(t)]^2 . dt \Big/ \int_0^\infty H^2(t) . dt \quad \cdots (12)$$

$$J = \int_0^\infty [G(t) - G_r(t)]^2 . dt \Big/ \int_0^\infty [G(t) - G(\infty)]^2 . dt \cdots (13)$$

Where, $H(t)$ and $G(t)$ are the impulse and step responses of original system, respectively, and $H_r(t)$ and $G_r(t)$ are that of their approximants.

In this paper, both the relative mapping errors '*I*' and '*J*' are calculated and plotted with respect to time for the proposed reduction algorithm. These relative mapping errors are also compared in the tabular form for the proposed reduction algorithm and the other well-known existing order reduction techniques.

## IV. NUMERICAL EXAMPLES

Two numerical examples are chosen from the literature to show the flexibility and effectiveness of the proposed reduction algorithm than other existing methods, and the response of the original and reduced models are compared.

**Example-1:** Consider an eighth-order system[15] described by the transfer function as:

$$G_8(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \cdots (14)$$

If a second order reduced model $R_2(s)$ is desired, then the steps to be followed are as under:

**Step-1:** Expand $G_8(s)$ about s=0, gives the time moment proportional's $(C_i)$ where i=0,1,2,....which are shown below

$$G_8(s) = 1 + 1.889286s - 2.556337s^2 + 2.786301s^3 - 2.890797s^4 + 2.943704s^5 + \cdots \quad \cdots (15)$$

Similarly, Expand $G_8(s)$ about s= ∞ gives the markov parameters($M_j$) where j= 1,2,3....... which are shown below

$$G_8(s) = 18s^{-1} - 134s^{-2} + 978s^{-3} - 7312s^{-4} + 55650s^{-5} \cdots \quad \cdots (16)$$

**Step-2:** Taking t=3 time moment proportional's and m=1 markov parameters of $G_8(s)$ in Eq.(7) gives the reduced denominator coefficients as the solution of :

$$\begin{bmatrix} C_2 & C_1 \\ C_1 & C_0 \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \end{bmatrix} = \begin{bmatrix} -C_0 \\ M_1 \end{bmatrix} \quad \cdots (17)$$

i.e. $$\begin{bmatrix} -2.556337 & 1.889286 \\ 1.889286 & 1 \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \end{bmatrix} = \begin{bmatrix} -1 \\ 18 \end{bmatrix} \cdots (18)$$

**Step-3:**The reduced denominator coefficients from Eq.(18) are obtained using Eq.(8):Therefore,

$$D_r(s) = s^2 + 7.203178s + 5.714764$$

**Step-4:** By using PSO to minimize the objective function 'E', as described earlier, we have

$N_r(s) = 17.889124s + 5.714764$ .

Therefore, finally $R_2(s)$ is given as:

$$R_2(s) = \frac{17.889124s + 5.714764}{s^2 + 7.203178s + 5.714764} \quad \cdots (19)$$

The proposed method produces quite different reduced models gives the results as shown in Table II, where 't' time moments and 'm' markov parameters are used to calculate the denominators and the numerators are determined by minimizing the integral square error using PSO technique.

TABLE II
COMPARISON OF SECOND ORDER MODELS BY PROPOSED

| *t* | *m* | $d_0$ | $d_1$ | $e_0$ | $e_1$ | *I* | *J* |
|---|---|---|---|---|---|---|---|
| 4 | 0 | 4.820 | 15.00 | 4.820 | 5.993 | 0.0145 | 0.0026 |
| 3 | 1 | 5.714 | 17.88 | 5.714 | 7.203 | 0.0011 | 0.0015 |
| 2 | 2 | 5.427 | 18.56 | 5.427 | 7.745 | 0.0013 | 0.0028 |
| 5 | 0 | 4.632 | 14.41 | 4.632 | 5.750 | 0.0214 | 0.0042 |
| 4 | 1 | 5.747 | 17.72 | 5.747 | 7.093 | 0.0014 | 0.00209 |

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order reduced model is given in Table III. Figure 4(a)-(c) Presents diagrams of step, impulse and frequency responses of $G_8(s)$ and $R_2(s)$, respectively.

TABLE III
COMPARISON OF REDUCED ORDER MODELS

| Method of Reduction | Reduced Models | I | J |
|---|---|---|---|
| Proposed method (t=3 andm=1) | $\frac{17.8891s + 5.7147}{s^2 + 7.203178s + 5.7147}$ | 0.00119 | 0.00157 |
| PSO [15] | $\frac{16.8517s + 5.1379}{s^2 + 6.8976s + 5.1379}$ | 0.00178 | 0.00070 |
| PSO and Eigen Spectrum[17] | $\frac{22.836s + 8}{S^2 + 9s + 8}$ | 0.04630 | 0.02508 |
| Pole Clustering and Pade [5] | $\frac{16.51145s + 5.45971}{s^2 + 6.19642s + 5.45971}$ | 0.00834 | 0.00959 |
| Mukherjee et al.[21] | $\frac{11.3909s + 4.4357}{s^2 + 4.2122s + 4.4357}$ | 0.08993 | 0.03881 |
| Pal[22] | $\frac{151776.576s+40320}{65520s^2+75600s+40320}$ | 0.72967 | 1.12609 |
| Chen et al[25] | $\frac{0.72046s + 0.36669}{s^2 + 0.02768s + 0.36669}$ | 1.03179 | 4.91813 |

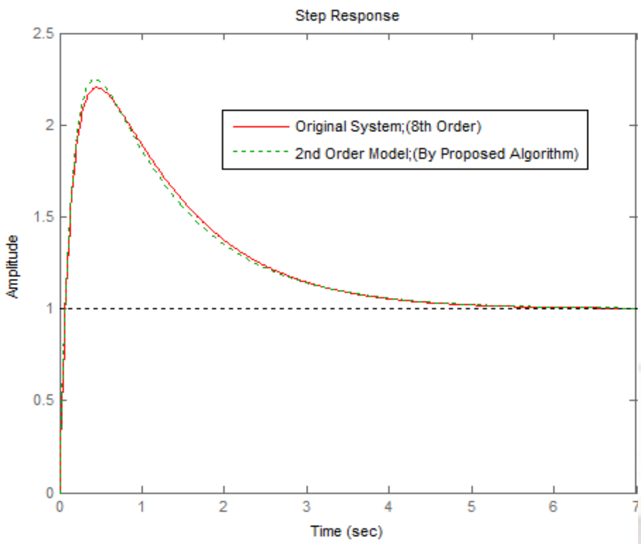


Fig. 4(a) Comparison of step responses of $G_8(s)$ & $R_2(s)$

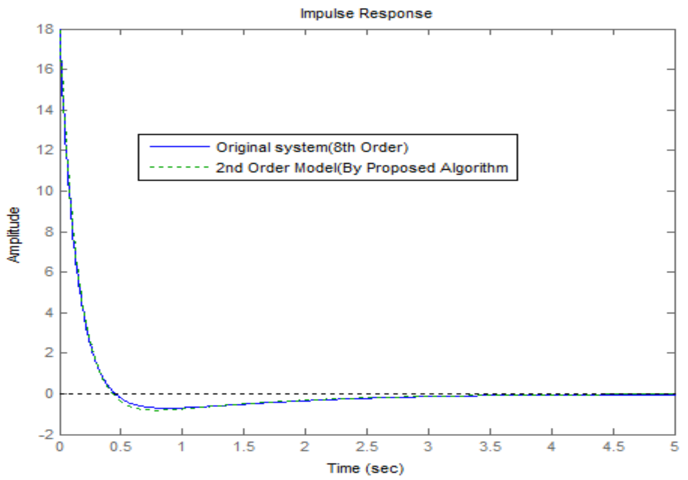


Fig.4(b) Comparison of Impulse responses of $G_8(s)$ &$R_2(s)$

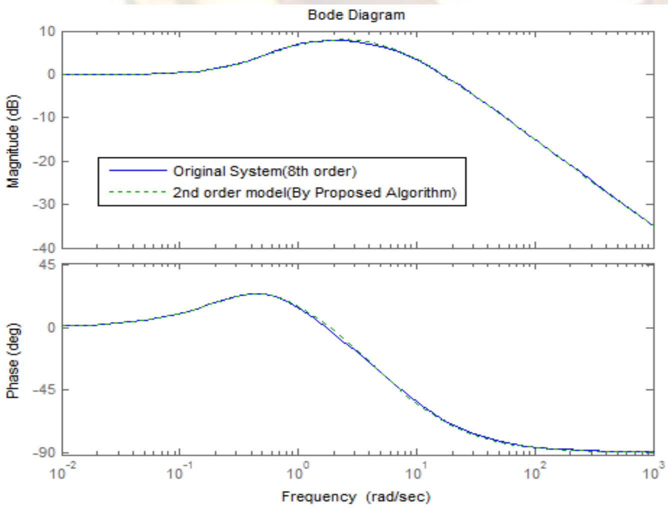


Fig. 4(c) Comparison of Bode Plots of $G_8(s)$ and $R_2(s)$

**Example-2:** Consider an Eighth order system transfer function taken from [23]:

$$G_8'(s) = \frac{35s^7 + 1086s^6 + 13285s^5 + 82402s^4 + 278376s^3 + 511812s^2 + 482964s + 194480}{s^8 + 21s^7 + 220s^6 + 1558s^5 + 7669s^4 + 24469s^3 + 46350s^2 + 45952s + 17760} \cdots (20)$$

By Using the proposed algorithm gives the result as shown below, where t=0 time moment proportional's and m=12 markov parameters of $G_8'(s)$ are used in Eq.(7) and Reduced denominator coefficients from Eq.(8) are obtained as :

$$D_r(s) = s^2 + 1.947888s + 35.92345 \cdots (21)$$

By using PSO to minimize the objective function 'E', as described earlier, we have

$$N_r(s) = 40.389042s + 393.378021 \quad \cdots (22) .$$

Therefore, finally $R_2'(s)$ is given as:

$$R_2'(s) = \frac{40.389042s + 393.378021}{s^2 + 1.947888s + 35.923454} \quad \cdots (23)$$

The Second order reduced models generated by the proposed method to Eq. (20) gives the results as shown in Table IV, where *'t'* time moments and *'m'* markov parameters are used to calculate the denominators and the numerators by minimizing the integral square error using PSO technique.

TABLE IV
COMPARISON OF SECOND ORDER MODELS BY PROPOSED

| $t$ | $m$ | $d_0$ | $d_1$ | $e_0$ | $e_1$ | $I$ | $J$ |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 401.2 | 29.83 | 36.63 | 1.43 | 0.028 | 0.057 |
| 2 | 3 | 394.3 | 32.73 | 36.00 | 1.5 | 0.019 | 0.047 |
| 3 | 3 | 374.1 | 40.58 | 34.16 | 1.67 | 0.012 | 0.039 |
| 0 | 10 | 400.4 | 42.84 | 36.56 | 2.21 | 0.023 | 0.049 |
| 0 | 12 | 393.3 | 40.38 | 35.92 | 1.94 | 0.006 | 0.0031 |
| 0 | 13 | 404.8 | 36.09 | 36.97 | 1.86 | 0.005 | 0.034 |

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order reduced model is given in Table V. Figure 4(d)-(f) Presents diagrams of step, impulse and frequency responses of $G_8'(s)$ and, $R_2'(s)$ respectively.

TABLE V
COMPARISON OF REDUCED ORDER MODELS

| **Method of Reduction** | **Reduced Models:** $R_2'(s)$ | **I** | **J** |
|---|---|---|---|
| Proposed Algorithm (t=0&m=12) | $\frac{40.389042s + 393.378021}{s^2 + 1.947888s + 35.923454}$ | 0.0066 | 0.0318 |
| S.N.Sivanandam et al [23] | $\frac{35s + 401.21}{s^2 + 1.436s + 36.63}$ | 0.0311 | 0.0640 |
| C.S.Hsieh et al[3] | $\frac{1.6666s + 61.271473}{0.047619s^2 + s + 5.595338}$ | 0.5594 | 0.8629 |
| R.Prasad et al[6] | $\frac{8.690832s + 4.498007}{s^2 + 0.836381s + 0.41076}$ | 0.9327 | 2.9583 |

| | | | |
|---|---|---|---|
| Y.Shamas [24] | $\frac{13.09095s + 5.271465}{s^2 + 1.245549s + 0.481393}$ | 0.8841 | 2.3253 |

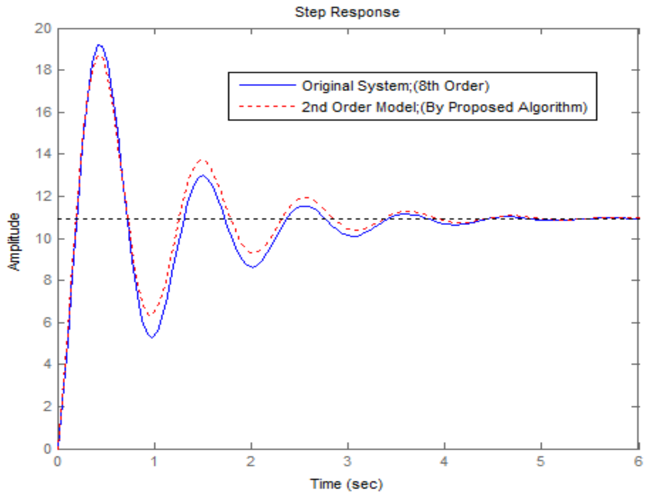


Fig. 4(d) Comparison of step responses of $G_8'(s)$ &$R_2'(s)$

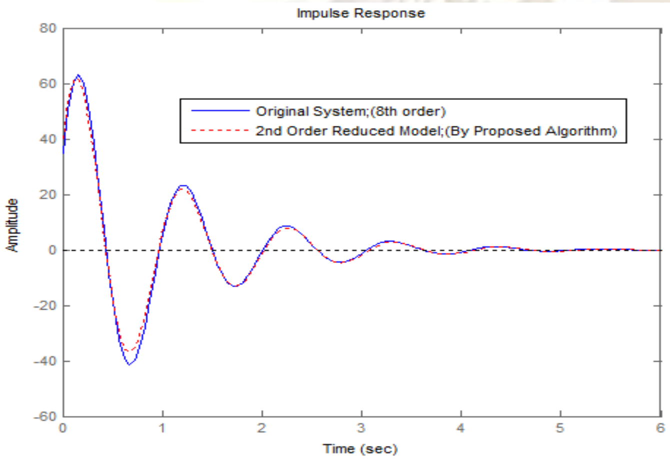


Fig. 4(e) Comparison of Impulse responses of $G_8'(s)$ & $R_2'(s)$

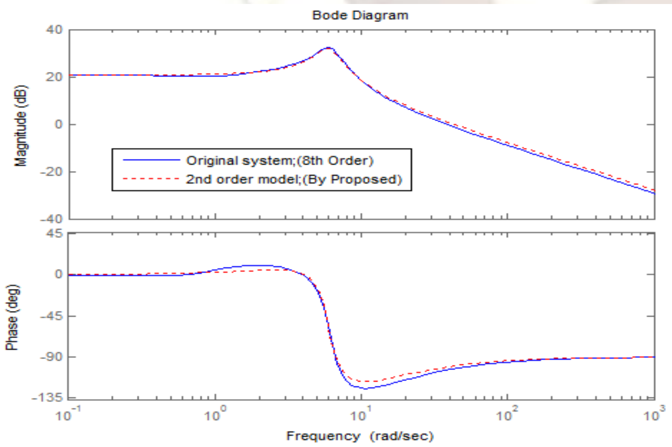


Fig. 4(f) Comparison of Bode Plots of $G_8'(s)$ & $R_2'(s)$

## V. CONCLUSIONS

The authors proposed a mixed algorithm for reducing the order of linear dynamic SISO systems. In this algorithm, the concept of order reduction by a generalised least-squares method has been improved and employed to determine the coefficients of reduced denominator while the coefficients of the reduced numerator are obtained by minimizing the integral square error between the transient responses of original and reduced models using PSO technique pertaining to unit step input. The matching of the unit step and impulse responses is assured reasonably well in the algorithm. The algorithm is simple, rugged and computer oriented.

The relative step and impulse mapping errors between the original and reduced order models are also determined and plotted with respect to time. A comparison of these mapping errors for the proposed reduction algorithm and the other well known existing order reduction techniques is also given in Tables III and V, from which it is clear that the proposed algorithm compares well with the other techniques of model order reduction.

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