

## **Channel Estimation for MIMO-OFDM Systems**

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### **ABSTRACT**

**Multiple-input multiple (MIMO) communication system combined with the orthogonal frequency division multiplexing (OFDM) modulation technique can achieve reliable high data rate transmission over broadband wireless channels. In this paper performance analysis of channel estimation through different algorithms for estimating channel using different modulation scheme are investigated. The estimation of channel at pilot frequencies is based on Least Square, Minimum mean square, Least Mean Square and Recursive Least Square channel estimation algorithm. We have compared the performances of channel estimation algorithm by measuring bit error rate vs. SNR with BPSK, QPSK 16-PSK and 256-PSK modulation schemes. Recursive Least Square estimation has been shown to perform much better than LS, MMSE and LMS but is more complex than other channel estimation algorithm.**

*Keywords – MIMO-OFDM Least square, Minimum mean square, Least Mean Square and Recursive Least Square*

### **I. INTRODUCTION**

It is a well-known fact that the amount of information transported over communication systems grows rapidly. Not only the file sizes increase, but also large bandwidth-required applications such as video on demand and video conferencing require increasing data rates to transfer the information in a reasonable amount of time or to establish real-time connections. To support this kind of services, broadband communication systems are required. Large-scale penetration of wireless systems into our daily lives will require significant reductions in cost and increases in bit rate and/or system capacity. Recent information theoretical studies have revealed that

the multipath wireless channel is capable of huge capacities, provided that multipath scattering is sufficiently rich and is properly exploited through the use of the spatial dimension. Appropriate solutions for exploiting the multipath properly, could be based on new techniques that recently appeared in literature, which are based on Multiple Input Multiple Output (MIMO) technology. Basically, these techniques transmit different data streams on different transmit antennas simultaneously. By designing an appropriate processing architecture to handle these parallel streams of data, the data rate and/or the Signal-to-Noise Ratio (SNR) performance can be increased. Multiple Input Multiple Output (MIMO) systems are often combined with a spectrally efficient transmission technique called Orthogonal Frequency Division Multiplexing (OFDM) to avoid Inter Symbol Interference (ISI).[5]

Channel estimation is a crucial and challenging issue in coherent demodulation. Its accuracy has significant impact on the overall performance of the MIMO-OFDM system. The digital source is usually protected by channel coding and interleaved against fading phenomenon, after which the binary signal is modulated and transmitted over multipath fading channel. Additive noise is added and the sum signal is received. Due to the multipath channel there is some intersymbol interference (ISI) in the received signal. Therefore a signal detector needs to know channel impulse response (CIR) characteristics to ensure successful removal of ISI.

The channel estimation in MIMO-OFDM system is more complicated in comparison with SISO system due to simultaneous transmission of signal from different antennas that cause co-channel interference. This issue highlights that developing channel

algorithm with high accuracy is an essential requirement to achieve full potential performance of the MIMO-OFDM system. A considerable number of channel estimation methods have already been studied by different researchers for MIMO systems.

A wideband channel is normally frequency selective and time variant. For an MIMO OFDM mobile communication system, the channel transfer function at different subcarriers appears unequal in both frequency and time domains. Therefore, a dynamic estimation of the channel is necessary. Pilot-based approaches are widely used to estimate the channel properties and correct the received signal. There are two types of pilot arrangements: Block type and Comb type.[8]

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all carriers. fast fading channel, where the channel changes between adjacent OFDM symbols, the pilots are transmitted at all times but with an even spacing on the subcarriers, representing a comb type pilot channel estimation [9].

Several Channel Estimation techniques have been proposed to mitigate interchannel interference (ICI) in OFDM systems. The least square (LS) CE has been proposed to minimize the squared differences between the received and estimated signal. The LS algorithm, which is independent of the channel model, is commonly used in equalization and filtering applications. But the statistics of channels in real world change over time and inversion of the large dimensional square matrix turns out to be ill-conditioned. To further improve the accuracy of the estimator, Wiener filtering based iterative CE has been investigated; However, this scheme also requires high complexity and knowledge of channel correlations. The most important research topic in the wireless communications is the adaptive CE where the channel is rapidly time-varying. The time-varying multipath channel can be represented by a tap-delayed line with time varying coefficients and fixed tap spacing. An adaptive algorithm is a process that

changes its parameters as it gain more information of its possibly changing environment. Among numerous iterative techniques that exist in the open literature, the popular category of approaches which are obtain from the minimization of the mean square error (MSE) between the output of the filter and desired signal to perform CE .MIMO OFDM systems normalized least mean (NLMS) square and recursive least squares (RLS) adaptive channel estimator are described for MIMO OFDM systems)[13].

## II. MIMO OFDM SYSTEM

We consider MIMO-OFDM systems with two transmit antennas and two receive antenna as shown in fig.2.

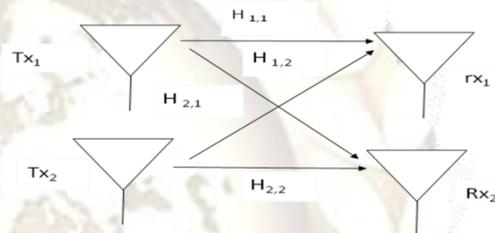


Fig 1.MIMO-OFDM System with 2 transmitting antenna and two receiving antenna

The total number of subcarriers is N. Basically; the MIMO-OFDM transmitter has  $N_t$  parallel transmission paths which are very similar to the single antenna OFDM system. In OFDM system the binary data is first grouped and mapped according to the modulation in “signal mapper”. After modulation the symbol rate reduced to  $R = (R / \log_2 M)$ , where M is constellation size. Then this serial data is fed to serial to parallel convertor. This reduces data rate by N times, where N is number of parallel streams. Each of parallel streams constitutes tiny bandwidth in the spectrum. So these streams almost undergo flat fading in the channel. After inserting pilots either to all subcarriers with a specific period of blocks or within a uniform period of frequency bins in all blocks, IDFT block is used to transform the data sequence of length into time domain signal

$$x(n)=\text{IDFT}\{X(k)\}, \quad n=0, 1, 2, \dots, N-1$$

$$= \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad (1)$$

At the receiver's end, after passing to discrete domain through A/D and low pass filter, guard time is removed and the signal  $Y^g(n+n_g)$  for  $-N_g \leq n \leq N-1$  will become as  $y(n)$  for  $n=0,1,2,\dots,N-1$

Then  $y(n)$  is sent to DFT block for the following operation:

$$Y(k) = \text{DFT}\{y(n)\}, \quad k=0,1,2,\dots,N-1 \quad (2)$$

As a matter of convenience we can write the entire operation as

$$Y(k) = X(k)H(k) + W(k) \quad (3)$$

Where  $X(k) = \text{DFT}\{x(n)\}$  and  $W(k) = \text{DFT}\{w(n)\}$ .

Then the binary information data is obtained back in "signal Demapper" block, where  $N$  is DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time is a copy of the last part which is prepended to OFDM symbol. This makes the transmitted symbol periodic, which plays a key role in identifying frames correctly, so as to avoid ISI and intercarrier interference (ICI). The resultant OFDM symbol is given as follows:

$$X^g(n) = x(n_g + n) \quad n_g = N_g, N_g + 1, \dots, N-1$$

$$n = 0, 1, 2, \dots, N-1 \quad (4)$$

Where  $N_g$  is the length of the guard interval. The transmitted signal will pass through the frequency selective time varying fading channel with additive noise. Then the received signal is

$$y^g(n) = \{x^g(n) \cdot h(n)\} + w(n) \quad (5)$$

The channel response  $h$  can be represented by

$$h(n) = 1/\sqrt{N} \sum_m \alpha_m e^{-j\pi N(k+N-1)\tau_m} \sin(\pi \tau_m) / \sin(\pi/N(\tau_m k))$$

As a MIMO signalling technique,  $N_t$  different signals are transmitted simultaneously over  $N_t \times N_r$

transmission paths and each of those  $N_r$  received signals is a combination of all the  $N_t$  transmitted signals and the distorting noise. It brings in the diversity gain to enhance system capacity as we desire. The data stream from each antenna undergoes OFDM Modulation with the encoding matrix represented as

$$X = \begin{bmatrix} X_1 & -X_2^* \\ X_2 & X_1^* \end{bmatrix} \quad (6)$$

$$X_1 = (X[0] \ X[1] \ X[2] \ \dots \ X[N-1])$$

$$X_2 = (X[1] \ X^*[0] \ X[3] \ X^*[2] \ \dots \ X^*[N-2])$$

The vectors  $X_1$  and  $X_2$  are modulated using the inverse fast Fourier transform (IFFT) and after adding a cyclic prefix as a guard time interval, two modulated blocks  $X_{g1}$  and  $X_{g2}$  are generated and are then transmitted by the first and second Transmit antennas respectively. Assuming that the guard time interval is more than the expected largest delay spread of a multipath channel. The received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{N_k} \end{bmatrix} = \begin{bmatrix} H_{1,1} & H_{1,2} & \dots & H_{1,N_r} \\ H_{2,1} & H_{2,2} & \dots & H_{2,N_r} \\ \vdots & \vdots & \dots & \vdots \\ H_{N_k,1} & H_{N_k,2} & \dots & H_{N_k,N_r} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_N \end{bmatrix} \quad (7)$$

In the equation 7,  $[W_1, W_2, \dots, W_{N_T}]$  denotes AWGN and  $H_m, n$  is the (single-input single-output) channel gain between the  $m^{\text{th}}$  receive and  $n^{\text{th}}$  transmit antenna pair. The  $n^{\text{th}}$  column of  $\mathbf{H}$  is often referred to as the spatial signature of the  $n^{\text{th}}$  transmit antenna across the receive antenna array.

Knowing the channel information at the receiver, Maximum Likelihood (ML) detection can be used for decoding of received signals for two antenna transmission system, which can be written as

$$\begin{aligned} \bar{S}[2k] &= \sum_{i=1}^{Nk} H_{i,1}^* [2k] Y_i [2k] + H_{i,2} [2k] Y_i^* [2k+1] \\ \bar{S}[2k+1] &= \sum_{i=1}^{Nk} H_{i,2}^* [2k+1] Y_i [2k] - H_{i,1} [2k+1] Y_i^* [2k+1] \end{aligned} \quad (8)$$

Where  $k=0, 1, 2, \dots, (N/2)-1$

Assuming that the channel gains between two adjacent sub channels are approximately equal. i.e.

$$\begin{aligned} H_{i,1}[2k] &= H_{i,1}[2k+1] \quad \text{and} \\ H_{i,2}[2k] &= H_{i,2}[2k+1] \end{aligned} \quad (9)$$

At the end, the elements of block  $s[k]$  are demodulated to take out the information data.

### III. TRAINING SEQUENCE BASED CHANNEL ESTIMATION

Based on the assumptions such as perfect synchronization and block fading, a MIMO-OFDM system is designed. In training based channel estimation algorithms, training symbols or pilot tones that are known to the receiver, are multiplexed along with the data stream for channel estimation. The idea behind these methods is to develop knowledge of transmitted pilot symbols at the receiver to estimate the channel. For a block fading channel, where the channel is constant over a few OFDM symbols, the pilots are transmitted on all subcarriers in periodic intervals of OFDM blocks. The channel estimates from the pilot subcarriers are interpolated to estimate the channel at the data subcarrier. This type of pilot arrangement, given in Fig.3 is called the block type arrangement

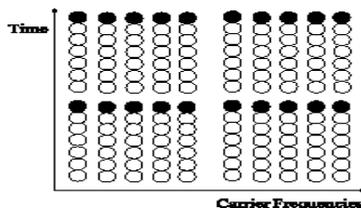


Fig.3. Block type

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted

periodically, in which all subcarriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all carriers.

For a fast fading channel, where the channel changes between adjacent OFDM symbols, the pilots are transmitted at all times but with an even spacing on the subcarriers, representing a comb type pilot placement. This type of pilot arrangement, given in Fig.4 is called the comb type arrangement

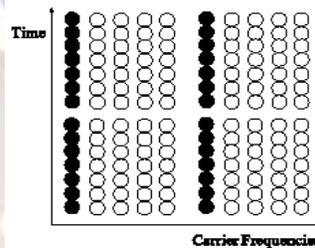


Fig.4 comb type

The estimation can be performed by using either LS or MMSE. In comb-type pilot based channel estimation, the  $Np$  pilot signals are uniformly inserted into  $X(k)$  according to the following equation:

$$X(k) = X(mL+l) \quad l=1, 2, \dots, L-1 \quad (10)$$

Where  $L = \text{No. of subcarriers} / Np$  and  $m$  is pilot carrier index. If inter symbol interference is eliminated by the guard interval, we write in matrix notation

$$Y = XFh + W \quad (11)$$

Where  $X = \text{diag}\{X(0), X(1), \dots, X(N-1)\}$

$$Y = [Y(0), Y(1), \dots, Y(N-1)]^T$$

$$W = [W(0), W(1), \dots, W(N-1)]^T$$

$$H = [H(0), H(1), \dots, H(N-1)]^T = \text{DFT}\{h\}$$

$$F = \begin{bmatrix} W_N^{00} & \dots & W_{N-1}^{0(N-1)} \\ \vdots & \dots & \vdots \\ W_N^{(N-1)0} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

$$F = \begin{bmatrix} W_N^{00} & \dots & W_{N-1}^{0(N-1)} \\ \vdots & \dots & \vdots \\ W_N^{(N-1)0} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

is the DFT matrix

#### IV. PERFORMANCE EVALUATION

##### Adaptive channel estimation

The most important research topic in the wireless communications is the adaptive channel estimation where the channel is rapidly time-varying. An adaptive algorithm is a process that changes its parameters as it gain more information of its possibly changing environment.

The channel estimation methods like least square estimation and recursive least square which uses adaptive estimator which are able to update parameters of the estimator continuously, so that knowledge of channel and noise statistics are not required. The LMS and RLS CE algorithm requires knowledge of the received signal only. This can be done in a digital communication system by periodically transmitting a training sequence that is known to the receiver.

The adaptive channel estimation scheme is given in fig.4.1

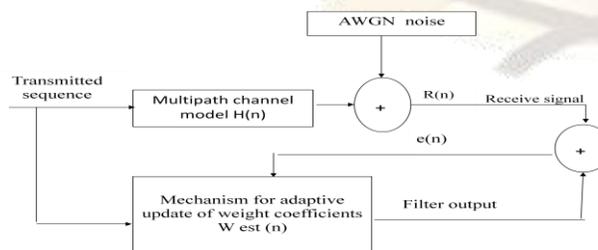


Fig 5. adaptive channel estimation scheme

##### Least Mean Square

The signal X(n) is transmitted via a time-varying channel H(n), and corrupted by an additive noise. The main aim of most channel estimation algorithms is to minimize the mean squared error i.e., between the received signal and its estimate. In the Fig 4.1, we have unknown multipath fading channel, that has to be estimated with an adaptive filter whose weight are updated based on some criterion so that coefficients of adaptive filter should be as close as possible to the unknown channel. The output from the channel can be expressed as:

$$Y(n) = X(n)H(n) + W(n)$$

Output of adaptive filter is given as

$$P(n) = W_{\text{esti}}(n)X(n)$$

Where  $W_{\text{esti}}$  = estimated channel coefficient at time n.

The priori estimated error signal needed to update the weights of the adaptive filter is

$$e(n) = Y(n) - P(n) = X(n)H(n) + W(n) - W_{\text{esti}}(n)X(n)$$

Where e(n) minimized the mean square error.

Now Cost function for adaptive filter structure given as  $j(n) = E[e(m)e^*(m)]$

$$j(n) = \sigma_r^2 - C(n)W_{\text{esti}}(n) - W_{\text{esti}}^T(n)C(n) + D(m)W_{\text{esti}}(n)$$

Where  $\sigma_r^2$  is variance of received signal.

$C(m) = [X(n)Y(n)]$  is the cross correlation vector between input vector and received vector.

$D(m) = E[X(n)X^T(n)]$  is the correlation matrix between input Gradient of cost function j(n) is given as

$$\begin{aligned} \Delta j(n) &= -2C(n) + 2D(m)W_{\text{esti}}(n) \\ &= -2X(n)Y^*(n) + 2X(n)X(n)W_{\text{esti}}(n) \end{aligned}$$

By using this least mean square equation is given as  $W_{\text{esti}}(n+1) = W_{\text{esti}}(n) - 1/2\eta X(n)e^*(n)$

Where,  $W_{esti}^{(n+1)}$ =weighted vector and  $\eta$ =LMS step size

**Recursive Least Square**

RLS algorithm required all the past sample of input and estimated output at each iteration. The objective function of a RLS CE algorithm is defined as an exponential weighted sum of errors square.

$$C(m) = \sum \lambda^{n-m} e^H(n)e(n) + \delta \lambda^m H^H(n)H(n)$$

Where  $\delta$ =positive real no. called regularization parameter,  $e(n)$  is the prior estimation error, and  $\lambda$  is the exponential forgetting factor with  $0 < \lambda < 1$ . The prior estimation error is the difference between the desired response and estimation signal. Prior estimation error is given as

$$e(n) = W^H(n)x(n)$$

The objective function is minimized by taking the partial derivatives with respect to  $W(n)$  and setting the results equal to zero.

$$W(n) = R^{-1}(n) R_{sh}(n)$$

Where  $R^{-1}(n)$  =auto covariance matrix

$R_{sh}(n)$  = cross covariance matrix

Now from this **Recursive Least Square** equation is given as

$$H(n) = H(n-1) + K(m)[W(n) - H^H(n-1)X(n)]^H$$

$$= H(n-1) + K(m)\epsilon^H(n)$$

Where  $\epsilon(n) =$

$$W(n) - H^H(n-1)X(n) \text{ and } K(m) = R^{-1}(n)X(n)$$

OFDM system parameters used in the simulation are

indicated in Table I. We assume to have perfect synchronization since the aim is to observe channel estimation performance. We have chosen the guard interval to be greater than the maximum delay spread in order to avoid inter-symbol interference. The simulation parameters to achieve those results are shown in the table.

TABLE I.

Simulation Parameter

Parameter	Specification
Code length	100
Pilot length	20
IFFT	64
Tx and Rx Antenna	2 x 2
Guard time	1

The simulation results for the channel estimation for the MIMO-OFDM system are given fig.5. In this paper we compare the least mean square and recursive least square channel estimation techniques on the basis of different modulation schemes. Comparison of LMS and RLS are shown in following fig by using above parameter. The complexity of RLS estimator is larger than LMS estimator but give better performance than LMS.

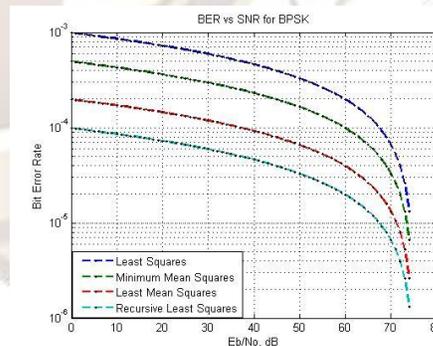
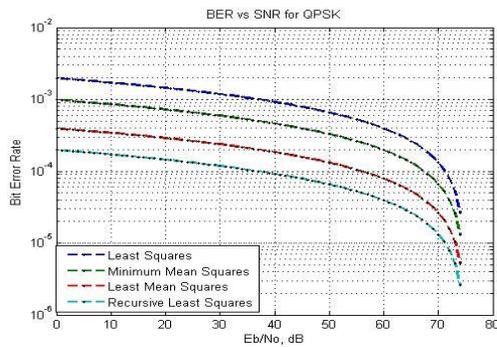
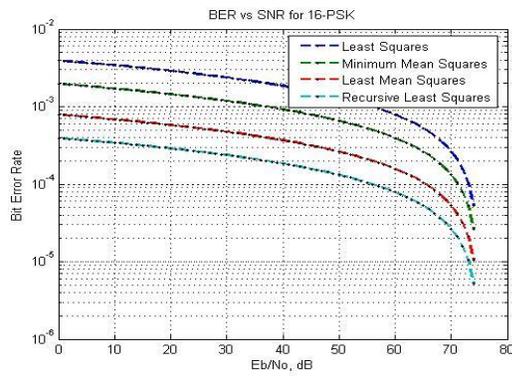


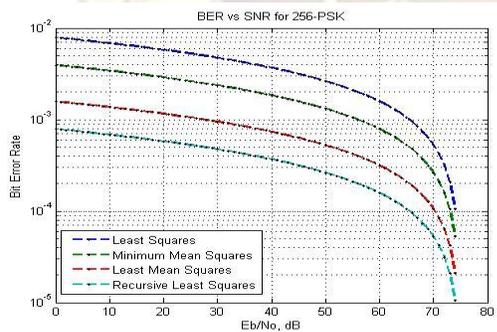
Fig BER vs SNR of BPSK or SNR 75



**Fig BER vs SNR of QPSK or SNR 75**



**Fig BER vs SNR of 16- PSK or SNR 75**



**Fig BER vs SNR of 256- PSK or SNR 75**

**V. CONCLUSION**

In this paper channel estimation based Least square, Minimum Mean Square Least mean square(LMS) and Recursive Least square of MIMO OFDM based systems are studied.. The complexity of RLS is larger than other estimators. The RLS estimator has good performance but high complexity. The LS,MMSE and LMS estimator has low complexity but its performance is not as good as that RLS at low SNRs. Simulation results show that estimation

for MIMO OFDM provides less BER than other systems. Lastly by comparing the performance of RLS with LMS, it is observed that the RLS is more resistant to the noise in terms of the channel estimation

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