Signal Flow Graph Analysis of Linearized Fuzzy PI Controller

B. Amarendra Reddy
Asst. Professor
Dept. of Electrical Engineering,
Andhra University, Visakhapatnam

Nammi Nagarjun
PG Scholar
Dept. of Electrical Engineering,
Andhra University, Visakhapatnam

Praveen Adimulam
PG Scholar
Dept. of Electrical Engineering,
Andhra University, Visakhapatnam

ABSTRACT: A systematic procedure for developing the signal flow graph model of a linearized fuzzy PI controller is presented in this paper. This proposed method provides ease of model formulation and avoids the mathematical complexity involved in obtaining the linearized model from a non-linear model. As a first step in constructing the signal flow graph, the analytical structures of fuzzy PI controller are needed. In view of this triangular/trapezoidal membership functions for inputs variables, singleton or triangular/trapezoidal membership functions are considered for output variables, Zadeh fuzzy logic AND operator, Lukasiewicz fuzzy logic OR operation and centroid defuzzifier are considered. A fuzzy PI controller is represented as a non-linear PI controller which is linearized around an operating point using perturbation method. For the linearized fuzzy PI controller signal flow graphs are developed.

1. Introduction:

Fuzzy controllers are important primarily because it provides insightful information about what a fuzzy controller is how it works, and how it relates to and differs from a classical controller. Given the dominance of conventional PI control in industrial control, it is significant both in theory and in practice if a controller can be found that is capable of outperforming the PI controller with comparable ease of use. We begin our study with an analytical structure of some simplest PI fuzzy controllers and reveal their connections with PI control and variable gain control. Compared with other more complex fuzzy controllers, these simplest fuzzy controllers have fewer design parameters and hence are more practically useful.

"Analytical structure" we mean the mathematical expression of a fuzzy controller that represents precisely the fuzzy controller without any approximation. Note that this is never an issue for conventional control because the analytical structure of a conventional controller, linear or nonlinear, is always readily available for analysis and design. Thus the design goal is to design the controller structure and Parameters on the basis of the given system model so that resulting control system performance will meet user's performance specifications. For fuzzy control, in addition to this usual requirement, there exist few more major difficulties pertinent only to fuzzy control and irrelevant to conventional control. One of them is that the input-output structure of a fuzzy controller is usually mathematically unavailable after the controller is constructed; most fuzzy controllers are constructed via so called intelligent system approaches as opposed to the mathematical approaches exclusively used in conventional controller. The fuzzy controller have been treated and used as black-box controllers without the analytical structure information, precise and effective mathematical analysis and design are very difficult to achieve.

Hence the foremost issue is revealing the analytical structure is sensible in the context of conventional control theory. This is to say that merely deriving the structure is not useful enough and the structure must be represented in a form clearly understandable from control theory stand point. Once the structure is well understood, analytical issues can be explored using the well developed conventional control theory. Theoretical analysis coupled with signal flow graph depiction involving various system models demonstrates the effectiveness and superior performance of these simplest fuzzy controllers in comparison with linear PI controller. Theoretical analysis coupled with signal flow graph representation involving various system models demonstrates the effectiveness and superior performance of these simplest fuzzy controllers in comparison with the comparable non-linear PI controller.

This paper organized as follows, section II describes the configuration of non-linear fuzzy PI controller, Section III describes fuzzification algorithm and fuzzy control rules, section IV describes Fuzzy logic for the evaluation of fuzzy control rules. Section V describes Defuzzification Module and Structural analysis of the fuzzy PI controller. Section VI describes Linearization using

Perturbation Theory section VII describes the SFG analysis of the fuzzy PI controller.

2. Configuration of Non - Linear Fuzzy PI controller:

The fuzzy PI controller being developed is a nonlinear fuzzy PI controller. PI or Fuzzy PI controller is the most used controller in the industry, because the proportional (P) with the Integral actions in the proportional- integral (PI) controller eliminates the steady state error.

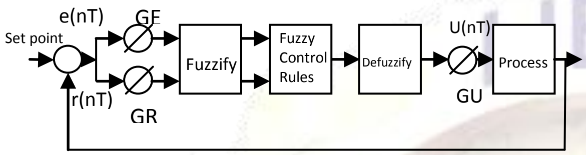


Fig 1 : Block Diagram of a typical fuzzy controller

The inputs of the fuzzification module are scaled error (GE) and scaled rate (GR) .

GE.e(nT)=GE(y(nT)-set point)

GR.r(nT)=GR(e(nT)-e(nT-T)).

Where GE and GR are scalars for the error and rate, respectively (nT) is the process output at sampling time nT and e(nT-T) is error at previous sampling time. The set point is a target value for the process output.

3.Fuzzification Algorithm:

The following input membership functions are selected to transform inputs data of the FLC into two linguistic values, "P" and "N" for positive and negative input membership functions, as shown in fig.3 (a)

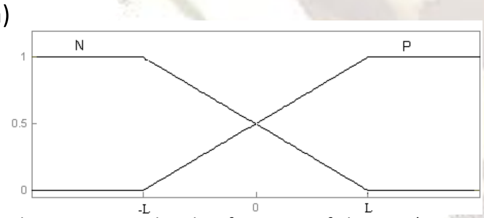


Fig.2 (a) The inputs membership functions of the FLC (error and rate)

The input membership functions of the FLC:

For error e(nT):

$$\mu_{\tilde{P}}(e) = \begin{cases} \mathbf{0}, & \boldsymbol{GEe(nT)} < -L \\ \dfrac{\boldsymbol{GEe(nT)+L}}{\mathbf{2L}}, & -\boldsymbol{L} \le \boldsymbol{GEe(nT)} \le \boldsymbol{L} \\ \mathbf{1} & \boldsymbol{GEe(nT)} > L \end{cases}$$

$$\mu_{\tilde{N}}(e) = \begin{cases} \mathbf{0}, & \boldsymbol{GEe(nT)} < -L \\ \dfrac{\boldsymbol{-GEe(nT)+L}}{\mathbf{2L}}, & -\boldsymbol{L} \le \boldsymbol{GEe(nT)} \le \boldsymbol{L} \\ \mathbf{1} & \boldsymbol{GEe(nT)} > L \end{cases}$$

where $\mu_P(e)$ and $\mu_N(e)$ are the positive and negative membership functions of the error.

For rate r(nT):

$$\mu_{\tilde{P}}(r) = \begin{cases} \mathbf{0}, & \boldsymbol{GR\,r(nT)} < -L \\ \dfrac{\boldsymbol{GR\,r(nT)+L}}{\mathbf{2L}}, & -\boldsymbol{L} \le \boldsymbol{GR\,r(nT)} \le \boldsymbol{L} \\ \mathbf{1} & \boldsymbol{GR\,r(nT)} > L \end{cases}$$

$$\mu_{\tilde{N}}(r) = \begin{cases} \mathbf{0}, & \boldsymbol{GR\,r(nT)} < -L \\ \dfrac{\boldsymbol{-GR\,r(nT)+L}}{\mathbf{2L}}, & -\boldsymbol{L} \le \boldsymbol{GR\,r(nT)} \le \boldsymbol{L} \\ \mathbf{1} & \boldsymbol{GR\,r(nT)} > L \end{cases}$$

Where $\mu_P(r)$ and $\mu_N(r)$ are the positive and negative membership functions of the rate (change of error).

The scaled output membership functions of FLC are represented by three membership functions, labeled as "zero", "pos" and "neg" for zero, positive, and negative output membership functions respectively, as shown in fig.3 (b)

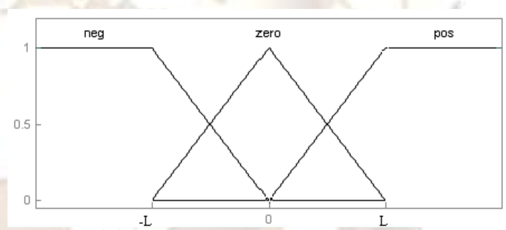


Fig.2(b) The output membership functions of the FLC

3.1 Fuzzy Rules base Module:

The four fuzzy control rules are:

R1: if error is positive and rate is positive thenoutput is positive.

R2: if error is positive and rate is negative then output is zero.

R3: if error is negative and rate is positive then output is zero.

R4: if error is negative and rate is negative then output is negative.

Here AND is Zadeh’s logical “AND” defined by

$$\mu_A \text{ AND} \mu_B = \min\{\mu_A, \mu_B\}$$

for any membership value μ_A and μ_B on the fuzzy subsets A and B, respectively.

4. FUZZY LOGIC FOR EVALUATION OF THE FUZZY CONTROL RULES:

Zadeh fuzzy logic AND operator is used to realize the AND operations in antecedent part of the rules. Due to the use of Zadeh AND operator, the input space must be divided into number of regions in such a way that in each region a unique analytical inequality relationship can be obtained for each fuzzy rule between the two membership functions being ANDed.

Consider the first rule antecedent parts which contain two membership functions the boundary on which the membership value is same between two MFs is obtained by letting them equal.

Boundary division for rule1 is given by:

$$\mu_{\tilde{P}}(e) = \mu_{\tilde{P}}(r)$$

$$\frac{K_e e(n) + L}{2L} = \frac{K_r r(n) + L}{2L}$$

$$r(n) = e(n) \ldots [1]$$

Boundary division for rule2 is given by:

$$\mu_{\tilde{P}}(e) = \mu_{\tilde{N}}(r)$$

$$\frac{K_e e(n) + L}{2L} = \frac{-K_r r(n) + L}{2L}$$

$$r(n) = -e(n) \ldots [2]$$

Boundary division for rule3 is given by:

$$\mu_{\tilde{N}}(e) = \mu_{\tilde{P}}(r)$$

$$\frac{-K_e e(n) + L}{2L} = \frac{K_r r(n) + L}{2L}$$

$$r(n) = -e(n) \ldots [3]$$

Boundary division for rule3 is given by:

$$\mu_{\tilde{N}}(e) = \mu_{\tilde{N}}(r)$$

$$\frac{-K_e e(n) + L}{2L} = \frac{-K_r r(n) + L}{2L}$$

$$e(n) = r(n) \ldots [4]$$

Equations [1]-[4] are used to generate individually the space division (plane division) between error and rate.

Superimposing the four input space divisions from the expression [1]-[4] to form a total of 20 input combinations.

They are labeled from ICI to IC20, as shown in *Fig 4*

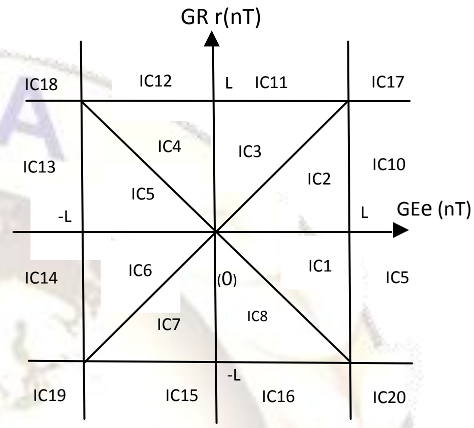
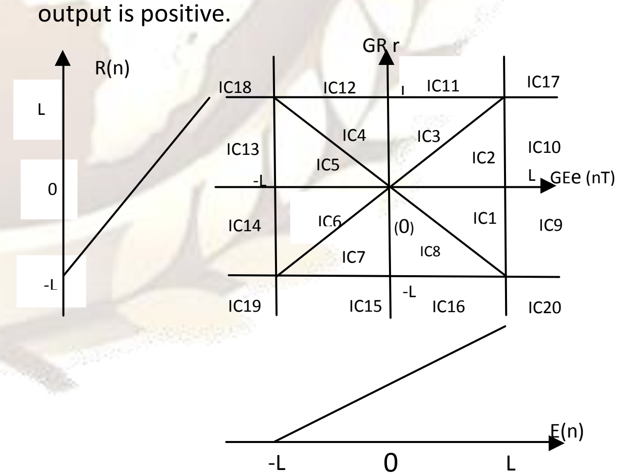


Fig.3 Possible input combinations (IC) of scaled error and rate of change of error, which must be considered when the fuzzy controller rules are evaluated as shown in Table. 1.

Now consider IC1 region and rule1, i.e.

R1: if error is positive and rate is positive then output is positive.



The rule1 is associated with error positive and rate positive membership functions. Consider any point in the IC1 region, i.e., from 0 to L [0, L] error positive membership function will take the values from 0.5 to 1.0 and rate positive will take the values from 0 to 0.5. Applying Zadeh AND operation i.e., min (e.p, r.p), minimum of error positive and rate positive is rate positive. Similarly if we apply the minimum operation (Zadeh AND) to all regions and all rules, the results are tabulated as shown in Table 1.

Table-1evaluation results for the four fuzzy control rules R1-R4 for all combinations of inputs using Zadeh AND fuzzy logic when scaled error and rate of process output are within the interval [-L, L] of the fuzzification algorithm. The input combinations of scaled error and rate are shown graphically in fig 4

Input combinations of scaled error and rate as shown in fig 3.5	Memberships obtained by evaluating fuzzy control rules R1, R2, R3 and R4 using Zadeh AND fuzzy logic.			
	R1	R2	R3	R4
IC1	R.P	R.N	E.N	E.N
IC2	R.P	R.N	E.N	E.N
IC3	E.P	R.N	E.N	R.N
IC4	E.P	R.N	E.N	R.N
IC5	E.P	E.P	R.P	R.N
IC6	E.P	E.P	R.P	R.N
IC7	R.P	E.P	R.P	E.N
IC8	R.P	E.P	R.P	E.N

These regions are necessary because they will result in, in each of the 20 ICs, a unique inequality between error and rate when each of the four fuzzy rules evaluated by a Zadeh logic AND. After applying defuzzification algorithm to each region with resulting memberships, we obtain the expressions for different IC's which are in table2 in the next section.

5. Defuzzification Module:

The defuzzification means the fuzzy to crisp conversions. The fuzzy results generated cannot be used such as to the applications hence it is necessary to convert the fuzzy quantities into crisp quantities for further processing. This can be achieved by using defuzzification process. Defuzzification method canalso be called as "rounding off" method. The defuzzification reduces the collection of membership function values into a single scalar quantity.

The centroid defuzzification method is the mostly used method to convert the inference fuzzy control action to real number. The fuzzy logic controller output is obtained by,

$$\Delta u(nT) = \frac{\sum \Delta u_i \ \mu_{ri}}{\sum \mu_{ri}}$$

Where Δu_i is the value of the output member for i^{th} rule, μ_{ri} corresponding inferred input member for i^{th} rule.

5.1Structural analysis of the fuzzy PI controller in defuzzification method:

The structure of the fuzzy PI controller from the defuzzification method is

$$\Delta u(nT) = \sum_{i=1}^{4} K_I{}^i(e,r)e(nT) + K_P{}^i(e,r)r(nT)$$
$$= [K_P(e,r)r(nT) + K_I(e,r)e(nT)]$$

Where $K_P(e, r)$ and $K_I(e, r)$ are the dynamic proportional gain and integral gain respectively, they change with e(nT) and r(nT). This is to say the fuzzy controller is a nonlinear PI controller with variable proportional gain and integral gain. We have the structure related to PI controller. Since the only difference between them are the gains.

We now derive the analytical expressions for $K_P(e, r)$ and $K_I(e, r)$. First we need to divide the error-rate input space into 20 different input combinations as shown in fig.4.

Table.2 Mathematical input-output relations of the fuzzy controller for the overall input space division

IC No.	$\Delta u(nT)$ =
IC1 ,IC2	$\frac{0.5L(GRr(nT) + GEe(nT)}{2L - GEe(nT)}$
IC3 ,IC4	$\frac{0.5L(GRr(nT) + GEe(nT)}{2L - GRr(nT)}$
IC5,IC6	$\frac{0.5L(GRr(nT) + GEe(nT)}{2L + GEe(nT)}$
IC7 ,IC8	$\frac{0.5L(GRr(nT) + GEe(nT)}{2L + GRr(nT)}$
IC9,IC10	$\frac{L + GRr(nT)}{2}$
IC11,IC12	$\frac{L + GEe(nT)}{2}$
IC13 ,IC14	$\frac{GRr(nT) - L}{2}$
IC15,IC16	$\frac{GEe(nT) - L}{2}$
IC17	L
IC18	0
IC19	-L
IC20	0

It is clear from the equations which are given in table 3.2, the controller may switch automatically from one control formula to another form time to time, depending on the locations of the states of the input (i.e., e(nT) and r(nT)). However such switching is always continuous in time and, moreover, smooth on the boundaries of any two adjacent regions, as can be verified by matching any two adjacent formulas on their boundary.The proportional gain and integral gain are chosen as the respective values of K_P and K_I when the error and rate of error are both zero: $e(nT) = r(nT) = 0$. In this case

$$K_P = \frac{GU \cdot GR}{4} \qquad K_I = \frac{GU \cdot GE}{4}$$

5. Linearization using Perturbation Theory:

Linearization is a method for assessing the local stability of a nonlinearsystem at an equilibrium point. Linearization makes it possible to use tools for studying linear systems to analyze the behavior of a nonlinear system near a given point. The linearization of a function is the first order term of its power series expansion around the point of interest. In this paper we linearize the non linear system using Perturbation theory. Perturbation theory leads to an expression for the desired solution in terms of a formal power series. The leading term in this power series is the solution of the exactly solvable non-linear problem.

We now consider incremental output of IC1 and apply perturbation theory to linearize it.,

$$u(nT) = \frac{0.5LGEe + 0.5LGRr}{2L - GEe} \dots (1)$$

$$u = u_0 + \Delta u \;\; ; \; e = e_0 + \Delta e \;\; ; \; r = r_0 + \Delta r \dots (2)$$

Now put (2) in (1), we get

$$u(2L - GEe) = 0.5LGEe + 0.5LGRr$$

$$(u_0 + \Delta u)\big(2L - GE(e_0 + \Delta e)\big) = 0.5LGE(e_0 + \Delta e) + 0.5LGR(r_0 + \Delta r)$$

$$2Lu_0 + 2L\Delta u - GEe_0u_0 - GE\Delta eu_0 - GEe_0\Delta u - GE\Delta u\Delta e = 0.5LGEe_0 + 0.5LGE\Delta e + 0.5LGRr_0 + 0.5LGR\Delta r$$

$$(2L - GEe_0)\Delta u = GEu_0\Delta e + 0.5LGE\Delta e + 0.5LGR\Delta r$$

$$\Delta u = \frac{GEu_0 + 0.5LGE}{(2L - GEe_0)}\Delta e + \frac{0.5LGR}{(2L - GEe_0)}\Delta r \dots (3)$$

Equation (3) is now linearized form of incremental output of IC1.

IC No.	$\Delta u(nT) =$
IC1,IC2	$\left(\frac{GEu_0 + 0.5LGE}{2L - GEe_0}\right) * \Delta e + \left(\frac{0.5LGR}{2L - GEe_0}\right) * \Delta r$
IC3.IC4	$\left(\frac{GRu_0 + 0.5LGR}{2L - GRr_0}\right) * \Delta r + \left(\frac{0.5LGE}{2L - GRr_0}\right) * \Delta e$
IC5 ,IC6	$\left(\frac{GEu_0 + 0.5LGE}{2L + GEe_0}\right) * \Delta e + \left(\frac{0.5LGR}{2L + GEe_0}\right) * \Delta r$
IC7 ,IC8	$\left(\frac{GRu_0 + 0.5LGR}{2L + GRr_0}\right) * \Delta r + \left(\frac{0.5LGE}{2L + GRr_0}\right) * \Delta e$
IC9,IC10	$\frac{L + GR\Delta r}{2}$
IC11,IC12	$\frac{L + GE\Delta e}{2}$
IC13 ,IC14	$\frac{GR\Delta r - L}{2}$
IC15,IC16	$\frac{GE\Delta e - L}{2}$
IC17	L
IC18	0
IC19	-L
IC20	0

Table 3 shows all the linearized incremental outputs of IC1 – IC20.

For the linearized IC's we now draw the signal flow graphs which provides ease of model formulation and avoids the mathematical complexity involved in obtaining the linear fuzzy controller.

6.Signal Flow Graph Analysis:

Signal-flow graph is a graphical representation of relationships between variables of a set of linear algebraic equations in a system. It is a directed graph consisting of nodes and branches. Its nodes are the variables of a set of linear algebraic relations. An SFG can only represent multiplications and additions. Multiplications are represented by the weights of the branches; additions are represented by multiple branches going into one node. A signal-flow graph has a one-to-one relationship with a system of linear equations. It can also be used to solve for ratios of these signals.

Key elements of a signal flow graph are:

1.The system must be linear,

2. Nodes represent the system variables,
3. Branches represent paths for signal flow.
4. Signals travel along branches only in the direction of the arrows.

Signal flow graphs (SFGs) can form an intuitive picture of the signal flow in a system. As an application, we will develop SFGs to all ICs from *Table 3.3*. The SFGs are shown below in fig-a, b, c, d, e, f, g, h.

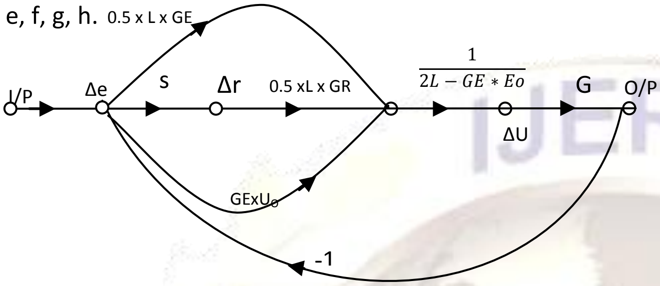


Fig-(a): signal flow graph model for IC1,IC2

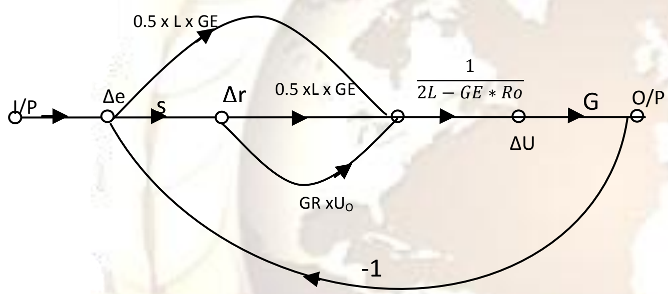


Fig-(b): signal flow graph model for IC3,IC4

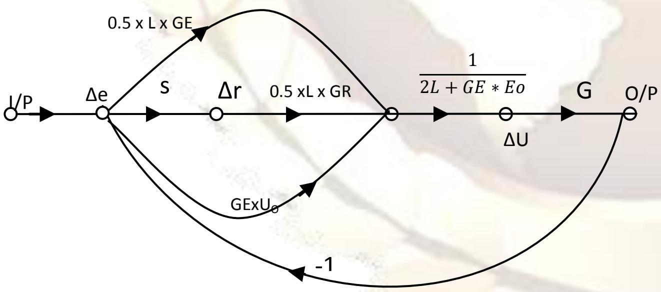


Fig-(c): signal flow graph model for IC5,IC6

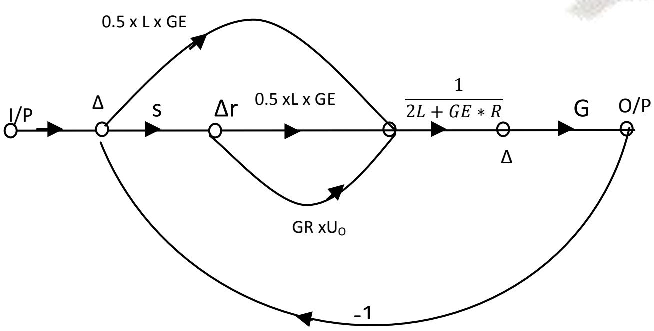


Fig-(d): signal flow graph model for IC7,IC8

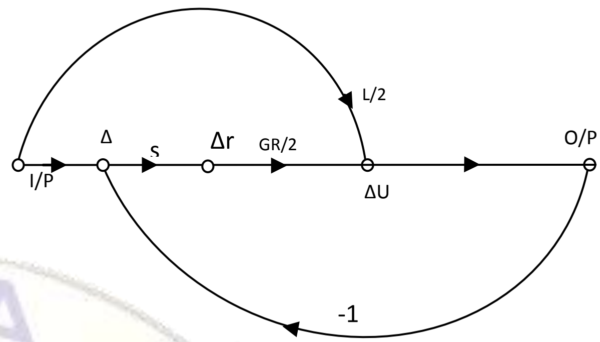


Fig-(e): signal flow graph model for IC9,IC10

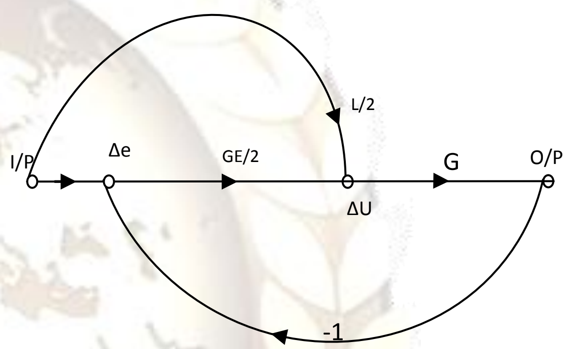


Fig-(f): signal flow graph model for IC11,IC12

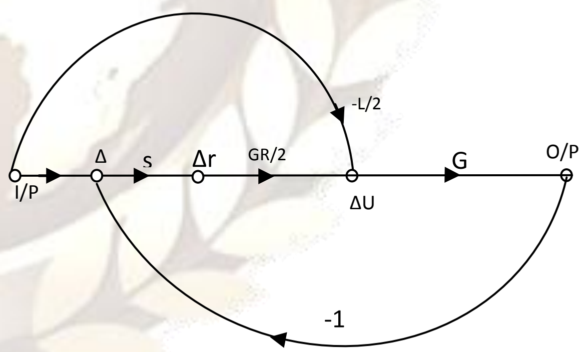


Fig-(g): signal flow graph model for IC13,IC14

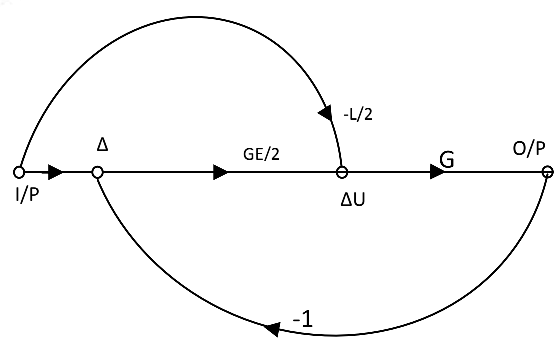


Fig-(h): signal flow graph model for IC15,IC16

The Transfer functions from the signal flow graphs are develop using mason's gain formula and are tabulated in table-4.

Table-4 transfer functions of the signal flow graphs

IC	Transfer function from SFG
IC!,IC2	$\frac{0.5GEG(0.5L+U_0)+0.5SLGRG}{2L+GE(-E_0+0.5LG+U_0G)+0.5LGRG}$
IC3,IC4	$\frac{0.5GEG(1+S)+SGRU_0}{2L+GE(-R_0+0.5LG+0.5LG)+SGRU_0G}$
IC5,IC6	$\frac{0.5GEG(0.5L+U_0)+0.5SLGRG}{2L+GE(E_0+0.5LG+U_0G)+0.5LGRG}$
IC7,IC8	$\frac{0.5GEG(1+S)+SGRU_0}{2L+GE(R_0+0.5LG+0.5LG)+SGRU_0G}$
IC9,IC10	$\frac{G(GR+L)}{2+GRG}$
IC11,IC12	$\frac{G(GE+L)}{2+GEG}$
IC13,IC14	$\frac{G(GR-L)}{2+GRG}$
IC15,IC16	$\frac{G(GE-L)}{2+GEG}$

7. Conclusion:

In this paper we have derived the analytical input and output relationships for the linearized fuzzy PI controller having two input variables and one output variable. There are two triangular/trapezoidal membership functions in each input variable and three triangular/trapezoidal membership functions in output variable. Zadeh AND operator is used to evaluate the antecedent part of the each of the rule. Since Zadeh AND is used, the input space is divided into 20 regions. Then the non-linear PI controller is linearized around an operating point using perturbation method. For the linearized fuzzy PI controller signal flow graphs are developed for each IC and Transfer function from SFG for linearized fuzzy PI controllers was developed.

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