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Signal Flow Graph Modeling of Linear Fuzzy PI Controller

B. Amarendra Reddy
Asst. Professor,
Dept. of Electrical Engineering,
Andhra University, Visakhapatnam

Revanth Kumar P
PG Scholar,
Dept. of Electrical Engineering,
Andhra University, Visakhapatnam

M Sujatha
PG Scholar,
Dept. of Electrical Engineering,
Andhra University, Visakhapatnam

ABSTRACT: A systematic procedure for developing the unified signal flow graph model of a fuzzy PI controller is presented in this paper. From this unified model it is possible to determine the complete behavior of the Fuzzy PI controller. As a first step in constructing the signal flow graph the analytical structures oflinear fuzzy PI controller are needed. In view of this by considering triangular/trapezoidal membership functions for input variables, singleton or triangular/trapezoidal membership functions for output, Zadeh fuzzy logic AND operator, Zadeh OR operator or Lukasiewicz fuzzy logic OR operation and linear defuzzifier. A fuzzy PI controller can be represented as a linear PI controller and piece-wise linear fuzzy PI controller. In these cases a systematic signal flow graph analysis is presented in this paper.

1. Introduction:

Since the first successful application of the idea of the fuzzy sets [1] to the control of dynamic plant by mamdani and assilian [2] there has been considerable worldwide interest in the subject of "Fuzzy Control System Engineering". It has been known that it is possible to control many complex systems effectively by human operators who have no knowledge of their underlying dynamics, while it is difficult to achieve the same with conventional controllers. It is this fact which has ultimately led to the prospective development of fuzzy control in avariety of applications [3] most of these applications have been based on the intuitive implementation of domain experts' experience.

"Analytical structure" we mean the mathematical expression of a fuzzy controller that represents precisely the fuzzy controller without any approximation. Note that this is never an issue for conventional control because the analytical structure of a conventional controller, linear or nonlinear, is always readily available for analysis and design. Thus the design goal is to design the controller structure and Parameters on the basisof thegiven system model so that resulting controlsystem performance will meet user's performance specifications. For fuzzy control, in addition to this usual requirement, there exist few more major difficulties pertinent only to fuzzy control and irrelevant to conventional control. One of them is that the input-output structure of a fuzzy controller is usually mathematically unavailable after the controller is constructed; most fuzzy controllers are constructed via so called intelligent system approaches as opposed to the mathematical approaches exclusively used in conventional controller. The fuzzy controller have been treated and used as black-box controllers without the analytical structure information, precise and effective mathematical analysis and design are very difficult to achieve.

Hence the foremost issue is revealing the analytical structure is sensible in the context of conventional control theory. This is to say that merely deriving the structure is not useful enough and the structure must be represented in a form clearly understandable from control theory stand point. Once the structure is well understood, analytical issues can be explored using the well-developed conventional control theory.Theoretical analysis coupled with signal flow graph depiction involving various system models demonstrates the effectiveness and superior performance of these simplest fuzzy controllers in comparison with linear PI controller.

This paper organized as follows, in section IIdescribes the configuration of fuzzy PI controller. Section III describes fuzzification algorithm and fuzzy control rules, section IV describes fuzzy logic for evaluation of fuzzy controlrules, section V describes the defuzzification algorithm, section VI describes the SFG analysis of the fuzzy PI controller

2. Configuration of Linear Fuzzy PI controller:

A typical fuzzy controller can be described by the block diagram in *Fig 1*. Most fuzzy controllers developed so far employs error and rate about a set point as their inputs. K_e and $K_r$are gains for error and rate. K_u is the gain for incrementaloutput, fuzzy controller output $\Delta u(nT)$ is the input to Process/plant. An analytical structure of some simplest PI fuzzy controllers and reveal their connections with PI control and variable gain control. Compared with other more complex fuzzy controllers, these simplest fuzzy controllers have fewer design parameters and hence are more practically useful.

A fuzzy controller is called a linear fuzzy controller if its output variable is a linear function of its input variables. The structures of linear fuzzy controller are easier to derive and understand hence they provide an excellent stepping stone towards understanding an analysis of more complicated fuzzy controllers.

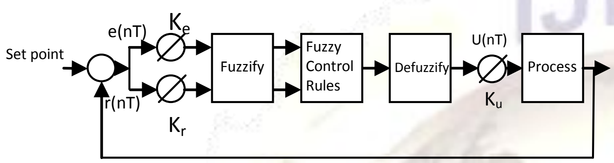

Fig 1 : Block Diagram of a typical fuzzy controller

3. Fuzzification Algorithm:

In the fuzzification step, we employ two input variables namely error *E (n)*and rate R *(n)* with input fuzzy sets namely Positive and Negativeas shown in *fig.2a.Each* fuzzy variable error and rate has two membership functions.The mamdani fuzzy PI controller has one output variable which has three fuzzy membership functions which are singleton membership functions.

Using $\tilde{P}$ and $\tilde{N}$ to represent Positive and Negativemembership functions, respectively.

Fuzzymembership functionsfor*E (n)* are defined as

$$\mu_{\tilde{P}}(e) = \begin{cases} 0, & E(n) < -L \\ \dfrac{K_e e(n) + L}{2L}, & -L \leq E(n) \leq L \\ 1 & E(n) > L \end{cases}$$

and

$$\mu_{\tilde{N}}(e) = \begin{cases} 0, & E(n) < -L \\ \dfrac{-K_e e(n) + L}{2L}, & -L \leq E(n) \leq L \\ 1 & E(n) > L \end{cases}$$

AndFuzzy membership functions for R (n) are defined as

$$\mu_{\tilde{P}}(r) = \begin{cases} 0, & R(n) < -L \\ \dfrac{K_r r(n) + L}{2L}, & -L \leq R(n) \leq L \\ 1 & R(n) > L \end{cases}$$

and

$$\mu_{\tilde{N}}(r) = \begin{cases} 0, & R(n) < -L \\ \dfrac{-K_r r(n) + L}{2L}, & -L \leq R(n) \leq L \\ 1 & R(n) > L \end{cases}$$

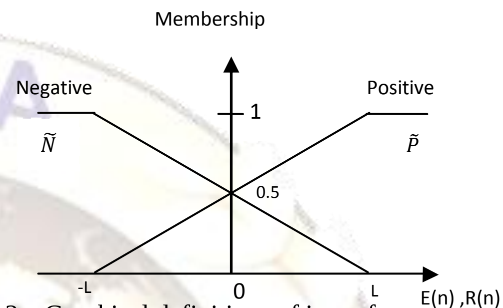

Fig 2a: Graphical definitions of input fuzzy sets used by the linear fuzzy PI controller: (a) two input fuzzy sets Positive and Negative for *E(n)* and *R(n)*

At any point in $(-\infty, \infty)$, it should satisfy

$$\mu_{\tilde{P}}(e) + \mu_{\tilde{N}}(e) = 1, \text{ for } E(n) \in (-\infty, \infty)$$
$$\mu_{\tilde{P}}(r) + \mu_{\tilde{N}}(r) = 1, \text{ for } R(n) \in (-\infty, \infty)$$

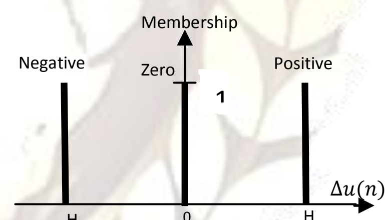

Fig 2b: Graphical definitions of output fuzzy sets used by the linear fuzzy PI controller: three singleton output fuzzy sets, Positive, Zero, and Negative.

3.1Fuzzy ControlRules:

The fuzzy PI controller uses the following four fuzzy rules:
IF *E(n)* is Positive AND *R(n)* is Positive THEN$\Delta u(n)$is Positive …… (r1)
IF *E(n)* is Positive AND *R(n)* is Negative THEN $\Delta u(n)$is Zero ……. (r2)
IF *E(n)* is Negative AND *R(n)* is Positive THEN $\Delta u(n)$is Zero ……. (r3)
IF *E(n)* is Negative AND *R(n)* is Negative THEN $\Delta u(n)$is Negative … (r4)

Here the output fuzzy sets are of the singleton type and their nonzero values are at *H*, 0, and-*H*, respectively for Positive, Zero, and Negativeas shown in *fig.2b*.

4. FUZZY LOGIC FOREVALUATION OF THE FUZZY CONTROL RULES:

Zadeh fuzzy logic AND operator is used to realize the AND operations in antecedent part of the rules.

Due to the use of Zadeh AND operator, the input space must be divided into number of regions in such a way that in each region a unique analytical inequality relationship can be obtained for each fuzzy rule between the two membershipfunctions being ANDed.

Consider the first rule, antecedent parts which contain two membership functions the boundary on which the membership value is same between two MFs is obtained by letting them equal.

Boundary division for rule1 is given by:

$$\mu_{\tilde{P}}(e) = \mu_{\tilde{P}}(r)$$
$$\frac{K_e e(n) + L}{2L} = \frac{K_r r(n) + L}{2L}$$
$$r(n) = e(n) \ldots\ldots\ldots\ldots (1)$$

Boundary division for rule2 is given by:

$$\mu_{\tilde{P}}(e) = \mu_{\tilde{N}}(r)$$
$$\frac{K_e e(n) + L}{2L} = \frac{-K_r r(n) + L}{2L}$$
$$r(n) = -e(n) \ldots\ldots\ldots\ldots (2)$$

Boundary division for rule3 is given by:

$$\mu_{\tilde{N}}(e) = \mu_{\tilde{P}}(r)$$
$$\frac{-K_e e(n) + L}{2L} = \frac{K_r r(n) + L}{2L}$$
$$r(n) = -e(n) \ldots\ldots\ldots\ldots (3)$$

Boundary division for rule4 is given by:

$$\mu_{\tilde{N}}(e) = \mu_{\tilde{N}}(r)$$
$$\frac{-K_e e(n) + L}{2L} = \frac{-K_r r(n) + L}{2L}$$
$$e(n) = r(n) \ldots\ldots\ldots\ldots (4)$$

Equations (1) to (4) are used to generate individually the space division (plane division) between error and rate.

Superimposing the four input space divisions from the expressions (1) to (4) to form a total of 12 input combinations. They are labeled from ICI to ICI2, as shown in *Fig4*

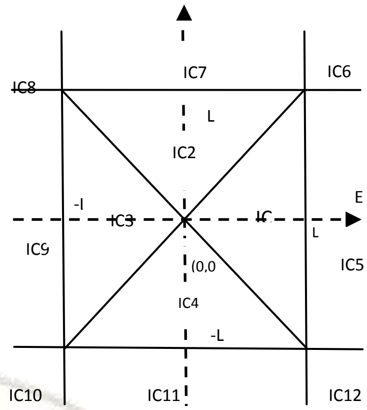

Fig 4 : Division of the *E(n)* - *R(n)* input space into 12 regions for applying the Zadeh fuzzy AND operation in the four fuzzy

In each IC for rule 1 the result of Zadeh AND operation is obtained from figure 4a

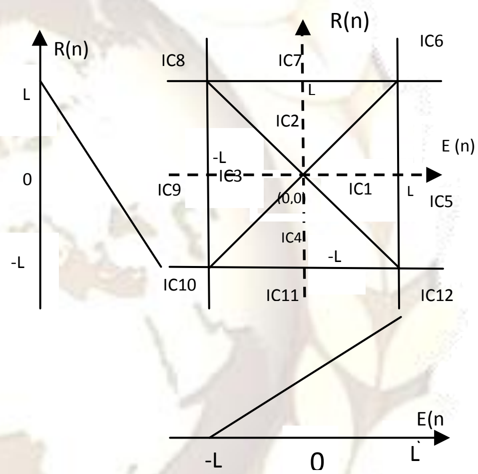

Fig.4a: Division of input space for evaluation of Zadeh AND operation for fuzzy rule r1 .

In each IC for rule 2 the result of Zadeh AND operation is obtained from figure 4b

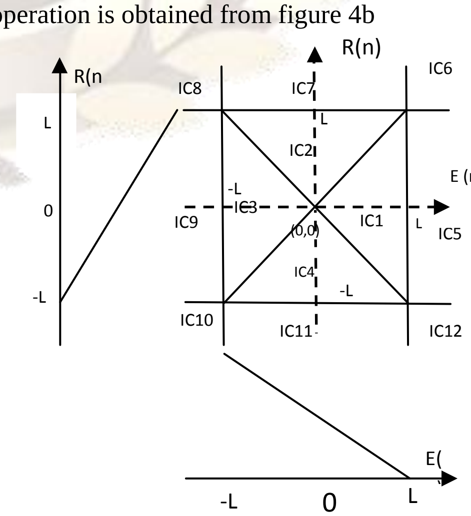

Fig.4b: Division of input space for evaluation of Zadeh AND operation for fuzzy rule r2 .

In each IC for rule 3 the result of Zadeh AND operation is obtained from figure 4c

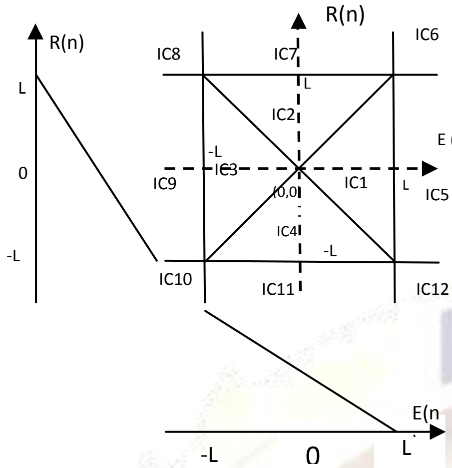

Fig.4c: Division of input space for evaluation of Zadeh AND operation for fuzzy rule r3 .

In each IC for rule 4 the result of Zadeh AND operation is obtained from figure 4d

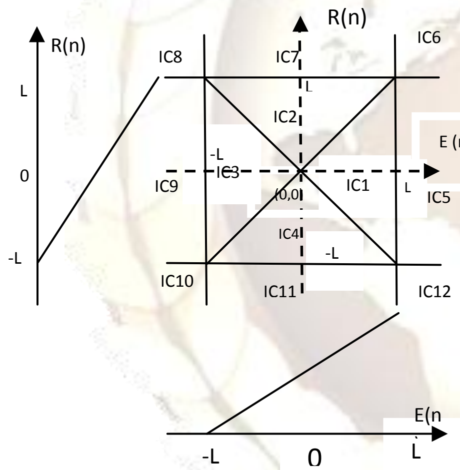

Fig.4d: Division of input space for evaluation of Zadeh AND operation for fuzzy rule r4 .

The results of fuzzy logic AND operation for each rule in each ICs are shown in *Table 1*. They are the membership values, and are used to obtain analytical expression using the output fuzzy sets.

TABLE 1: Application of the Zadeh Fuzzy AND Operator* for evaluating antecedent parts for all the rules in each IC

IC No.	r1	r2	r3	r4
1	$\mu_{\tilde{P}}(r)$	$\mu_{\tilde{N}}(r)$	$\mu_{\tilde{N}}(e)$	$\mu_{\tilde{N}}(e)$
2	$\mu_{\tilde{P}}(e)$	$\mu_{\tilde{N}}(r)$	$\mu_{\tilde{N}}(e)$	$\mu_{\tilde{N}}(r)$
3	$\mu_{\tilde{P}}(e)$	$\mu_{\tilde{P}}(e)$	$\mu_{\tilde{P}}(r)$	$\mu_{\tilde{N}}(r)$
4	0	$\mu_{\tilde{P}}(e)$	$\mu_{\tilde{P}}(r)$	$\mu_{\tilde{N}}(e)$
5	0	$\mu_{\tilde{N}}(r)$	0	0
6	$\mu_{\tilde{P}}(e)$	0	$\mu_{\tilde{N}}(r)$	0
7	0	0	$\mu_{\tilde{P}}(r)$	$\mu_{\tilde{N}}(r)$
8	0	$\mu_{\tilde{P}}(e)$	0	$\mu_{\tilde{N}}(e)$
9	1	0	0	0
10	0	0	1	0
11	0	0	0	1
12	0	1	0	0

*These membership values are for the output fuzzy sets in the four rules r1 to r4.

5.DEFUZZIFICATION ALGORITHMS

Case A: Linear PI Controller

In evaluating the fuzzy logic AND operator in the antecedent part of the fuzzy rules, the product fuzzy logic AND operator isused. The results of product AND operations in the four fuzzy rules are

$$\mu_{\tilde{P}}(e).\mu_{\tilde{P}}(r) \text{ for } \mathbf{H}$$
$$\mu_{\tilde{P}}(e).\mu_{\tilde{N}}(r) \text{ for } 0$$

$$\mu_{\tilde{N}}(e).\mu_{\tilde{P}}(r) \text{ for } \mathbf{0}$$
$$\mu_{\tilde{N}}(e).\mu_{\tilde{N}}(r) \text{ for } \mathbf{-H}$$

The Lukasiewicz fuzzy logic OR operation is applied to combine the membership values fromrules r2 and r3, as there exists an implied OR between the two rules for the same output fuzzyset, Zero. Since

$$\mu_{\tilde{P}}(e).\mu_{\tilde{N}}(r) + \mu_{\tilde{N}}(e).\mu_{\tilde{P}}(r) = 1 - \mu_{\tilde{P}}(e).\mu_{\tilde{P}}(r) - \mu_{\tilde{N}}(e).\mu_{\tilde{N}}(r)$$

Due to the use of the singleton output fuzzy sets, the fuzzy inference result is the sameno matter which one of the four inference methods is employed. The fuzzy controller output is obtained by using centroid defuzzifier

$$\Delta U(n) = K_{\Delta u} \frac{\mu_{\tilde{P}}(e).\mu_{\tilde{P}}(r).H + \mu_{\tilde{P}}(e).\mu_{\tilde{N}}(r).(-H)}{\mu_{\tilde{P}}(e).\mu_{\tilde{N}}(r) + \mu_{\tilde{P}}(e).\mu_{\tilde{N}}(r) + \mu_{\tilde{N}}(e).\mu_{\tilde{P}}(r) + \mu_{\tilde{N}}(e).\mu_{\tilde{N}}(r)}$$

Case B: Piece-wise Linear PI Controller

A fuzzy piece-wise linear controller that differs from the linear fuzzy PI controller in: (1)the Zadeh fuzzy logic AND operator is used, (2) either the Zadeh orthe Lukasiewicz fuzzy logic OR operator is used, and (3) the linear defuzzifier isutilized.

Lukasiewicz or the Zadeh fuzzy OR operator is used to combine the output fuzzy set zero for rules r2 and r3.Using the linear defuzzifier, the analytical structures for the fuzzy controller structure for the 12 ICs are given in *Table2.*

For IC1, the AND results are: $\mu_{\tilde{P}}(r)$for r1, $\mu_{\tilde{N}}(r)$for r2, and $\mu_{\tilde{N}}(e)$for r3 and r4. Using the defuzzifier,

$$\Delta U(n) = K_{\Delta u}\left(\mu_{\tilde{P}}(r).H + \mu_{\tilde{N}}(r) * 0 + \mu_{\tilde{N}}(e) * 0 + \mu_{\tilde{N}}(e).(-H)\right)$$

$$= K_{\Delta u}\left(\frac{K_r r(n) + L}{2L} * H + \frac{-K_r r(n) + L}{2L} * 0 + \frac{-K_e e(n) + L}{2L} * 0 + \frac{-K_e e(n) + L}{2L} * (-H)\right)$$

$$= \frac{K_{\Delta u} K_e H}{2L} e(n) + \frac{K_{\Delta u} K_r H}{2L} r(n)$$

Table 2: Incremental Output of the Linear Fuzzy PI Controller in All 12 Regions after the Linear Defuzzifier is employed to combine the Results in

IC No.	Increment Output of the linear Fuzzy PI Controller, $\Delta U(n) =$
1,2,3 and 4	$\frac{K_{\Delta u} K_e H}{2L} e(n) + \frac{K_{\Delta u} K_r H}{2L} r(n)$
5	$\frac{K_{\Delta u} K_r H}{2L} r(n) + \frac{K_{\Delta u} H}{2}$
6	$\frac{K_{\Delta u} K_e H}{2L} e(n) + \frac{K_{\Delta u} H}{2}$
7	$\frac{K_{\Delta u} K_r H}{2L} r(n) - \frac{K_{\Delta u} H}{2}$
8	$\frac{K_{\Delta u} K_e H}{2L} e(n) - \frac{K_{\Delta u} H}{2}$
9	$K_{\Delta u} H$
10 and 12	0
11	$-K_{\Delta u} H$

Table 1*

6. Signal Flow Graph :

Signal-flow graph is a graphical representation of relationships between variables of a set of linear algebraic equations in a system. It is a directed graph consisting of nodes and branches. Its nodes are the variables of a set of linear algebraic relations. An SFG can only represent multiplications and additions. Multiplications are represented by the weights of the branches; additions are represented by multiple branches going into one node. A signal-flow graph has a one-to-one relationship with a system of linear equations. It can also be used to solve for ratios of these signals.

Key elements of a signal flow graph are:

1.The system must be linear,

2. Nodes represent the system variables,

3. Branches represent paths for signal flow.

4. Signals travel along branches only in the direction of the arrows.

Signal flow graphs (SFGs) can form an intuitive picture of the signal flow in a system. As an application, we will develop SFGs to all ICs from *Table 2*. The SFGs are shown below in *fig 4a,4b,4c,4d,4e* .

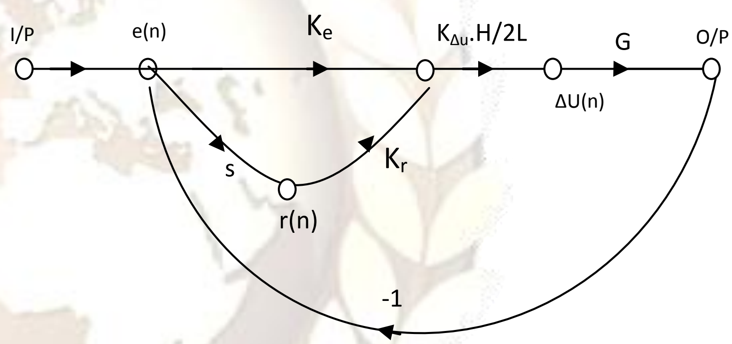

Fig 4a: Signal flow graph for IC1, IC2, IC3, IC4

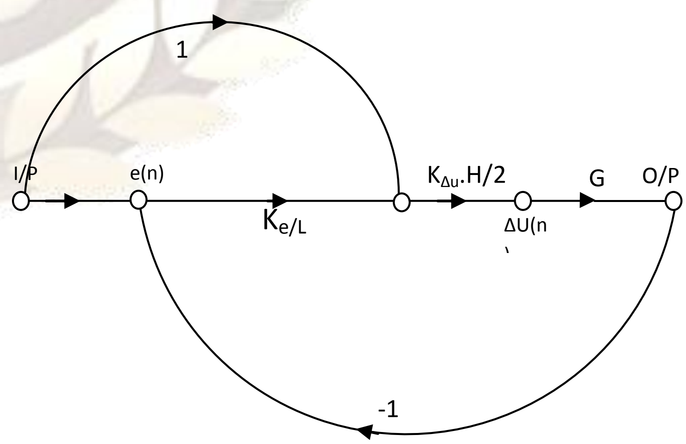

Fig 4b: Signal flow graph for IC5

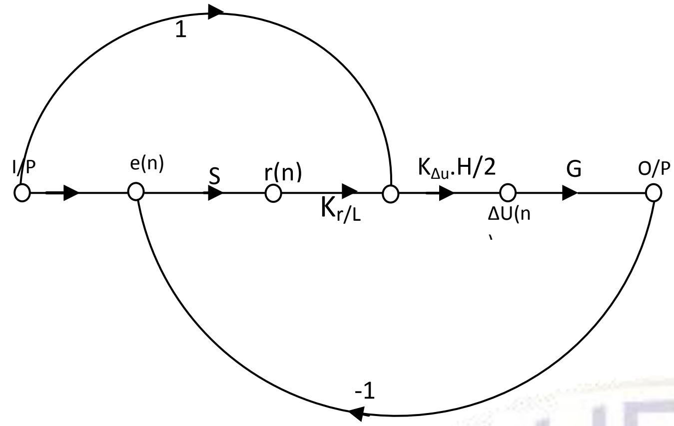

Fig 4c: Signal flow graph for IC6

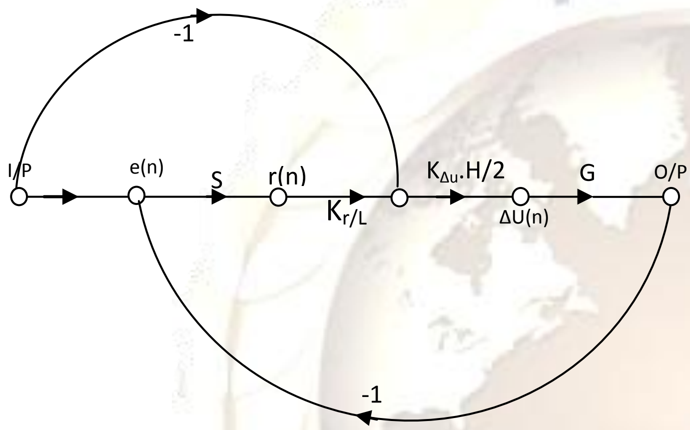

Fig 4d: Signal flow graph for IC7

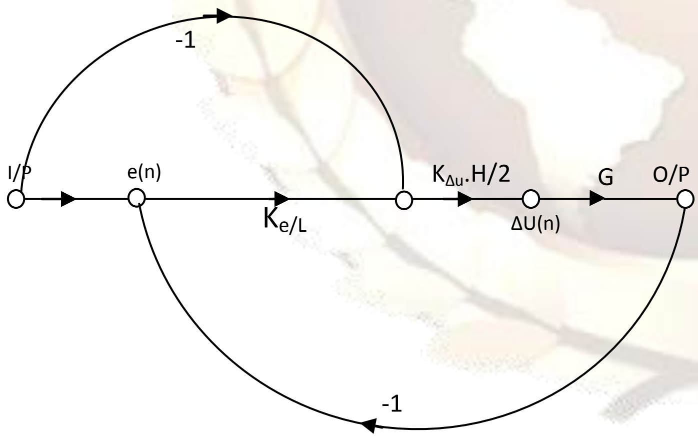

Fig 4e: Signal flow graph for IC8

The Transfer functions of the signal flow graphs are developed from Mason's Gain formula. Table 3 shows the transfer functions in all ICs.

IC No.	Transfer Function from Signal Flow Graph
1,2,3 and 4	$\frac{K_{\Delta u}K_e H.G + K_{\Delta u}H.GsK_r}{2L + K_e K_{\Delta u}.H.G + sK_r K_{\Delta u}.H.G}$
5	$\frac{K_{\Delta u}.H.G.L + K_{\Delta u}H.GsK_r}{2L + sK_r K_{\Delta u}.H.G}$
6	$\frac{K_{\Delta u}K_e H.G + K_{\Delta u}H.G.L}{2L + K_e K_{\Delta u}}$
7	$\frac{-K_{\Delta u}.H.G.L + K_{\Delta u}H.GsK_r}{2L + sK_r.K_{\Delta u}.H.G}$
8	$\frac{K_{\Delta u}K_e H.G - K_{\Delta u}H.G.L}{2L + K_e K_{\Delta u}.H.G}$
9	$K_{\Delta u}H$
10 and 12	0
11	$-K_{\Delta u}H$

7. Conclusion:

In this paper we have derived the analytical input and output relationships for the linear fuzzy PI controller having two input variables and one output variable. There are two triangular/trapezoidal membership functions in each input variable and three singleton output fuzzy sets in output variable. Zadeh AND operator is used to evaluate the antecedent part of the each of the rule. Since Zadeh AND is used, the input space is divided into 12 regions. The fuzzy OR operation is used to combine the output fuzzy set Zero for rules r2 and r3. The fuzzy controller is as linear or piece - wise linear fuzzy controller that depends on the defuzzification method, and evaluation method, Fuzzy Logical or method. For the linear Fuzzy PI Controller and Piece wise Linear Fuzzy PI Controller the Signal Flow Graphs are developed. As each IC having unique mathematical relationship the signal flow graphs are developed for each IC.

8. References :

[1] Zadeh, L. A (1965). Fuzzy sets. Inf. Control, 8 , pp-338-353
[2] E.H Mamdani, S. Assilian. An experiment in linguistic synthesiswith a fuzzy logic controller, internat. J. Man- machine stud. 7(1)(1974). Pp1-13.
[3] Sugeno, M, (1985). Industrial applications of fuzzy control,Amsterdam: North-Holland.
[4] Fuzzy control and modeling, analytical foundations and applications Hao Ying.
[5] Siler, W. and Ying. H "Fuzzy Control Theory: The linear case"Fuzzy Sets and Systems, Vol. 33. pp. 275-290, 1989.

[6] Ying. H "analytical structure of fuzzy controllers with linearcontrol rules" Inf. Sci., vol. 81, pp 213-227, 1994

[7] Sergey Edward Lyshevski, "Control Systems Theory withEngineering Applications", Springer International Edition.

[8] Ying. H "Deriving analytical input-output Relationship forfuzzy Controllers using Arbitrary input fuzzy sets and Zadeh FuzzyAND Operator " IEEE Trans. Fuzzy systems, vol. 14, No. 5pp. 654-662,2006

[9] A Simplest Fuzzy PID Controller: AnalyticalStructure and Stability AnalysisB. M. Mohan and Arpita Sinha IEEE INDU ANNUAL CONPERENCE 2004. INDICON 2oW

[10] Discussion on Mathematical Modeling of FuzzyTwo-Term (PI/PD) Controllers B.M.Mohan INTERNATIONAL JOURNAL OF COMPUTATIONAL COGNITION (HTTP://WWW.IJCC.US), VOL. 8, NO. 3, SEPTEMBER 2010.